System Dynamics (22.554 and 24.509)
Exam #1 Spring 2014

Problem #1a. (10 points)

Find the general solution to the following LTI system:

$$\frac{d^3}{dt^3}y + 2\frac{d^2}{dt^2}y + 5\frac{d}{dt}y = 15$$

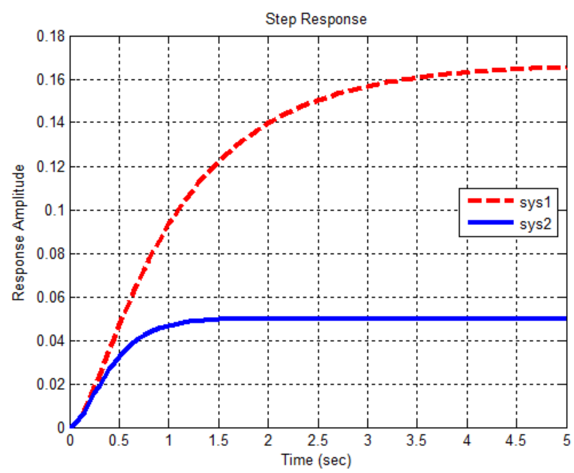
Use the classical approach for solving linear constant coefficient ODEs by finding separate homogeneous and part solutions (i.e. do NOT use the space-space approach here since this approach is much more tedious for this case).

Problem #1b. (10 points)

The plot here shows the step response of two different 2nd order systems. Identify which system has the following eigenvalues:

$$\lambda_1 = -1 \qquad \lambda_2 = -6$$

Clearly justify your selection between sys1 and sys2. Note that a guess without a valid argument that supports your choice will not be worth much!

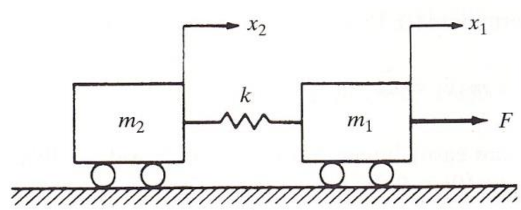


Problem #2. (20 points)

a. Two masses m_1 and m_2 are connected by a linear spring with spring constant k as shown in the sketch. Assuming negligible friction in the system, the equations of motion are

$$m_1\ddot{x}_1 = -k(x_1 - x_2) + F$$

$$m_2\ddot{x}_2 = -k(x_2 - x_1)$$



where x_1 and x_2 are the positions of the masses, the double dot notation represents the 2nd derivative with respect to time, and F(t) is a independent applied force on the system.

If the desired output is the distance between the masses, $y = x_1 - x_2$, put this LTI system into standard state space form

$$\frac{d}{dt}z = Az + Bu \qquad \text{and} \qquad y = Cz + Du$$

by explicitly defining the state vector z, input u, and the appropriate A, B, C, and D matrices.

Note: z is used here (instead of x) as the state vector so that it does not get confused with the positions of the masses, x_1 and x_2, as denoted in the sketch.

b. Now assume that the applied force F is proportional to the difference in the actual output y(t) and some desired output $y_d(t)$, where $y_d(t)$ is now the independent input to the system. In particular, if $F = \alpha(y - y_d)$, where α is a known constant, determine the new state-space matrices, A and B, for this modified system.

Also, for the new system, we wish to have two outputs,

$$y_1 = x_1 - x_2 \qquad \text{and} \qquad y_1 = \alpha(x_1 - x_2 - y_d)$$

Determine the new output matrices, C and D, for this situation.

Problem #3. (20 points)

Given the nonlinear system described by

$$\frac{d}{dt}x_1 = 2x_1x_2 + 4x_1u_1$$

$$\frac{d}{dt}x_2 = x_1x_2 + 4x_2 + 2u_2$$

a. Linearize the system about the reference state defined by $\underline{x}_r = \begin{bmatrix}-1 & 2\end{bmatrix}^T$ and $\underline{u}_r = \begin{bmatrix}-1 & -3\end{bmatrix}^T$. In particular, find the $\underline{\underline{A}}$ and $\underline{\underline{B}}$ matrices for the linearized system.

b. Is the linearized system stable? Justify your answer.

Problem #4. (40 points)

Linear stationary systems written in state form can be expressed as

$$\frac{d}{dt}\underline{x} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \qquad \text{where} \qquad \underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

a. Using the integrating factor technique for solving linear first-order equations, **formally derive** an analytic solution for this system for an arbitrary input with zero initial conditions.

b. If $\underline{u}(t)$ is a unit step function, **formally evaluate** the expressions developed in Part a to obtain a formal solution for $\underline{y}(t)$.

c. In a particular SISO system with two states, we have

$$\underline{\underline{A}} = \begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix}4\\ -5\end{bmatrix} \qquad \underline{\underline{C}} = \begin{bmatrix}1 & -1\end{bmatrix} \qquad \underline{\underline{D}} = 0$$

Using the results of Parts a and b, determine an explicit expression for the step response of this system (with zero initial conditions). Show your work!!!

Note: If you were not successful with Part b, you should a least compute $\underline{\underline{A}}^{-1}$ and $e^{\underline{\underline{A}}t}$ to get partial credit for Part c.