System Dynamics (22.554 & 24.509)

Mini-Project #1 -- Spring 2014

Analysis and Linearization of Nonlinear Systems

Consider the mechanical system shown in Fig. 1, where y(t) represents the compression of the shock absorbers in the vertical direction relative to the uncompressed initial position. This system is a simplified representation of the suspension mechanism for a Venetian landing craft.

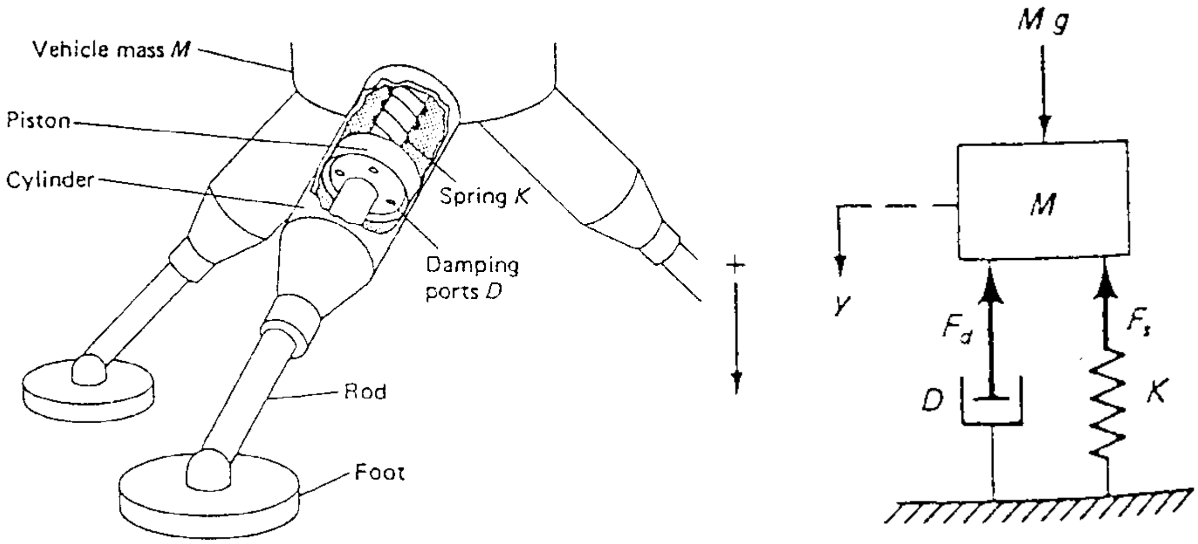

Fig. 1 Suspension System for a Venetian Lander.

A simplified model for the dynamics of the suspension system as the lander just touches the surface of Venus can be written as

$$My''(t) = Mg + F_s + F_d \qquad \text{with} \qquad y(0) = 0 \qquad \text{and} \qquad y'(0) = V_o \tag{1}$$

where M = mass of vehicle = 3000 kg

g = 8.92 m/s^2 (on surface on Venus)

F_s = equivalent restoring force of the springs in the three legs

$$F_s = -k\left(y + 8y^3\right) \qquad \text{with } k = 72000 \text{ N/m} \tag{2}$$

and F_d = equivalent damping of the three legs

$$F_d = -c\left\{y' + 0.05\left(y'\right)^2 \frac{y'}{|y'|}\right\} \qquad \text{with } c = 20000 \text{ N-s/m} \tag{3}$$

where $y'/|y'|$ is used to give the proper sign for the second damping term for both positive and negative directions of motion (note that downwards is positive in this development). Note also that c is used here to denote damping whereas, in the above figure, the symbol D is used.

Now with this mathematical model, perform the following manipulations/simulations:

1. Letting $x_1 = y$ and $x_2 = y'$, put the above system in state space form by identifying the **A**, **B**, **C**, **D**, and **f(x**,t) terms in the standard nonlinear methodology. In this formulation, $x_1(t) = y(t)$ is the desired output response and $u(t) = g$ is the input forcing function (where g is the gravitational acceleration on Venus).

2. Determine the equilibrium point (i.e. the value of the state vector after the lander has come to rest on Venus).

3. Compute and plot the spring force, F_s, versus deflection, y, as given in eqn. (2). Let y vary over the range $-0.6 \leq y \leq 0.6$ m. Also determine and plot the linear approximation to F_s at the equilibrium point determined in Step 2, where the first-order Taylor series gives the linear model as

$$F_s(y) \approx F_s(y_e) + \frac{d}{dy} F_s(y)\Big|_{y=y_e} (y - y_e)$$

From your observations, explain the overall linearization process. In particular, explain what we are really doing when we linearize a nonlinear term.

4. Now compare and contrast the nonlinear and linear damping force in a similar manner as done in Step 3 for the spring force. Again, explain what really happens when you linearize the nonlinear damping force!

5. Formally linearize the nonlinear model for this system about the equilibrium state (i.e. that is, let $\underline{x}_r = \underline{x}_e$ and $u_r = u_e = g$ from Step 2). Define the new **A**, **B**, **C**, and **D** matrices for the linearized problem and be sure to clearly identify the initial conditions for the linearized model. Discuss any assumptions inherent in this development.

6. Now simulate the linearized system using Matlab and determine a reasonable value for the initial velocity such that a near optimum landing is achieved. Note that the maximum compression of the lander legs is 0.5 m and it is desired that bouncing be minimized. Note also that V_o is inversely related to the amount of fuel needed. Thus, the design criteria are:

   a. The maximum compression is 0.5 m (beyond this value the lander legs buckle).

   b. To minimize bouncing, keep the maximum overshoot below 0.1 m relative to the equilibrium value.

   c. Minimize the fuel consumption during landing (i.e. use the largest V_o that is consistent with the above two constraints).

7. Now, using a value of V_o from Step 6, numerically integrate the actual nonlinear equations using Matlab's **ode45** function. Do the two simulations give similar results? Increase the initial velocity to near maximum (make sure the legs don't collapse). How do the linear and nonlinear simulations compare for this case where the velocity is somewhat larger? Explain your observations. How do the observed differences relate to the approximations made for the linearized damping force F_d and the linearized spring force F_s?

8. When analyzing low-order nonlinear systems (primarily 2nd order systems), one often plots the solutions in the so-called **phase plane** defined by the coordinates $x_1(t)$ and $x_2(t)$. In models involving Newtonian mechanics, these coordinates simply represent position and velocity -- and

the phase plane plots usually show v(t) versus y(t). For this project, you should generate the phase plane plots for the linear and nonlinear systems for both nominal V_o and maximum V_o. Comment, in general, on the resultant profiles. How do the linear and nonlinear profiles compare? Is this plot format informative? Can you see the relationship to the traditional time-domain plots? Comment...

Documentation

This problem should be treated as a mini-project that requires a much more formal report than a typical homework assignment. In particular, you should address the linear and nonlinear dynamics of this specific system in some detail and then discuss your findings in a professional summary report on your study. Do the development and answer the questions posed above as part of your report and be sure to thoroughly discuss your results and explain any unique or unusual physical phenomena that were observed. The point here is not to simply go through the motion of computing the results. Instead, you need develop the models, do the Matlab simulations, interpret the results, and then explain your observations in a well-written document. Be sure to properly label all the Matlab-generated plots and to discuss/explain them in as much detail as possible. You should insert the plots directly with your report, but include a listing of the Matlab code used in the simulations in an appendix. This should be a professional well-organized and well-written report!!!