System Dynamics (22.554 & 24.509)

Homework Assignment #5 -- Spring 2014

Transfer Function View of LTI Systems

Problem #1: Matrix Exponential via Laplace Transforms

Compute the matrix exponential for the following system matrix using the Laplace transform approach, where

$$e^{\underline{\underline{A}}t} = L^{-1}\left\{\left(s\underline{\underline{I}} - \underline{\underline{A}}\right)^{-1}\right\} \qquad \text{and} \qquad \underline{\underline{A}} = \begin{bmatrix} 0 & 0 & -1 \\ 2 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

Be sure to show your work! Does your final result agree with the result from HW #2 where we used Sylvester's Theorem? Which approach is easier to apply?

Problem #2: Inverse Laplace Transforms

Take the inverse Laplace transform of the following functions:

a. $\dfrac{s+1}{(s+2)^2(s+3)}$ b. $\dfrac{s+2}{s^2+2s+5}$

Problem #3: System Transfer Function Matrix and System Response

Given the following linear time-invariant system

$$\frac{d}{dt}\underline{x}(t) = \underline{\underline{A}}\,\underline{x} + \underline{b}u \qquad \text{with} \qquad \underline{y}(t) = \underline{\underline{C}}\,\underline{x}(t)$$

where $\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad$ and $\quad \underline{\underline{C}} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

a. Analytically determine the system transfer function matrix.

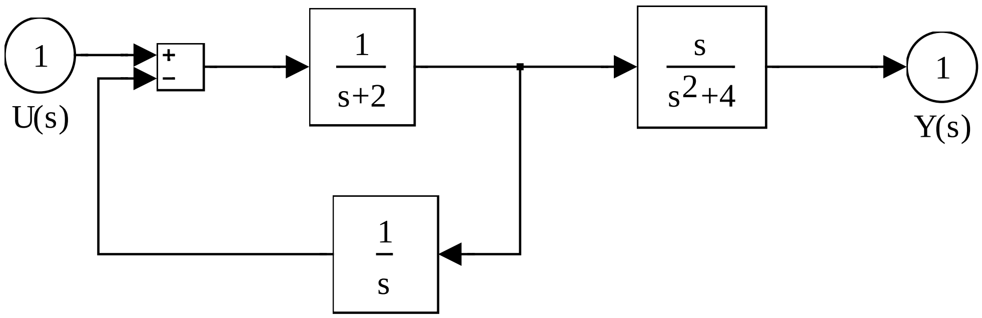
b. If u(t) is the unit step, what is $y_2(t)$? Answer this question using analytical means.

c. Is this a stable system? Explain.

d. Convert the state-space system to transfer function form using Matlab's **ss** and **tf** commands. Do you get the same transfer function matrix as in Part a? Now, simulate the system in Matlab and compare the analytical solution to $y_2(t)$ from Part b to the Matlab solution. Does everything make sense here?

Problem #4: Systems in Block Diagram Form

Consider the system shown below:



a. Using block diagram arithmetic, determine the overall transfer function for this SISO system.

b. Analytically determine the impulse response for this system using the transfer function, G(s), developed in Part a.

c. Now create a transfer function object using Matlab's **tf** command starting with the G(s) result from Part a, and then determine the impulse response of this LTI transfer function object using the **impulse** command. Is your result the same as from Part b.

d. Based on the results of Parts a and b, what is the order, n, of this SISO system?

e. Convert the transfer function in Part a into an n^{th} order ODE and put this into standard state form. This should be done by hand…

f. Finally, within Matlab, convert the transfer function object from Part c into a state space object with the **ss** command. Does this system resemble your result from Part e? Do they have similar eigenvalues? Simulate both these state space systems to compare their impulse responses -- are they indeed identical? Explain your observations…

Documentation

Documentation for this assignment should include the hand manipulations needed for each problem, a listing of any requested Matlab script file and comparison plots, and a good description of your procedure and results for all the problems. As usual, an overall professional job is expected!