System Dynamics (22.554 & 24.509)

Homework Assignment #4 -- Spring 2014

Solution Schemes for LTI Systems

Consider the following LTI system:

$$\frac{d}{dt}\underline{x} = \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u} \qquad \text{and} \qquad \underline{y} = \underline{\underline{C}}\underline{x} + \underline{\underline{D}}\underline{u}$$

where

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & -1 \\ 2 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \underline{\underline{C}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad \underline{\underline{D}} = 0 \qquad \underline{x}_o = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Note: The state matrix given here is the same one used as part of HW #2. Thus, you can simply use the results of HW #2 as needed -- no reason to repeat those computations here…

Problem #1: Impulse Response using Various Methods

Find the impulse response of the above SISO system using the following three methods. Show that the solutions are identical by comparing all three cases on a single plot and via a short summary table of numerical values for $0 \le t \le 2.5$ seconds. Explain any differences observed.

Case 1: Analytical solution using the matrix exponential approach. Obtain an explicit expression for y(t). This problem should be solved by hand manipulations and then the explicit y(t) result should be plotted in Matlab.

Case 2: Solution using Matlab's built-in LTI solvers (***impulse***, ***step***, and ***lsim***, as needed).

Case 3: Solution using numerical integration via Matlab's ***ode45*** routine.

Problem #2: Pulse Response using Various Methods

Find the response of the above SISO system using the three methods listed in Prob. #1 for the following pulse input:

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$

Show that the solutions are identical by comparing all three cases on a single plot and via a short summary table of numerical values for $0 \le t \le 2.5$ seconds. Pay particular attention at t = 1 sec.

Documentation

Documentation for this assignment should include the detailed hand manipulations needed for Case 1 of both Probs. #1 and #2, and a listing of the Matlab script and function files, output print, and resultant plots from both problems. Also, be sure to include a good description of your procedure and results -- since this will let me know if you really understand some of the subtle points that are embedded within these problems. As usual, an overall professional job is expected!