Compute $e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$ where $A = \begin{bmatrix} 0 & 0 & -1 \\ 2 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$

First form $sI-A$

$$sI-A = \begin{bmatrix} s & 0 & 1 \\ -2 & s-1 & -2 \\ 2 & 0 & s-1 \end{bmatrix}$$

Now, $(sI-A)^{-1} = \dfrac{[\text{cofactor of } (sI-A)]^T}{\det(sI-A)}$

$$\det(sI-A) = s\begin{vmatrix} s-1 & -2 \\ 0 & s-1 \end{vmatrix} + 0 + 1\begin{vmatrix} -2 & s-1 \\ 2 & 0 \end{vmatrix}$$

expand along row #1

$$= s(s-1)^2 - 2(s-1)$$

$$= (s-1)[s(s-1)-2] = (s-1)(s^2-s-2)$$

$$\Delta = (s-1)(s+1)(s-2)$$

$$\text{cofactor}(sI-A) = \begin{bmatrix} +(s-1)^2 & -[(-2)(s-1)+4] & +[-2(s-1)] \\ -0 & +[s(s-1)-2] & -0 \\ +-(s-1) & -(-2s+2) & +s(s-1) \end{bmatrix}$$

$2s-2+4 = 2s+6 = 2(s-3)$

$s^2-s-2 = (s+1)(s-2)$

$$\therefore (sI-A)^{-1} = \frac{1}{\Delta}\begin{bmatrix} (s-1)^2 & 0 & -(s-1) \\ 2(s-3) & (s+1)(s-2) & 2(s-1) \\ -2(s-1) & 0 & s(s-1) \end{bmatrix}$$

$$\text{or } (sI-A)^{-1} = \begin{bmatrix} \dfrac{s-1}{(s+1)(s-2)} & 0 & \dfrac{-1}{(s+1)(s-2)} \\ \dfrac{2(s-3)}{(s-1)(s+1)(s-2)} & \dfrac{1}{s-1} & \dfrac{2}{(s+1)(s-2)} \\ \dfrac{-2}{(s+1)(s-2)} & 0 & \dfrac{s}{(s+1)(s-2)} \end{bmatrix}$$

checked with Matlab

Now $e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$ so we need to take the inverse L.T. of each term in the matrix

There are several terms of the form

$$\frac{c}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$$

$$A = \left.\frac{c}{s-2}\right|_{s=-1} = -\frac{1}{3}c \qquad \Rightarrow \boxed{\frac{c}{3}\left(e^{2t} - e^{-t}\right)}$$

$$B = \left.\frac{c}{s+1}\right|_{s=2} = \frac{1}{3}c$$

We have one term of the form

$$\frac{s-1}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$$

$$A = \left.\frac{s-1}{s-2}\right|_{s=-1} = \frac{-2}{-3} = \frac{2}{3} \qquad \Rightarrow \boxed{\frac{1}{3}\left(2e^{-t} + e^{2t}\right)}$$

$$B = \left.\frac{s-1}{s+1}\right|_{s=2} = \frac{1}{3} = \frac{1}{3}$$

And one term of the form

$$\frac{2(s-3)}{(s-1)(s+1)(s-2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$A = \left.\frac{2(s-3)}{(s+1)(s-2)}\right|_{s=1} = \frac{-4}{(2)(-1)} = 2$$

$$B = \left.\frac{2(s-3)}{(s-1)(s-2)}\right|_{s=-1} = \frac{-8}{(-2)(-3)} = -\frac{4}{3} \qquad \Rightarrow \boxed{2e^{t} - \frac{4}{3}e^{-t} - \frac{2}{3}e^{2t}}$$

$$C = \left.\frac{2(s-3)}{(s-1)(s+1)}\right|_{s=2} = \frac{-2}{(1)(3)} = -\frac{2}{3}$$

missed one term see next pg

$$\therefore\ e^{At} = \begin{bmatrix} \frac{1}{3}\left(2e^{-t}+e^{2t}\right) & 0 & -\frac{1}{3}e^{2t} + \frac{1}{3}e^{-t} \\ 2e^{t} - \frac{4}{3}e^{-t} - \frac{2}{3}e^{2t} & e^{t} & \frac{2}{3}e^{2t} - \frac{2}{3}e^{-t} \\ -\frac{2}{3}e^{2t} + \frac{2}{3}e^{-t} & 0 & \frac{1}{3}e^{-t} + \frac{2}{3}e^{2t} \end{bmatrix}$$

Okay – This agrees with previous calculations the 3, 3 term (I messed this one)

$$\frac{s}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2}$$

$$A = \left.\frac{s}{s-2}\right|_{s=-1} = \frac{-1}{-3} = \frac{1}{3} \quad \Rightarrow \boxed{\frac{1}{3}e^{-t} + \frac{2}{3}e^{2t}}$$

$$B = \left.\frac{s}{s+1}\right|_{s=2} = \frac{2}{3}$$

(1) find $f(t)$ given $F(s) = \dfrac{s+1}{(s+2)^2(s+3)}$ — repeated roots

$$F(s) = \frac{s+1}{(s+2)^2(s+3)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+3}$$

$$B = \left.\frac{s+1}{s+3}\right|_{s=-2} = \frac{-1}{1} = \boxed{-1}$$

$$C = \left.\frac{s+1}{(s+2)^2}\right|_{s=-3} = \frac{-2}{(-1)^2} = \boxed{-2}$$

also could simply evaluate at some convenient value of s

now clearing fractions

$$s+1 = A(s+2)(s+3) + B(s+3) + C(s+2)^2$$

$$= As^2 + 5As + 6A - s - 3 - 2s^2 - 8s - 8$$

$$= (A-2)s^2 + (5A-9)s + (6A-11)$$

$\therefore$ $A - 2 = 0$ $\quad$ $5A - 9 = 1$ $\quad$ $6A - 11 = 1$

$\boxed{A = 2}$ $\quad$ $10 - 9 = 1$ ✓ $\quad$ $12 - 11 = 1$ ✓

$$\therefore \quad F(s) = \frac{2}{s+2} - \frac{1}{(s+2)^2} - \frac{2}{s+3}$$

and $\boxed{f(t) = 2e^{-2t} - te^{-2t} - 2e^{-3t}}$ ans

(2) find $f(t)$ given $F(s) = \dfrac{s+2}{s^2+2s+5}$ — quadratic factor

$$F(s) = \frac{s+2}{s^2+2s+5} = \frac{s+2}{(s+1)^2+(2)^2}$$

$$s^2 + 2\alpha s + (\alpha^2 + \beta) = (s+\alpha)^2 + \beta^2$$

$\therefore$ $\alpha = 1$ $\quad$ $\beta = 2$

$$F(s) = \frac{s+1}{(s+1)^2+(2)^2} + \frac{\frac{1}{2}(2)}{(s+1)^2+(2)^2}$$

Then from table look up, we have

$$\boxed{f(t) = e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t}$$ ans

Given the following linear time-invariant system (LTI)

$$\frac{d}{dt}\underline{x} = \underline{A}\,\underline{x} + \underline{b}\,u \qquad \text{and} \qquad \underline{y} = \underline{C}\,\underline{x}$$

where $\underline{A} = \begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad$ and $\underline{C} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

(a) Determine the system transfer function vector

In general $\underline{Y}(s) = \underline{G}(s)\,U(s)$

where $\underline{G}(s) = \underline{C}\,(s\underline{I} - \underline{A})^{-1}\,\underline{b}$

note that this is a vector for this problem because there is only a single input $u(t)$

→ thus we simply need to perform the indicated operations

$$(s\underline{I} - \underline{A}) = \begin{bmatrix} s & -1 \\ -7 & s+4 \end{bmatrix}$$

$$(s\underline{I} - \underline{A})^{-1} = \frac{1}{\Delta}\begin{bmatrix} s+4 & 7 \\ 1 & s \end{bmatrix}^T = \frac{1}{\Delta}\begin{bmatrix} s+4 & 1 \\ 7 & s \end{bmatrix}$$

where $\Delta = s^2 + 4s - 7$

$$\therefore \underline{G}(s) = \frac{1}{\Delta}\,\underline{C}\,(s\underline{I} - \underline{A})^{-1}\,\underline{b}$$

$$= \frac{1}{\Delta}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ 7 & s \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{\Delta}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s+6 \\ 2s+7 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} 7s+27 \\ 2s+7 \end{bmatrix}$$

or $\boxed{G_1(s) = \dfrac{7s+27}{s^2+4s-7}}$ and $\boxed{G_2(s) = \dfrac{2s+7}{s^2+4s-7}}$

(b) If the input $u(t)$ is the unit step, what is $y_2(t)$?

$$y_2(t) = \mathcal{L}^{-1}\left\{ G_2(s)\,\frac{1}{s} \right\}$$

since $U(s) = \frac{1}{s}$ and $G_2(s)$ relates the input $U(s)$ to the second output $Y_2(s)$

$$\therefore \text{let } Y_2(s) = G_2(s)\,\frac{1}{s}$$

$$Y_2(s) = \frac{2s+7}{s(s^2+4s-7)}$$

Note, however, that the quadratic factor can be written as two linear factors

$$s^2+4s-7 \Rightarrow P_{1,2} = \frac{-4 \pm \sqrt{16-4(-7)}}{2} = -2 \pm \tfrac{1}{2}\sqrt{44} = -2 \pm \sqrt{11}$$

P_{poles}

$$P_{1,2} = \begin{cases} a = -2+\sqrt{11} = +1.3166 \leftarrow \text{unstable} \\ b = -2-\sqrt{11} = -5.3166 \end{cases}$$

$$\therefore\ Y_2(s) = \frac{2s+7}{s(s-a)(s-b)} = \frac{A}{s} + \frac{B}{s-a} + \frac{C}{s-b}$$

$$A = \left.\frac{2s+7}{(s-a)(s-b)}\right|_{s=0} = \frac{7}{(a)(b)} = \frac{7}{-7} = -1$$

$$B = \left.\frac{2s+7}{s(s-b)}\right|_{s=a} = \frac{2a+7}{a(a-b)} = \frac{2(1.3166)+7}{1.3166(6.6332)} = 1.1030$$

$$C = \left.\frac{2s+7}{s-a}\right|_{s=b} = \frac{2b+7}{b(b-a)} = \frac{2(-5.3166)+7}{-5.3166(-6.6332)} = -0.1030$$

Then $y_2(t) = A + Be^{at} + Ce^{bt}$

$$\boxed{y_2(t) = -1 + 1.103\,e^{1.3166t} - 0.1030\,e^{-5.3166t}}$$ ans

---

note that we could also write this as

$$y_2(t) = -1 + e^{-2t}\left[1.103\,e^{\sqrt{11}t} - 0.1030\,e^{-\sqrt{11}t}\right]$$

but $\sinh\sqrt{11}t = \frac{e^{\sqrt{11}t} - e^{-\sqrt{11}t}}{2}$ and $\cosh\sqrt{11}t = \frac{e^{\sqrt{11}t} + e^{-\sqrt{11}t}}{2}$

$$C_1\sinh\sqrt{11}t + C_2\cosh\sqrt{11}t = \left(\frac{C_2-C_1}{2}\right)e^{-\sqrt{11}t} + \left(\frac{C_2+C_1}{2}\right)e^{\sqrt{11}t}$$

$$\therefore\ \begin{aligned} C_2 + C_1 &= 2.206 \\ C_2 - C_1 &= -0.2060 \\ \hline 2C_2 &= 2.0 \end{aligned}$$

$\boxed{C_2 = 1}$ $\therefore\ \boxed{C_1 = 1.206}$

or $$\boxed{y_2(t) = -1 + e^{-2t}\cosh\sqrt{11}t + 1.206\,e^{-2t}\sinh\sqrt{11}t}$$ ans as a final note, we could also get the sinh & cosh form directly from the $Y_2(s)$ function ...

to see this, let's leave $Y_2(s)$ with the quadratic factor

$$Y_2(s) = \frac{2s+7}{s(s^2+4s-7)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s-7}$$

$$A = \left.\frac{2s+7}{s^2+4s-7}\right|_{s=0} = -1$$

Now, to find B and C we simply clear the fractions

$$2s+7 = (-1)(s^2+4s-7) + Bs^2 + Cs$$
$$= -s^2 - 4s + 7 + Bs^2 + Cs$$
$$= (B-1)s^2 + (C-4)s + 7$$

$\therefore$ $B - 1 = 0$ $\quad\boxed{B = 1}$ $\qquad$ $C - 4 = 2$ $\quad\boxed{C = 6}$ $\qquad$ $7 = 7$ ok

gives real roots

Also, we can write the denominator as a difference of two squares

$$s^2 + 2\alpha s + (\alpha^2 - \beta^2) = (s+\alpha)^2 - \beta^2$$

for this case $\boxed{\alpha = 2}$ $\qquad \alpha^2 - \beta^2 = -7$ $\qquad \boxed{\beta = \sqrt{11}}$

$$-\beta^2 = -7 - 4 = -11$$

$$\therefore \quad Y_2(s) = -\frac{1}{s} + \frac{s+6}{(s+2)^2 - (\sqrt{11})^2}$$

$$= -\frac{1}{s} + \frac{s+2}{(s+2)^2-(\sqrt{11})^2} + \frac{4}{\sqrt{11}}\frac{\sqrt{11}}{(s+2)^2-(\sqrt{11})^2}$$

or $$\boxed{y_2(t) = -1 + e^{-2t}\cosh\sqrt{11}\,t + \frac{4}{\sqrt{11}}\sinh\sqrt{11}\,t}$$

1.206

ok - same as above

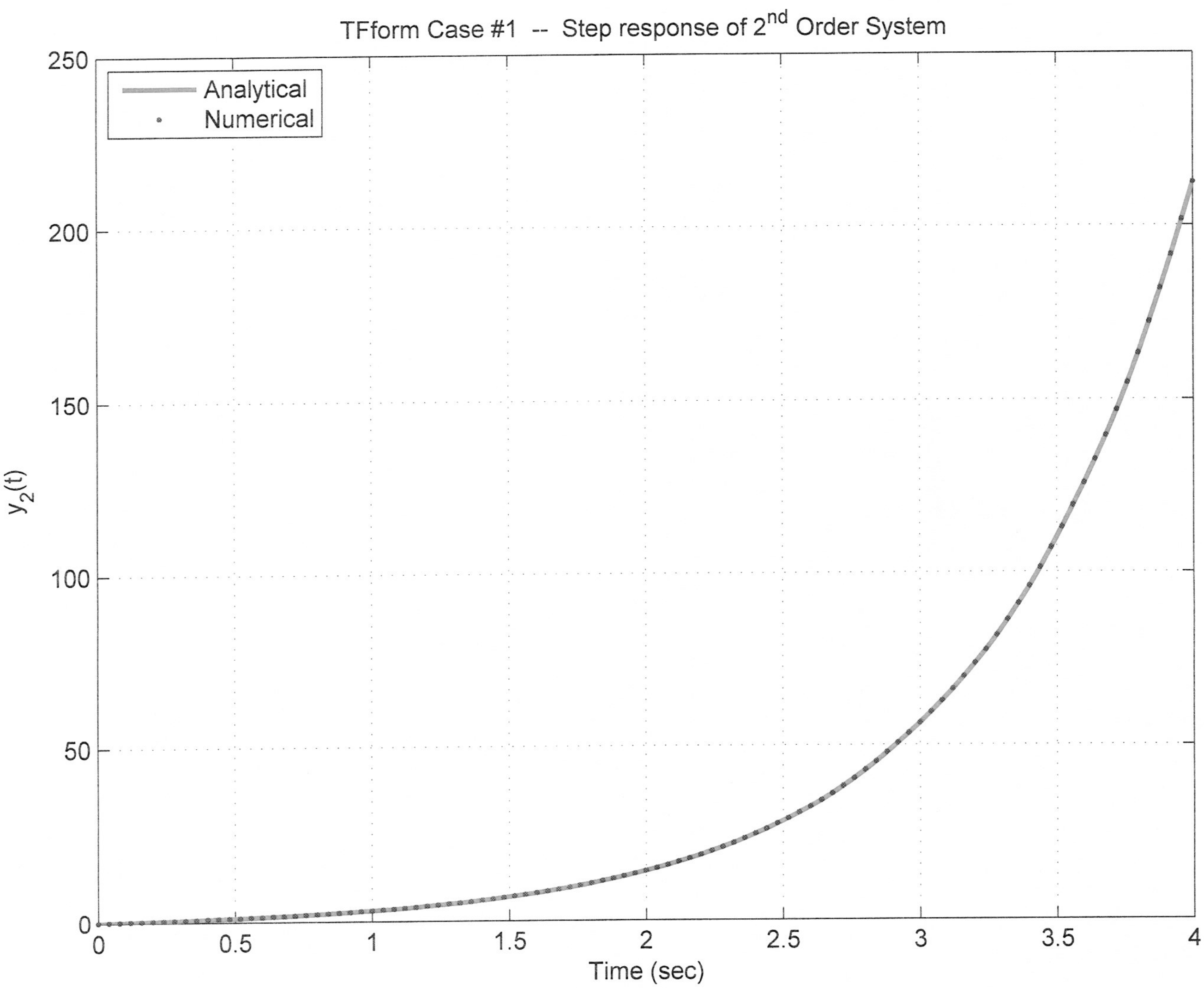
c. No - The system is un-stable - it has a pole in the right half plane of the s-plane

d. Now, model and simulate the system in Matlab $\quad$ see $\boxed{\text{TFform-ss1}}$ case #1

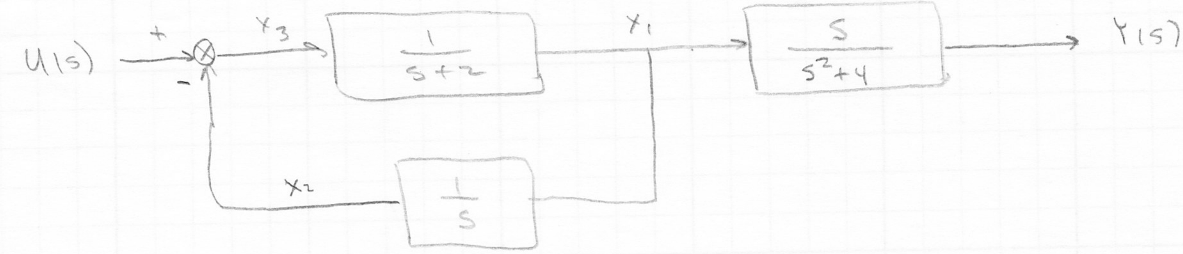
→ everything is self-consistent ...

```
%
%   TFFORM_CS1.M   Conversion and Simulation using Transfer Function View   (Case #1)
%
%   This file takes a simple 2x2 state space system (Case #1) and converts it into
%   transfer function form, simulates the step response, and compares the solution
%   to the analytical solution developed by hand.
%
%   File generated by J. R. White, UMass-Lowell (March 2010)
%

     clear all,    close all,    nfig = 0;
%
%   system definition (state space representation)
     format compact
     disp('State space representation for 2x2 system (Case #1):')
     A = [0 1; 7 -4],   B = [1 2]',   C = [1 3; 0 1],   D = [0 0]',
     disp('Eigenvalues of state matrix:');   e = eig(A)
%
%   convert the state-space representation to transfer function form
     disp('Transfer Functions form:')
     sysa = ss(A,B,C,D);     sysb = tf(sysa)
%
%   now simulate the time domain response for a step input
     t = linspace(0,4,101);    yb = step(sysb,t);
%
%   exact analytical solution from hand development (see notes)
     a = -2;    b = sqrt(11);    c = 4/sqrt(11);
     y2e = -1 + exp(a*t).*cosh(b*t) + c*exp(a*t).*sinh(b*t);
%
%   plot both numerical and exact results for y2(t)-- 2nd column of yb matrix
     nfig = nfig+1;    figure(nfig)
     plot(t,y2e,'r-',t,yb(:,2),'b.','LineWidth',2),grid
     title('TFform Case #1  --  Step response of 2^{nd} Order System')
     ylabel('y_2(t)'),xlabel('Time (sec)')
     legend('Analytical','Numerical','Location','NorthWest')
%
%   end of program
```

```
>> tfform_cs1
State space representation for 2x2 system (Case #1):
A =
     0     1
     7    -4
B =
     1
     2
C =
     1     3
     0     1
D =
     0
     0
Eigenvalues of state matrix:
e =
  1.3166e+000
 -5.3166e+000
Transfer Functions form:

Transfer function from input to output...
        7 s + 27
 #1:  -------------
      s^2 + 4 s - 7

         2 s + 7
 #2:  -------------
      s^2 + 4 s - 7

>>
```


TFform Case #1 -- Step response of 2nd Order System
Analytical
Numerical
250
200
150
100
50
0
$y_2(t)$
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec)

Consider the system shown below



(a) Determine the overall transfer function for this system

$$X_1(s) = \frac{1}{s+2} X_3(s) = \left(\frac{1}{s+2}\right)\left[U(s) - \frac{1}{s}X_1(s)\right]$$

$$\left[1 + \left(\frac{1}{s}\right)\left(\frac{1}{s+2}\right)\right] X_1(s) = \left(\frac{1}{s+2}\right) U(s)$$

$$\therefore \frac{X_1(s)}{U(s)} = \frac{\frac{1}{s+2}}{\frac{s^2+2s+1}{s(s+2)}} = \frac{s}{s^2+2s+1}$$

and

$$Y(s) = \frac{s}{s^2+4} X_1(s) = \left(\frac{s}{s^2+4}\right)\left(\frac{s}{s^2+2s+1}\right) U(s)$$

$$\therefore \boxed{G(s) = \frac{s^2}{(s^2+4)(s^2+2s+1)}} \Rightarrow \boxed{G(s) = \frac{s^2}{(s^2+4)(s+1)^2}}$$

(b) Determine the impulse response

$$y(t) = \mathcal{L}^{-1}\{G(s)\}$$

$$G(s) = \frac{s^2}{(s^2+4)(s+1)^2} = \frac{Bs+C}{(s+1)^2} + \frac{Ds+E}{s^2+4}$$

Let's try simply clearing fractions

$$s^2 = (Bs+C)(s^2+4) + (Ds+E)(s^2+2s+1)$$

$$= Bs^3 + 4Bs + Cs^2 + 4C + Ds^3 + 2Ds^2 + Ds + Es^2 + 2Es + E$$

$$= (B+D)s^3 + (C+2D+E)s^2 + (4B+D+2E)s + (4C+E)$$

(cont.)

Now equating coeffs of like terms

$\boxed{D = -B}$ $\quad C + 2D + E = 1 \quad 4B + D + 2E = 0 \quad \boxed{E = -4C}$

$C - 2B - 4C = 1 \quad 4B - B - 8C = 0$

$3B = 8C$

$-2B - 3C = 1 \quad \boxed{B = \frac{8}{3} C}$

now

$-\frac{16}{3} C - \frac{9}{3} C = 1$

$-\frac{25}{3} C = 1$

$\boxed{D = \frac{8}{25}} \quad \boxed{C = \frac{-3}{25}} \quad \boxed{B = \frac{-8}{25}} \quad \boxed{E = \frac{12}{25}}$

$$\therefore G(s) = -\frac{8}{25}\frac{s}{(s+1)^2} - \frac{3}{25}\frac{1}{(s+1)^2} + \frac{8}{25}\frac{s}{s^2+2^2} + \frac{6}{25}\frac{2}{s^2+2^2}$$

and (from table of L.T.)

$$\boxed{y(t) = -\frac{8}{25}(1-t)e^{-t} - \frac{3}{25}te^{-t} + \frac{8}{25}\cos 2t + \frac{6}{25}\sin 2t}$$

ans

can also be written as

$$y(t) = \frac{-8}{25}e^{-t} + \frac{5}{25}te^{-t} + \frac{8}{25}\cos 2t + \frac{6}{25}\sin 2t$$

(c) Simulate this system in Matlab and compare the numerical and analytical solns

→ use the tf command to generate the LTI object

→ same identical soln. ok

(d) based on above development, this system is clearly a 4th order system

4th order system

(e) Convert the transfer function into an n^{th} order ODE and put this into standard state form.

$$Y(s) = \frac{s^2}{(s^2+4)(s^2+2s+1)} U(s)$$

$$\begin{array}{r} s^2+2s+1 \\ s^2+4 \\ \hline s^4+2s^3+s^2 \\ +4s^2+8s+4 \\ \hline s^4+2s^3+5s^2+8s+4 \end{array}$$

$$(s^2+4)(s^2+2s+1)\,Y(s) = s^2 U(s)$$

$$(s^4+2s^3+5s^2+8s+4)\,Y(s) = s^2 U(s)$$

Now take the inverse L.T. of each term (zero ICs)

$$\boxed{y'''' + 2y''' + 5y'' + 8y' + 4y = u''} \longleftarrow \text{contains derivatives of the input}$$

Using the recipe from the Notes, we have

$$y'''' + a_1 y''' + a_2 y'' + a_3 y' + a_4 y = \underbrace{b_0}_{0} u'''' + \cancel{b_1}^{\,0} u''' + b_2 u'' + \underbrace{b_3}_{0} u' + \underbrace{b_4}_{0}$$

Now we define the state vector as follows:

only $b_2 \neq 0$ $\quad \boxed{b_2 = 1}$

$$x_1 = y - \beta_0 u$$

$$x_2 = \frac{dx_1}{dt} - \beta_1 u$$

$$x_3 = \frac{dx_2}{dt} - \beta_2 u$$

$$x_4 = \frac{dx_3}{dt} - \beta_3 u$$

where

$$\beta_0 = b_0 = 0$$

$$\beta_1 = b_1 - a_1 \cancel{\beta_0}^{\,0} = 0$$

$$\beta_2 = b_2 - a_1 \cancel{\beta_1}^{\,0} - a_2 \cancel{\beta_0}^{\,0}$$

$$= b_2 = 1 \quad \boxed{\beta_2 = 1}$$

$$\beta_3 = \cancel{b_3}^{\,0} - a_1 \beta_2 - a_2 \cancel{\beta_1} - a_3 \underbrace{\beta_0}_{0}$$

$$= -a_1 \beta_2 = -2 \quad \boxed{\beta_3 = -2}$$

$$\beta_4 = \cancel{b_4}^{\,0} - a_1 \beta_3 - a_2 \beta_2 - a_3 \underbrace{\beta_1}_{0} - a_4 \underbrace{\beta_0}_{0}$$

$$= -a_1 \beta_3 - a_2 \beta_2$$

$$= -2(-2) - 5(1)$$

$$= -1 \quad \boxed{\beta_4 = -1}$$

$\therefore$

$$x_1 = y$$

$$x_2 = y' = \frac{dx_1}{dt}$$

$$x_3 = y'' - u = \frac{dx_2}{dt} - u$$

$$x_4 = y''' - \frac{du}{dt} + 2u = \frac{dx_3}{dt} + 2u$$

$\therefore$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -8 & -5 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \\ -1 \end{bmatrix} u$$

ans

(f) Now, in Matlab, convert the transfer function object into state space form
- does this match the soln to Part e
- do the two state matrices have similar eigenvalues
- simulate the two systems - are they identical

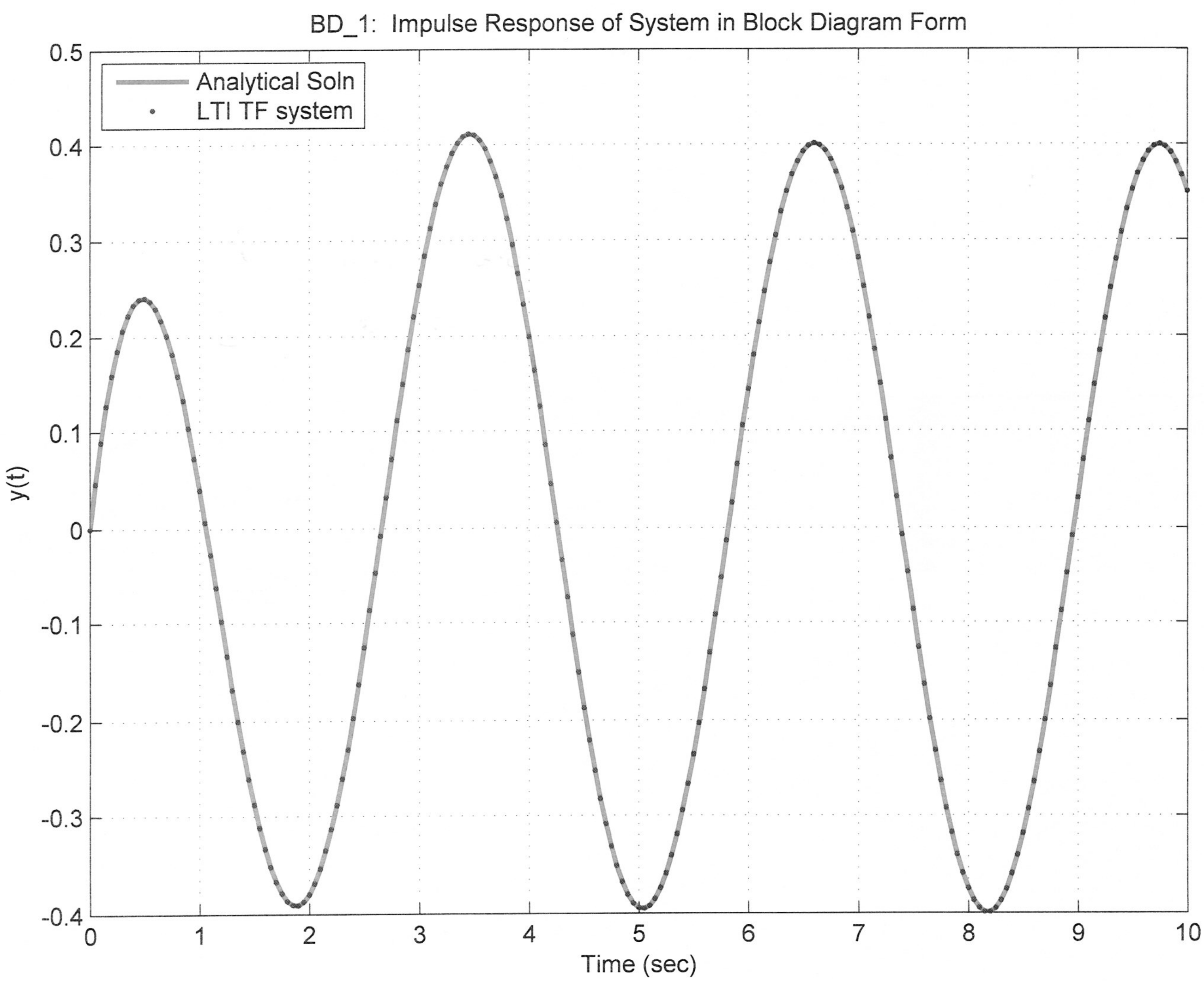
→ These definitely "look" different

A, B, and C matrices are different

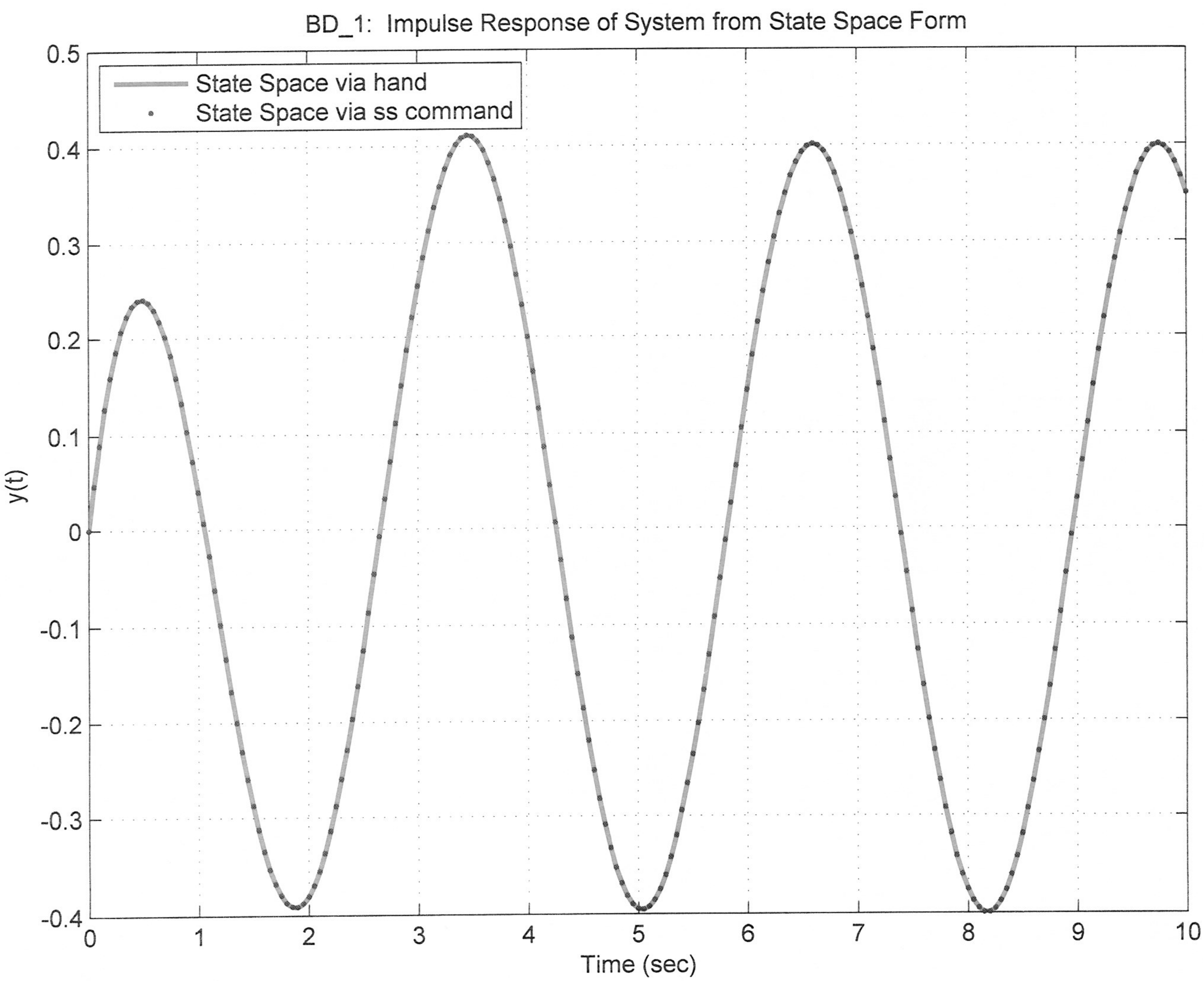
but the two states matrices have

(1) identical eigenvalues } or nearly so within roundoff error

(2) identical impulse responses



BD_1: Impulse Response of System in Block Diagram Form
Analytical Soln
LTI TF system
y(t)
Time (sec)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
1
2
3
4
5
6
7
8
9
10



BD_1: Impulse Response of System from State Space Form
State Space via hand
State Space via ss command
y(t)
Time (sec)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
1
2
3
4
5
6
7
8
9
10

```
>> bd_1
Hand generated state space representation (BD_1):
A =
     0     1     0     0
     0     0     1     0
     0     0     0     1
    -4    -8    -5    -2
B =
     0
     1
    -2
    -1
C =
     1     0     0     0
D =
     0
Eigenvalues of hand developed state matrix:
eA =
  5.5511e-017 +2.0000e+000i
  5.5511e-017 -2.0000e+000i
 -1.0000e+000
 -1.0000e+000
State space representation from the ss command(BD_1):
AA =
 -2.0000e+000 -1.2500e+000 -1.0000e+000 -1.0000e+000
  4.0000e+000            0            0            0
            0  2.0000e+000            0            0
            0            0  5.0000e-001            0
BB =
  5.0000e-001
            0
            0
            0
CC =
            0  5.0000e-001            0            0
DD =
     0
Eigenvalues of Matlab generated state matrix:
eAA =
  1.1102e-016 +2.0000e+000i
  1.1102e-016 -2.0000e+000i
 -1.0000e+000 +1.6746e-008i
 -1.0000e+000 -1.6746e-008i
>>
```

```
%
%   BD_1.M  Analysis of a System written in Block Diagram Form  (Case #1)
%
%   This file does some analysis of a system originally written in block diagram
%   form, as follows:
%   1. It is converted to transfer function form and we find the impluse response
%      of the system (all via hand manipulation -- see notes)
%   2. Here we create an LTI object from the base trasfer function using Matlab's
%      tf command and use the impulse function to compare to the analytical soln.
%   3. Generate the defining ODE from the transfer function and the state space
%      representation from the 4th order ODE (all via hand manipulation -- see notes)
%   4. Here we create the state space form with Matlab's ss command and compare it to
%      the hand development (base form, eigenvalues of the state matrix, and impulse
%      response of both systems).
%
%   File written by J. R. White, UMass-Lowell (March 2010)
%

     clear all,   close all,   nfig = 0;
%
%   show the block diagram created in Simulink (just used for presentation here)
     bd_1sl
%
%   create desired system
     num = [1 0 0];   d1 = [1 0 4];  d2 = [1 2 1];   denom = conv(d1,d2);
     systf = tf(num,denom);
%
%   now simulate the time domain response for an impulse input
     t = linspace(0,10,201);   ytf = impulse(systf,t);
%
%   analytical solution from hand development (see notes)
     ye = (1/25)*(8*cos(2*t) + 6*sin(2*t) - 8*(1-t).*exp(-t) - 3*t.*exp(-t));
%
%   plot both numerical and exact analytical results for y(t)
     nfig = nfig+1;   figure(nfig)
     plot(t,ye,'r-',t,ytf,'b.','LineWidth',2),grid
     title('BD\_1:  Impulse Response of System in Block Diagram Form')
     ylabel('y(t)'),xlabel('Time (sec)')
     legend('Analytical Soln','LTI TF system','Location','NorthWest')
%
%   now let;'s compare the state space developments (hand calc vs use of ss command)
     format compact
     disp('Hand generated state space representation (BD_1):')
     A = [0 1 0 0; 0 0 1 0; 0 0 0 1; -4 -8 -5 -2],
     B = [0 1 -2 -1]',  C = [1 0 0 0],  D = [0 ]
     disp('Eigenvalues of hand developed state matrix:');  eA = eig(A)
     sysA = ss(A,B,C,D);   yA = impulse(sysA,t);
%
     disp('State space representation from the ss command(BD_1):')
     sysAA = ss(systf);   [AA,BB,CC,DD] = ssdata(sysAA)
     disp('Eigenvalues of Matlab generated state matrix:');  eAA = eig(AA)
     yAA = impulse(sysAA,t);
```

```
%
%   plot both state space solutions  --  impulse responses
      nfig = nfig+1;    figure(nfig)
      plot(t,yA,'r-',t,yAA,'b.','LineWidth',2),grid
      title('BD\_1:  Impulse Response of System from State Space Form')
      ylabel('y(t)'),xlabel('Time (sec)')
      legend('State Space via hand','State Space via ss command','Location','NorthWest')
%
%   end of program
```