## ENGY. 4340 Nuclear Reactor Theory

Fall 2016

## HW \#2 -- Neutron Density, Flux, Current, Leakage and the Multigroup Balance Equation

## Problem 1 Plotting $\phi(E)$ and $\phi(u)$ (10 points)

The thermal flux spectrum generated (somewhat artificially) in the Lecture Notes to illustrate the general $\phi(\mathrm{E})$ behavior for thermal systems was produced at an assumed reactor temperature of T $=20 \mathrm{C}(293 \mathrm{~K})$. If the temperature increases, say to $300 \mathrm{C}(573 \mathrm{~K})$, the spectrum changes as follows:

$$
\phi(\mathrm{E})=\left\{\begin{array}{cc}
\phi_{1}(\mathrm{E})=\mathrm{c}_{1} \mathrm{e}^{-1.036 \mathrm{E}} \sinh \sqrt{2.29 \mathrm{E}} & \text { for } \mathrm{E}_{2} \leq \mathrm{E} \leq \mathrm{E}_{1} \\
\phi_{2}(\mathrm{E})=\mathrm{c}_{2} / \mathrm{E} & \text { for } \mathrm{E}_{3} \leq \mathrm{E} \leq \mathrm{E}_{2} \\
\phi_{3}(\mathrm{E})=\mathrm{c}_{3} \mathrm{Ee}^{-\mathrm{E} / \mathrm{kT}} & \text { for } 0 \leq \mathrm{E} \leq \mathrm{E}_{3}
\end{array}\right.
$$

where

$$
\begin{array}{ll} 
& \mathrm{E}_{1}=10 \mathrm{MeV}, \quad \mathrm{E}_{2}=0.05 \mathrm{MeV}, \quad \mathrm{E}_{3}=2 \times 10^{-7} \mathrm{MeV} \\
& \mathrm{c}_{1}=4.078 \times 10^{11}, \quad \mathrm{c}_{2}=6.676 \times 10^{9}, \quad \mathrm{c}_{3}=9.586 \times 10^{24} \quad \text { (to give units of } \mathrm{n} / \mathrm{cm}^{2}-\mathrm{s}-\mathrm{MeV} \text { ) } \\
\text { and } \quad \mathrm{k}=8.617 \times 10^{-11} \mathrm{MeV} / \mathrm{K} \quad \text { with } \quad \mathrm{T}=573 \mathrm{~K}
\end{array}
$$

a. Using the data given for $\mathrm{T}=300 \mathrm{C}$, carefully plot $\phi(\mathrm{E})$ vs. E and $\phi(\mathrm{u})$ vs. E for this situation, and clearly describe to the reader the behavior observed in each plot. Do the profiles make sense? Explain...
b. How does this flux spectrum compare to the one given in the Lecture Notes for $\mathrm{T}=20 \mathrm{C}--$ that is, "Is it a harder or softer spectrum (be sure to explain what is meant by this terminology)?

## Problem 2 Neutron Leakage in a Bare Cubical Reactor (10 points)

The 3-D flux distribution in a critical 1-group 1-region bare homogeneous cubical reactor is given as

$$
\phi(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{A} \cos \left(\frac{\pi \mathrm{x}}{\mathrm{a}}\right) \cos \left(\frac{\pi \mathrm{y}}{\mathrm{a}}\right) \cos \left(\frac{\pi \mathrm{z}}{\mathrm{a}}\right)
$$

where A is a constant, a is the length of a side of the cube (here we ignore the extrapolation distance by assuming it is small compared to $\mathrm{a} / 2$ ), and $\mathrm{x}, \mathrm{y}$, and z are measured from the center of the reactor (center of the cube). In this configuration, the range of each coordinate variable is from $-\mathrm{a} / 2$ to $\mathrm{a} / 2$, since each side has the same length (i.e. it is in the shape of a cube).
a. For this system, write a general expression for the neutron current assuming that Fick's Law is valid. Remember that current is a vector quantity and that, in Cartesian geometry, the gradient operator is given as

$$
\vec{\nabla} \phi=\left(\frac{\partial}{\partial \mathrm{x}} \hat{\mathrm{i}}+\frac{\partial}{\partial \mathrm{y}} \hat{\mathrm{j}}+\frac{\partial}{\partial \mathrm{z}} \hat{\mathrm{k}}\right) \phi
$$

b. Derive an expression for the neutron leakage from one face of the reactor. Show/explain your work.
c. What is the total leakage from the reactor? Explain your logic.

## Problem 3 Components of an Integral Neutron Balance (10 points)

A particular semi-infinite bare slab of moderator material of thickness H with a distributed exponentially varying source, $\mathrm{Q}(\mathrm{x})$, has the following flux profile,

$$
\phi(x)=c_{1} \cosh \frac{x}{L}+c_{2} \sinh \frac{x}{L}+c_{3} e^{-\alpha x} \quad \text { for } \quad 0 \leq x \leq H
$$

where all the constants ( $\alpha, L, H, c_{1}$, etc.) are known. For this system, develop an analytical expression in terms of the known constants for the following quantities:
a. The total external neutron source production rate per unit area in the $y-z$ plane within the full slab if the source distribution is given as $\mathrm{Q}(\mathrm{x})=\mathrm{Ae}^{-\alpha \mathrm{x}}$.
b. The absorption rate per unit area in the y-z plane within the full volume.
c. The neutron leakage per unit area across the $y$-z plane at $x=H$.
d. The leakage per unit area across the $y-z$ plane at $x=0$.

Note: In general, an integral balance should show that leakage + absorption = source in any desired volume associated with the system. However, to show this, we would need to have the numerical values of the constants, or their relationship to the basic system parameters. Thus, since these are not available, just develop explicit expressions for the individual components of the integral balance equation, as requested above -- that is, for this problem, you do not have to show that leakage + absorption = source.

## Problem 4 Understanding Neutron Leakage (Spherical Geometry Case) (10 points)

The flux distribution in a critical bare homogeneous spherical reactor is given as (see sketch)

$$
\phi(r)=A \frac{\sin B r}{r} \quad \text { where } \quad B=\frac{\pi}{R}
$$

and $\mathrm{R}=\mathrm{R}_{0}+\mathrm{d}$, with A as the flux normalization, $\mathrm{R}_{0}$ as the physical radius of the spherical reactor, and d is the extrapolation distance (recall that the vacuum BC in diffusion theory forces the flux to go to zero at some small distance, d , outside the physical boundary).

This expression will be derived in detail soon enough. For now, however, we simply want to use this expression to try to explain/understand

neutron leakage. So within this context, perform the following operations:
a. With D as the diffusion coefficient, write an explicit expression for the neutron current, $\vec{J}=-D \vec{\nabla} \phi$, for the system described above. Note that current is a vector quantity, so be sure to highlight this attribute.
b. Using the current from Part a, develop an expression for the leakage out of the reactor at $\mathrm{r}=$ $R_{0}$, where leakage is given by the surface integral $\int_{A} \vec{J} \cdot \hat{n} d A$. Again, be explicit in the treatment of the vector notation.
c. Develop and expression for the divergence of $\overrightarrow{\mathrm{J}}$-- that is, what is $\vec{\nabla} \cdot \overrightarrow{\mathrm{J}}$ for this system?
d. Develop an expression for the leakage out of the reactor at $\mathrm{r}=\mathrm{R}_{\mathrm{o}}$, using the volume integral $\int_{\mathrm{V}} \vec{\nabla} \cdot \overrightarrow{\mathrm{J}} \mathrm{dr}$.
e. Finally, compare your results from Parts b and d. Are they consistent -- that is do they satisfy the Divergence theorem that says that the surface and volume integral formulations for neutron leakage are equivalent?
Note: You may want to review some of the material from your Calculus III course before attempting this problem. Being familiar with various coordinate systems (Cartesian, spherical, and cylindrical geometry), performing area and volume integrals, and being comfortable with several concepts from vector calculus should prove to be quite useful here. In particular, one of the goals of this problem is for you to demonstrate your understanding of these subjects!

## Problem 5 Multigroup Diffusion Equation (10 points)

A particular 4-group energy structure that might be used for analysis of a high temperature gascooled reactor (HTGR) is given below. This thermal reactor is graphite moderated and helium cooled. The 4-group energy structure has one fast group, one epithermal group, and two thermal energy groups, as shown in the sketch (note that the sketch is not to scale):


Answer the following questions within the context of this group structure (use standard multigroup notation for the group fluxes and the group-averaged macroscopic cross sections, as needed, but you should refer explicitly to the group flux as $\phi_{1}$, or $\phi_{2}$, etc. and the group-averaged cross sections as $\Sigma_{a 1}$, etc.). That is, do not use any summation notation in your final results. For example, the total absorption rate per unit volume can be written as

$$
\sum_{\mathrm{g}=1}^{4} \Sigma_{\mathrm{ag}} \phi_{\mathrm{g}}=\Sigma_{\mathrm{a} 1} \phi_{1}+\Sigma_{\mathrm{a} 2} \phi_{2}+\Sigma_{\mathrm{a} 3} \phi_{3}+\Sigma_{\mathrm{a} 4} \phi_{4}
$$

but, for this problem, the second explicit representation on the RHS of the equal sign is the desired form for the result.

Assume that the macroscopic cross sections and fluxes are known and write any expressions that are requested in terms of these variables. This problem is all about notation and terminology, so be precise with the notation and state any assumptions that may be needed:
a. Is upscatter possible within this group structure? Explain...
b. Give explicit expressions for the inscatter rates to group 2 and to group 3.
c. What is the removal rate from group 2? What about from group 3?
d. What is a reasonable distribution of numerical values for the multigroup fission spectrum? State any assumptions.
e. Give explicit expressions for the fission rate in group 2 and the fission source in group 2. What is the difference in these two quantities?
f. Write an expression for the overall 1-group average absorption cross section.
g. In an infinite homogeneous system, the multiplication factor simply becomes a property of the materials within the system. Within this context, write an expression for $\mathrm{k}_{\infty}$ for a 4-group, 1-region problem. Explain any assumptions/simplifications that may be needed.
h. Write an expression for the power density within the system.

