ENGY.4340 Nuclear Reactor Theory

Fall 2016

HW #5: 1-Group 1-D Critical Reactors

Problem 1 Critical Size Calculations for 1-Group 1-D Critical Systems (10 points)

A particular oxide fueled fast reactor core consisting of 50% sodium, 30% fuel, and 20% stainless steel by volume was described in detail in HW #4. In that HW, you were asked to compute the material atom densities and several other quantities, including the k_∞ of the core material.

a. Using the results from HW #4 (or see the tabulated cross sections for Fast Reactor #1 in the **core_refl1g_gui** code), compute the critical size of a bare 1-D reactor using Cartesian, spherical, and cylindrical geometry. This should be done by hand (i.e. show your work).

b. Now run the **core_refl1g_gui** code, select the Reactor Type: Fast Reactor #1 and validate your hand calculations from Part a. Are your results as expected – that is, do things agree with your hand calculations and with your overall understanding of bare critical systems? In reviewing these results, tabulate the non-leakage probability (NLP) for the three geometries for the just critical case (i.e. when $k_{eff} = 1$). Explain your results…

Problem 2 Critical Size Calculations for a 1-Group 3-D Cubical Reactor (10 points)

Let's again consider the fast reactor core described in Prob. #1 of HW #4.

a. If the sodium-fuel combination described in that problem was put into a cubical bare reactor configuration, estimate the length of one side of the cube that would be needed to give a beginning of life k_{eff} of about 1.10. Describe any assumptions (see Note below).

b. Estimate the mass of uranium fuel needed for this system. Again, describe the logic used to obtain your result.

Note: This problem deals with a 3-D system -- which is not quite consistent with the above HW title. However, the same methods can be used to solve multi-dimensional bare 1-region homogeneous reactor problems as long as the correct B^2 is used. In particular, the appropriate buckling for a bare parallelepiped "critical" reactor is given as

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$$

where a, b, and c and the extrapolated lengths of the three sides (i.e. $a = a_o + 2d$, etc.) and a_o, b_o, and c_o and the physical dimensions.

Problem 3 Mode Shapes in 1-Group 1-D Spherical Reactor Geometries (10 points)

In developing the flux profiles for the 1-group bare reactor and the core-reflector system in **1-D spherical geometry**, the following results were obtained (see the Lecture Notes for the details):

Bare Core:

$$\phi_n(r) = \frac{1}{B_n}\frac{\sin B_n r}{r} \qquad \text{where} \quad B_n = \frac{n\pi}{R} \qquad \text{for} \quad n = 1, 2, \ldots$$

Core-Reflector System:

$$\phi_n(r) = \frac{1}{B_n}\frac{\sin B_n r}{r} \qquad \text{and} \qquad \phi_{rn}(r) = \frac{1}{B_n}\sin B_n R_o\left(\frac{e^{-(r-R_o)/L_r}}{r}\right) \qquad \text{for} \quad n = 1, 2, \ldots$$

where B_n is the nth zero of $f(B) = D_c\left(B\cos BR_o - \frac{1}{R_o}\sin BR_o\right) + D_r\left(\frac{1}{L_r} + \frac{1}{R_o}\right)\sin BR_o = 0$

where $R = R_o + d$ and the fluxes have been normalized to have a maximum value of unity at $r = 0$.

The above expressions represent an infinite number of solutions to the diffusion equation for the specific case of interest, where we argued that all the higher modes decay away leaving only the fundamental mode solution (n = 1) as the steady state profile for the critical system. The goal of this problem is to get some experience with these expressions and to visualize the spatial behavior of the higher modes relative to the fundamental mode solution.

a. Using the **material cross sections from Fast Reactor #1** from the **core_refl1g_gui** code, use Matlab (or any other suitable software package of choice) to evaluate and plot the first three modes (i.e. for n = 1, 2, and 3) for the case of the **bare critical spherical reactor**. You can use the **core_refl1g_gui** code to determine the critical value of R_o for the bare reactor case (or you can do this by hand if you prefer). Put all three curves on the same plot and annotate properly. Are your results as expected? Explain…

b. Now do the same exercise for the **core-reflector problem in spherical geometry** -- again using the critical R_o for the core-reflector case and material cross sections from the **core_refl1g_gui** code. This problem is a little more complicated since you need to find the first three zeros of the nonlinear equation, f(B) = 0, given above. One approach here is to plot f(B) vs. B for a range of B values and visually observe the location of the first three zero crossings. Then, using a root-finding tool such as Matlab's **fzero** routine, you should be able to bracket the root and find accurate values for B_1, B_2, and B_3. Once these have been obtained, simply evaluate and plot the flux profiles given above. Put all three curves for both the core and the reflector regions on the same plot and observe the behavior. Again, do these make sense? Are they as expected? Are the boundary conditions for this 2-region problem satisfied? Explain…

Problem 4 Power Distribution in a 1-Group RZ Reactor Model (10 points)

A bare cylindrical reactor of height 120 cm and diameter 150 cm is operating at a steady-state power of 50 MWt. If the origin is taken at the center of the reactor, what is the power density at the point r = 15 cm, z = -25 cm?

Hint: Here you can assume that the extrapolation distance, d, is small, since no information about the core material properties is given -- thus, there is no way to accurately estimate d in this case. Note also that an explicit fission cross section is not needed since this cancels from the final expression when combining the normalization factor with the unnormalized power density term. Finally, to numerically evaluate the Bessel function expression, you should use either Matlab or some other appropriate online resource...

Problem 5 1-Group 1-D Cylindrical Reactor with an Infinite Reflector (10 points)

This problem involves a formal derivation of the flux profile and critical condition in a 1-D cylindrical 1-group homogeneous reactor that has an infinite cylindrical reflector. In particular, assuming that the core radius is R_o,

a. Use 1-group theory to formally derive expressions for the fluxes in the core and reflector regions.

b. Formally show that the criticality condition can be written as

$$D_c B \frac{J_1(BR_o)}{J_0(BR_o)} = \frac{D_r}{L_r} \frac{K_1(R_o / L_r)}{K_0(R_o / L_r)}$$

c. Finally, also develop an expression for the proper flux normalization if P is the power per unit length of the reactor.

Hint: Be sure to read the appropriate Lecture Notes (on Bessel functions and the other 1-D critical reactor models) before you attempt this problem. The development here should follow that done in class and in the Lecture Notes. Be formal in your development and explain all the key steps in the process…