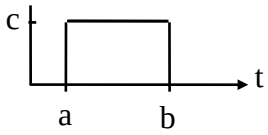
System Dynamics (22.554 & 24.509)

Homework Assignment #6 -- Spring 2014

Time Delays

Problem #1: Laplace Transform of a Delayed Pulse Function

Determine the Laplace transform for the following time-domain function given in graphical form:



Problem #2: Treatment of Delayed Input Functions

A simple SISO system has a transfer function given by

$$G(s) = \frac{3s}{s^2 + 9}$$

a. Analytically determine the system output for all time if the input is given by

$$u(t) = \begin{cases} 0 & 0 \le t < 1 \\ 2 & 1 \le t \le 4 \\ 0 & t > 4 \end{cases}$$

b. Simulate the dynamics of this system with Matlab and compare to the analytical solution in Part a. Note that the discrete nature of u(t) requires care in performing this simulation. If you use multiple calls to **lsim**, remember that only the state space form allows input of the initial conditions. Thus, you may want to use Matlab's **ss** state-space conversion routine to put the system into the standard state space form for use with **lsim**.

c. Set up and solve this problem with Matlab's **ode45** routine. To do this you will need to convert the given system into an appropriate ODE and carefully address how to treat the input and initial conditions for this problem. Compare the **ode45** solution to the analytical and **lsim** results from Parts a and b. Be sure to explain/justify your approach here.

Problem #3: Treatment of Delays within the Transfer Function

Consider the following SISO system with G(s) given by

$$G(s) = \frac{5e^{-2s}}{s + 3}$$

a. If u(t) = sin(t), determine the response of the system using analytical means.

b. Using the same input function, determine the response of the system using Matlab's **lsim** function and several low order Pade′ approximations for the e^{-2s} factor. See the help associated with Matlab's **pade** and **conv** functions. Try approximations of order 1, 2, and 3.

c. Develop the differential equation that describes this system. Using Matlab's **ode45** function, determine the response for u(t) = sin(t). Be careful with the treatment of the time delay here…

d. Plot and compare the solutions obtained in Parts a, b, and c. Does everything here make sense?

Documentation

Documentation for this assignment should include the hand manipulations needed for each problem, a listing of any requested Matlab script file and comparison plots, and a good description of your procedure and results for all the problems. As usual, an overall professional job is expected!