System Dynamics (22.554 & 24.509)

Homework Assignment #1 -- Spring 2014

Solution of Linear Constant-Coefficient Differential and Difference Equations

Problem #1: Continuous LTI 2nd Order System

To illustrate the behavior associated with typical 2nd order systems, let's study a simple mechanical device containing a spring, mass, and dashpot (for viscous damping). The defining differential balance equation is given as follows (assuming no applied force):

$$m\frac{d^2}{dt^2}x(t)+c\frac{d}{dt}x(t)+kx(t)=0$$

We want to analyze this system for specific values of c, m, and k, as follows:

Case 1.	m = 2.5 kg	c = 10.0 kg/s	k = 10 kg/s^2
Case 2.	m = 2.5 kg	c = 12.5 kg/s	k = 10 kg/s^2
Case 3.	m = 2.5 kg	c = 6.0 kg/s	k = 10 kg/s^2
Case 4.	m = 2.5 kg	c = 0 kg/s	k = 10 kg/s^2

a. Solve the above IVP analytically for all four cases with the same initial displacement and velocity, x(0) = 1 m and v(0) = 0 m/s, in each case.

b. Plot the time response, x(t), for the four cases using Matlab. Use one curve per plot and put all four plots on one page -- see **help plot** and **subplot** in Matlab. Also properly label your plots with Matlab's **title**, **xlabel**, and **ylabel** commands. In all cases, let the time vary from 0 to 10 s.

c. Briefly discuss the resultant plots. In particular, discuss the terms overdamping, critical damping, underdamping, and no damping and explain how these terms relate to the physical system and to the type of mathematical solution obtained from the above four specific cases. Also identify how the time domain solutions relate to the type of roots obtained from the characteristic polynomial. In this context, create a simple hand plot showing the roots plotted in the complex plane (i.e. imaginary vs. real), and explain how the root location affects the observed time domain behavior. In general, you should show as much insight as possible into this important class of problems…

Problem #2: 2nd Order Difference Equation

a. Analytically solve the following 2nd order difference equation:

$$y(k+2)-3y(k+1)+2y(k)=3 \qquad \text{for} \quad k=1,2,\cdots$$

with $y(1) = 0$ and $y(2) = 1$

b. Using Matlab, plot the analytical solution to Part a and compare this solution to the one obtained by simply evaluating the recursive form of the original equation. Put both curves (using discrete points) on the same plot. Do your solutions agree?

Documentation

Documentation for this assignment should include detailed solutions to the problems given (be sure to show all the steps), a listing of any Matlab script and function files generated, the resultant Matlab plots, and a brief description of the data and results of your analyses. An overall professional job is expected!