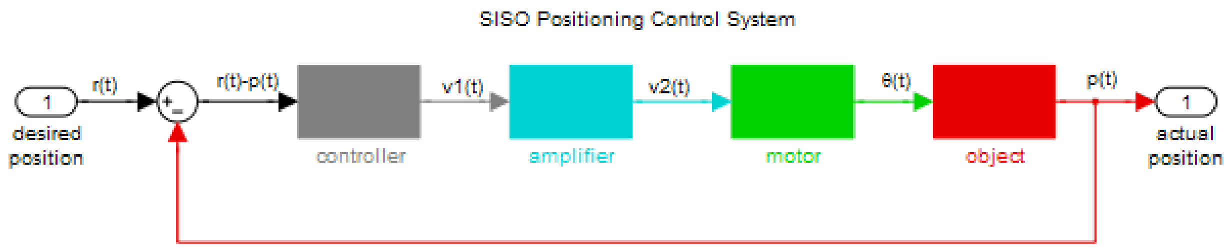
System Dynamics (22.554 & 24.509)

Homework Assignment #3 -- Spring 2014

Conversion to Standard State Form

Problem #1: A SISO Positioning Control System

Consider the SISO positioning control system shown in the sketch.



The system is defined explicitly by the following equations:

$$v_1 = k_1(r - p) \qquad v_2 = k_2 v_1$$

$$\frac{d\theta}{dt} = 0.4 v_2 \qquad \frac{d^2p}{dt^2} + \frac{dp}{dt} + 4p = \theta$$

where $v_1(t)$ = amplifier voltage input $\qquad v_2(t)$ = amplifier voltage output

k_1 = controller gain $\qquad k_2$ = amplifier gain

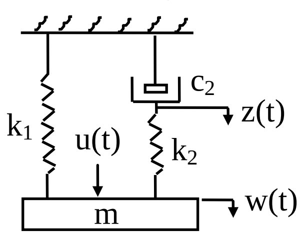
$\theta(t)$ = motor shaft position $\qquad p(t)$ = actual position of the object (output)

$r(t)$ = desired position of the object (input)

Your job here is to put this LTI system into the standard state form (i.e., find the A, B, C, and D matrices for this system).

Problem #2: Various Representations of a 3rd Order Mechanical System

Consider the mechanical system shown below:



m = 1000 kg

k_1 = 60 kN/m $\qquad k_2$ = 40 kN/m

c_2 = 4 kN-s/m $\qquad u(t)$ = ??? kN

The balance equation(s) for this system can be written in a variety of forms:

a. $$\frac{d^2}{dt^2}w(t) + 60w(t) + 40\left[w(t) - z(t)\right] = u(t)$$

$$40\left[w(t)-z(t)\right]=4\frac{d}{dt}z(t)$$

with the deviation from equilibrium, y(t) = w(t), as the desired response.

b. $$\frac{d^3}{dt^3}z(t)+10\frac{d^2}{dt^2}z(t)+100\frac{d}{dt}z(t)+600z(t)=10u(t)$$

with $y(t)=\frac{1}{10}\frac{d}{dt}z(t)+z(t)$ as the desired response.

c. $$\frac{d^3}{dt^3}w(t)+10\frac{d^2}{dt^2}w(t)+100\frac{d}{dt}w(t)+600w(t)=\frac{d}{dt}u(t)+10u(t)$$

with y(t) = w(t) as the desired response.

Assuming zero initial conditions, put each of these systems into state form and show that they are indeed equivalent representations of the same mechanical system. In showing this equivalency, verify that the eigenvalues of the resultant state matrices are identical and that each system has the same impulse and step response (do this verification in Matlab). Also use Matlab to plot the impulse and step responses -- can you explain the observed behavior?

Note: Matlab's ***Control System Toolbox*** has several functions for working with LTI systems written in standard state space form. In particular, the ***ss*** command creates a state-space object from the A, B, C, and D matrices that define an LTI system. Once an LTI object is available, the ***impulse***, ***step***, and ***lsim*** commands can be used to simulate the system for an impulse, step, and arbitrary input, respectively. Note that the ***impulse*** and ***step*** commands require zero initial conditions, whereas the ***lsim*** function allows user-specified initial conditions for the state vector.

Documentation

Documentation for this assignment should include the detailed hand manipulations needed for both Probs. #1 and #2, a listing of any Matlab script and function files, any pertinent output print, and the resultant Matlab plots as requested in the individual problem statements. Also, be sure to include a good description of your results and of any explicit comparisons that were requested -- since this will let me know if you really understand the key points associated with these problems.

As usual, an overall professional job is expected for both problems!