

System Dynamics (22.554 & 24.509)

Homework Assignment #2 -- Spring 2014

Matrix Methods and Functions of a Square Matrix

Problem #1: Matrix Methods via Hand Calculations

Given the 3x3 matrix, $\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & -1 \\ 2 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$, perform the following operations by hand:

- Calculate $\det \underline{\underline{A}}$.
- Calculate $\underline{\underline{A}}^{-1}$.
- Calculate the eigenvalues and eigenvectors of $\underline{\underline{A}}$.
- Calculate $e^{\underline{\underline{A}}t}$, $\frac{d}{dt}[e^{\underline{\underline{A}}t}]$, and $\int e^{\underline{\underline{A}}t} dt$.
- Calculate $\underline{\underline{A}}^2$ and $\sqrt{\underline{\underline{A}}}$.
- Compute the expression $\underline{\underline{D}} = \underline{\underline{M}}^{-1} \underline{\underline{A}} \underline{\underline{M}}$ for the specific case given here, where $\underline{\underline{M}}$ is the matrix whose columns are the eigenvectors of $\underline{\underline{A}}$. The resultant matrix, $\underline{\underline{D}}$, should be a diagonal matrix with the corresponding eigenvalues of $\underline{\underline{A}}$. Is this what you get?

Problem #2: Matrix Methods within Matlab

Perform the same operations as requested in Prob. 1 within Matlab. Type *help matfun* to see some of the available Matlab commands for working with matrix functions. For part d, let $t = 1$ for the numerical calculations. Put the sequence of Matlab commands into a single m-file with plenty of comments to describe what you are doing. Echo the key results back to the screen and print these as part of the documentation for this problem. Do your hand calculations in Prob. 1 agree exactly with the Matlab solutions? Explain any observed differences.

Note: For Part d, you can also use the symbolic capability in Matlab to get continuous analytical relationships for the matrix exponential. Use the *syms t* statement to make t a symbolic variable.

Documentation

Documentation for this assignment should include the detailed hand computations for Prob. #1 and a listing of the Matlab script file and output from Prob. #2. Also, include a brief description of the results of your comparison of Probs. 1 and 2. An overall professional job is expected!