System Dynamics (22.554 & 24.509)

Optional Extra Credit Homework Assignment #8 -- Spring 2014

State Feedback Control

Problem #1: System Analysis via Hand Calculations

A 3rd order SISO plant is described by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{8}{(s+1)^2(s+2)}$$

a. Determine the 3rd order ODE that describes the plant dynamics (assume zero ICs).

b. Determine the state space representation of the plant if y(t) is the desired output (i.e. find the A, B, C, and D matrices for the system). Assume that y(t) is measurable by direct means. Show your work here.

c. Do you think this plant is state controllable? Address this qualitatively first by analyzing the structure of the A and B matrices from Part b. Discuss your observations and make a preliminary decision on controllability based on the observed structure. Now, formally determine the controllability issue by finding the rank of the controllability matrix -- do this via hand calculation.

d. Assuming state controllability (from Part c) and the control law, $u = r_d - Kx$, determine the state gain matrix, K, that gives the following closed loop poles (again, do this via hand calculations): $\mu_{1,2} = -2 \pm 2\sqrt{3}j$ and $\mu_3 = -6$

e. With the desired closed loop poles given in Part d and the Design Aids handout, estimate the rise time, settling time, and maximum overshoot that you expect for the closed loop response (do this by hand and show/explain your work). Now, with the state space closed loop model developed above, simulate and plot the step response of the closed loop system within Matlab and compare the estimated time-domain response characteristics (rise time, settling time, etc.) with the values obtained from the actual simulation of the closed loop system. Did the use of the Design Aids handout give a reasonable approximation of the expected behavior?

Problem #2: System Analysis via Matlab Calculations

Now use Matlab to solve the above problem (i.e. with no hand manipulations needed). In particular,

a. Use the various conversion routines in Matlab to convert the given transfer function for the plant into state space form.

b. Determine the rank of the controllability matrix associated with the state space equations (see Matlab's **crtb** and **rank** functions).

c. Find the state feedback gain matrix given the desired closed loop poles from above (see the **place** command).

d. Simulate the step response of the closed loop system due to a unit step change in the setpoint, r_d.

e. Is the response of the system as expected? How does it compare to the results from Prob. #1. In particular, carefully compare the Matlab-generated system with the one generated by hand. Noting that state space systems are not unique, does your comparison make sense? Explain...

Problem #3: Add a Setpoint Gain to the System

Now, in the above simulations, you probably noticed that the steady state response did not come close to the unity setpoint change that was desired (this is common with a proportional controller). To remedy this situation, calculate an appropriate setpoint gain, N_r, and re-do your simulations and plots. Do this using both the hand-generated system matrices and the Matlab-generated matrices -- but do everything in Matlab. Do your results make sense?

Documentation

Documentation for this assignment should include specific answers to the above questions, the detailed hand calculations from Prob. #1, a listing of all your Matlab script files and the simulation results for Probs. #2 and #3, and a good description of the general results and comparisons of your analyses. As usual, an overall professional job is expected!

Extra Credit

This HW is optional. If you choose to do this assignment it can be worth up to **20 extra points** towards your HW grade for this course.