

The core of a fast reactor is a finite cylinder with diameter $D = 100 \text{ cm}$ and height $H = 100 \text{ cm}$. The composition of the core by volume is as follows: 18% fuel, 25% stainless steel cladding and structure, and 57% liquid sodium. The fuel consists of a mixture of U238 and Pu239 having a density of 19.1 g/cm^3 , with the plutonium making up 15 w/o of the fuel mixture.

- Calculate the multiplication factor, k , for a bare system with the core size and composition noted here.
- If a reflector is added with an effective reflector savings of 20 cm, estimate the multiplication factor for the fully reflected configuration.
- If the control system requires a total of 12% $\Delta k/k$ of reactivity control within twenty (20) B_4C control rods, estimate the mass of B_4C needed per rod.

Hint: This problem deals with the design of a control system for a fast reactor. In particular, you need to determine the bare and reflected core multiplication factors, the excess reactivity associated with the reflected core, and the amount of B_4C required in each control rod to give the desired total worth (to override the initial excess reactivity and to safely shutdown the reactor). Use 1-group theory for a homogeneous system as an approximate methodology for this problem. Also, for ease in the calculations, assume that stainless steel is primarily iron (Fe) and that a reasonable appropriately averaged cross section for natural boron for this fast system is about 0.27 b. You should get the remainder of the needed microscopic cross section data from Lamarsh Table 6.1 (within the Appendix to the Lecture Notes on "Cross Section Data for Preliminary Calculations"). Note also that the density of iron is about 7.9 g/cm^3 and that of liquid sodium is about 0.93 g/cm^3 .

First let's compute the homogenized atom densities

$$N_{\text{Pu239}} = \frac{19.1 \text{ g fuel}}{\text{cm}^3} \times \frac{0.15 \text{ g Pu239}}{\text{g fuel}} \times \frac{0.6022 \text{ at Pu239}}{239} \times \frac{\text{g Pu239}}{9 \text{ g Pu239}} \times \frac{\text{cm}^2}{5}$$

$$= 7.219 \times 10^{-3} \text{ at/b-cm of fuel}$$

$$N_{\text{Pu239}}^{\text{hom}} = \left(0.18 \frac{\text{cm}^3 \text{ fuel}}{\text{cm}^3 \text{ b core}} \right) \left(7.219 \times 10^{-3} \right) = 1.299 \times 10^{-3} \frac{\text{at}}{\text{b-cm of core}}$$

$$N_{\text{U238}}^{\text{hom}} = (19.1)(0.85) \left(\frac{0.6022}{238} \right) (0.18) = 7.394 \times 10^{-3} \frac{\text{at}}{\text{b-cm}}$$

$$N_{\text{Fe}}^{\text{hom}} = (7.9) \left(\frac{0.6022}{55.85} \right) (0.25) = 2.130 \times 10^{-2} \text{ at/b-cm}$$

$$N_{\text{Na}}^{\text{hom}} = (0.93) \left(\frac{0.6022}{23} \right) (0.57) = 1.388 \times 10^{-2} \text{ at/b-cm}$$

Now calculate microscopic data from Lamarsh Table 6.1

	σ_{tot}	σ_{str}	η
Pb239	2.11	6.8	2.61
U238	0.255	6.9	0.97
Fe	0.006	2.7	-
Na	0.0008	3.3	-

boron

Macroscopic Data

$$\Sigma_a^F = \frac{(1.299 \times 10^{-3})(2.11) + (7.394 \times 10^{-3})(0.255)}{4.626 \times 10^{-3} \text{ cm}^{-1}}$$

Pu239 + U238

Fe + Nu

$$\Sigma_a^m = \frac{(2.130 \times 10^{-2})(0.006) + (1.388 \times 10^{-2})(0.0008)}{1.389 \times 10^{-4} \text{ cm}^{-1}}$$

$$\Sigma_a = \Sigma_a^F + \Sigma_a^m = 4.765 \times 10^{-3} \text{ cm}^{-1}$$

$$f = \frac{\Sigma_a^F}{\Sigma_a} = 0.971$$

$$\bar{\eta} = \frac{\eta \Sigma_a|_{Pu239} + \eta \Sigma_a|_{U238}}{\Sigma_a^F}$$

$$= \frac{2.61(2.741 \times 10^{-3}) + (0.97)(1.885 \times 10^{-3})}{4.626 \times 10^{-3}}$$

$$\bar{\eta} = 1.942$$

$$\therefore K_{\infty} = \bar{\eta} f = (1.942)(0.971) = 1.886$$

$$\Sigma_{cr} = (1.299 \times 10^{-3})(6.8) + (7.394 \times 10^{-3})(6.9) + (2.130 \times 10^{-2})(2.7) + (1.388 \times 10^{-2})(3.3)$$

$$= 0.1632 \text{ cm}^{-1}$$

$$D = \frac{1}{3 \Sigma_{cr}} = 2.043 \text{ cm} \quad d = 2.130 \\ = 4.35 \text{ cm}$$

$$L^2 = \frac{D}{\Sigma_a} = \frac{2.043}{4.765 \times 10^{-3}} = 429 \text{ cm}^2$$

Geometry and Buckling

Base core

$$D_0 = H_0 = 100 \text{ cm} \quad R_0 = D_0/2 = 50 \text{ cm}$$

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \left(\frac{2.405}{R_0+d}\right)^2 + \left(\frac{\pi}{H_0+2d}\right)^2$$

@

$$\therefore B^2 = \left(\frac{2.405}{54.35} \right)^2 + \left(\frac{\pi}{108.7} \right)^2 \\ = 1.958 \times 10^{-3} + 8.35 \times 10^{-4} = 2.793 \times 10^{-3} \text{ cm}^{-2}$$

$$K_{\text{eff}} = \frac{K_{\text{ao}}}{1 + B^2 L^2} = \frac{1.886}{1 + (2.793 \times 10^{-3})(429)} = 1.886 (0.4549)$$

$K_{\text{bone}} = 0.858$ —
well subcritical

K_{eff} for Reflected syst.

AMPAD® (b)

for $\delta = 20 \text{ cm}$ the effective dimensions become

$$R_{\text{eff}} = R_0 + \delta \\ = 70 \text{ cm}$$

$$H_{\text{eff}} = H_0 + 2\delta \\ = 140 \text{ cm}$$

$$\therefore B_{\text{refl}}^2 = \left(\frac{2.405}{70} \right)^2 + \left(\frac{\pi}{140} \right)^2 \\ = 1.180 \times 10^{-3} + 5.036 \times 10^{-4} = 1.684 \times 10^{-3} \text{ cm}^{-2}$$

$$K_{\text{eff}} = \frac{1.886}{1 + (1.684 \times 10^{-3})(429)} = 1.886 (0.581) \\ = 1.095 = K_{\text{eff}}$$

Design of Control Syst

(c)

$$\rho_w = \frac{\Sigma_a^P}{\Sigma_a^F + \Sigma_a^M} \quad \text{where} \quad \rho_w = 0.12 \Delta K/K$$

↑
needed
to remove
this and to
shut down
reactor

$$\therefore \Sigma_a^P = 0.12 (4.765 \times 10^{-3} \text{ cm}^{-1}) = 5.718 \times 10^{-4} \text{ cm}^{-1}$$

and with $\sigma_a^B = 0.27 \text{ b}$ given, we have

$$N_B = \frac{\Sigma_a^P}{\sigma_a^B} = 2.118 \times 10^{-3} \frac{\text{at}}{\text{b-on}}$$

This assumes that
 $\Sigma_a^B \gg \Sigma_a^C$ so that

$$\Sigma_a^{B+C} \approx \Sigma_a^B$$

a great
assumption

$$\text{core vol.} = \frac{\pi D_o^2}{4} H_o = \frac{\pi}{4} (100)^2 = 7.854 \times 10^5 \text{ cm}^3$$

$$\# \text{ atoms} = \left(2.118 \times 10^{-3} \frac{\text{at}}{\text{b-on}} \right) \left(\frac{1 \text{ b}}{10^{24} \text{ cm}^2} \right) \left(7.854 \times 10^5 \text{ cm}^3 \right) = 1.663 \times 10^{27} \frac{\text{at}}{\text{B}}$$

$$\text{mass B} = \left(1.663 \times 10^{27} \text{ at/B} \right) \left(\frac{10.8 \text{ g/B}}{0.6022 \times 10^{24} \text{ at/B}} \right) = 29,833 \text{ g/B} \\ = 29.83 \text{ kg/B}$$

for 20 rods

$$\frac{\text{mass } B}{\text{rod}} = \frac{29.83 \text{ kg}}{20 \text{ rods}} = 1.49 \text{ kg of } B/\text{rod}$$

and mass B_{4C} per rod becomes

$$\frac{1.49 \text{ kg } B}{\text{rod}} \times \frac{(4 + 10.8 + 12) \text{ g } B_{4C}}{4 \times 10.8 \text{ g of } B} = \boxed{\frac{1.90 \text{ kg } B_{4C}}{\text{rod}}} \\ \underline{\underline{\text{ans}}}$$

Blade Worths within the UMLRR -- 2014 Results

In this problem you are asked to address several items of interest that require knowledge of the blade worth curves (here we will assume that they do not change significantly with burnup -- which is actually a pretty good approximation). Thus, using the **bw_display** GUI with data from Feb. 2014, answer the following questions:

- a. At the beginning of life (BOL), the UMLRR was critical with Blades 1-4 banked at 14.9 inches out with the regulating blade (RegBlade) at 10 inches out. With this information, estimate the **excess reactivity in the BOL startup core**.

	Blade 1	Blade 2	Blade 3	Blade 4	Reg Blade		
Total Blade Worth	2.595	2.129	3.301	3.419	0.332	%Dk/k	Total Worth B1-4
Critical Ht. Start Height	14.900	14.900	14.900	14.900	10.000	in	11.776
All out							
End Height	24.550	25.150	24.950	26.090	25.090	in	Total Reactivity Change
Reactivity Change	0.822	0.668	1.058	1.204	0.198	%Dk/k	3.950
Data Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready		

BOL
Excess P

- b. The **shutdown margin** for the UMLRR is the amount of negative reactivity that can be inserted into the core with the blade with the most worth stuck in its fully withdrawn position. Estimate the **shutdown margin for the BOL core**.

	Blade 1	Blade 2	Blade 3	Blade 4	Reg Blade		
Total Blade Worth	2.595	2.129	3.301	3.419	0.332	%Dk/k	Total Worth B1-4
Critical Ht. Start Height	14.900	14.900	14.900	14.900	10.000	in	11.776
All in with BL 4 out							
End Height	0.000	0.000	0.000	26.090	0.000	in	Total Reactivity Change
Reactivity Change	-1.773	-1.457	-2.243	1.204	-0.133	%Dk/k	-4.402
Data Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready		

BOL
shutdown margin

- c. The current critical height of Blades 1-4 with the RegBlade at 10 inches out is about 16.7 inches withdrawn. Estimate the **excess reactivity and shutdown margin for the current M-2-5 core configuration**. Also, estimate the amount of reactivity loss due to depletion and fission product buildup since the fuel was loaded at the BOL.

$$3.95 - 2.70 = 1.25 \text{ % } \Delta k/k$$

	Blade 1	Blade 2	Blade 3	Blade 4	Reg Blade	%Dk/k	Total Worth B1-4
Total Blade Worth	2.595	2.129	3.301	3.419	0.332		
Start Height	16.700	16.700	16.700	16.700	10.000	in	11.776
End Height	24.550	25.150	24.950	26.090	25.090	in	Total Reactivity Change
Reactivity Change	0.553	0.441	0.704	0.807	0.198	%Dk/k	2.703
Data Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready		A 2014 excess p

	Blade 1	Blade 2	Blade 3	Blade 4	Reg Blade	%Dk/k	Total Worth B1-4
Total Blade Worth	2.595	2.129	3.301	3.419	0.332		
Start Height	16.700	16.700	16.700	16.700	10.000	in	11.776
End Height	0.000	0.000	0.000	26.090	0.000	in	Total Reactivity Change
Reactivity Change	-2.042	-1.684	-2.597	0.807	-0.133	%Dk/k	-5.649
Data Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready		A 2014 shutdown margin

- d. The average coolant $\Delta T = T_{out} - T_{in}$ across the UMLRR core is about 36°F when the reactor is operating in natural convection mode at about 100 kW (so the average coolant temperature increase in the core is roughly $36/2 = 18^{\circ}\text{F}$). The combined (fuel + coolant) temperature coefficient for the UMLRR is about $-0.0033 \text{ % } \Delta k/k \text{ per } ^{\circ}\text{F}$. If the reactor is just critical at low power (say 10 W) with the regulating blade at 10 inches withdrawn, estimate the critical position of the RegBlade for critical steady state operation at 100 kW in natural convection mode. Assume that all other blades are fixed and ignore any Xe poisoning effects (xenon would only become important with extended operation in this state).

	Blade 1	Blade 2	Blade 3	Blade 4	Reg Blade	%Dk/k	Total Worth B1-4
Total Blade Worth	2.595	2.129	3.301	3.419	0.332		
Start Height	16.700	16.700	16.700	16.700	10.000	in	11.776
End Height	16.700	16.700	16.700	16.700	12.600	in	Total Reactivity Change
Reactivity Change	0.000	0.000	0.000	0.000	0.059	%Dk/k	0.059
Data Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready	<input checked="" type="checkbox"/> Ready		needed to cancel neg feedbacks near location after compensation

$$\rho_f = \alpha_t \Delta T = (-0.0033 \frac{\% \Delta k}{\text{per } ^{\circ}\text{F}})(18)$$

$$= -0.0594 \text{ % } \Delta k/k$$

This is the inherent reactivity due to the temperature increase in going from 0 $\rightarrow 100 \text{ kW}$

- Thus, to maintain criticality, the RegBlade must be moved outward to add positive ρ to compensate for the neg. temperature feedback
 → As seen here, the RegBlade must be moved to 12.6" out to achieve this

From eqn 11, the equilibrium worth is

$$\rho_{\infty} = \frac{(\gamma_I + \gamma_X)}{\sqrt{P_E P_F P_T}} \frac{\varepsilon_f}{\varepsilon_{f_2}} \frac{\phi_{\infty}}{\phi_X + \phi_{\infty}}$$

Now to estimate the maximum Xe worth in the UMLRR, we let $\phi_{\infty} \gg \phi_X$

thus $\frac{\phi_{\infty}}{\phi_X + \phi_{\infty}} \rightarrow 1$

which gives

$$\therefore \rho_{\infty \text{ max}} = \frac{-(\gamma_I + \gamma_X)}{\sqrt{P_E P_F P_T}} \frac{\varepsilon_f}{\varepsilon_{f_2}}$$

Note that this assumption implies that a change in power level simply means an increase in ϕ_{∞} - with no other changes

Also from the data on pg 6 and the defn of ε_f in eqn (3), we have

$$\begin{aligned} \varepsilon_f &= \varepsilon_{f_1} \frac{d_1}{d_2} + \varepsilon_{f_2} \\ &= (1.21 \times 10^{-3})(2.75) + 5.04 \times 10^{-2} \\ &= 5.37 \times 10^{-2} \text{ cm}^{-1} \end{aligned}$$

$$\therefore \frac{\varepsilon_f}{\varepsilon_{f_2}} = 1.065$$

This is a pretty good assumption

Also $P_E = (0.879)(1.067) = 0.938$

and $P_F P_T = (0.665)(0.909) = 0.644$

or $\rho_{\infty \text{ max}} = \frac{0.06627}{(2.43)(0.938)(0.644)} (1.065) = 0.048$

$$\rho_{\infty \text{ max}} = 4.8 \% \frac{\Delta k}{k}$$

Summary XeWorth
in SMLRR

$\% \Delta K/L$

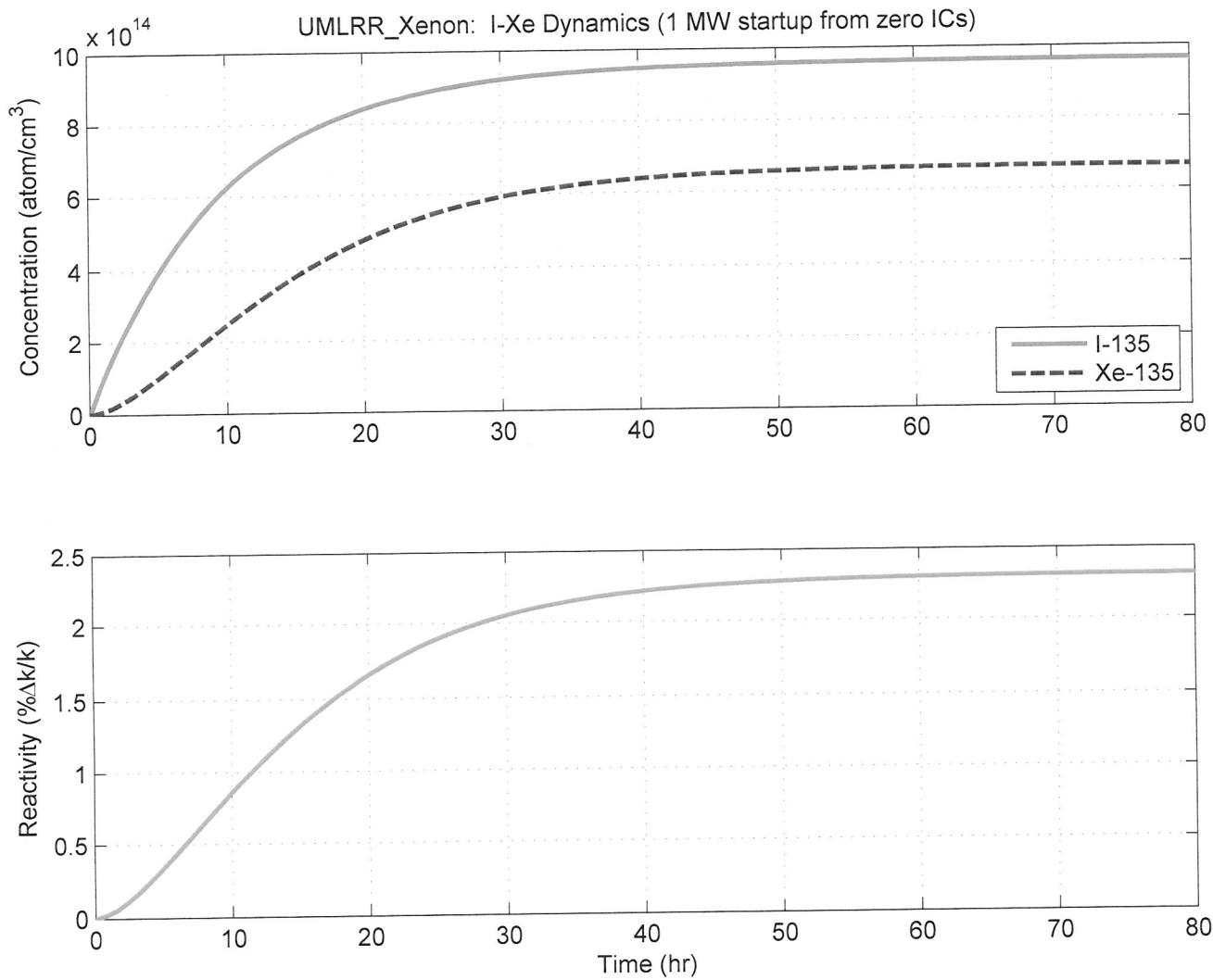
	1mW	2mW	5mW	20mW
P_{00}	~ 2.3	~ 3.1	~ 3.9	~ 4.5
peak shutdown	~ 2.7	~ 4.4	—	—
practical max	< 1.5	~ 2.5	—	—

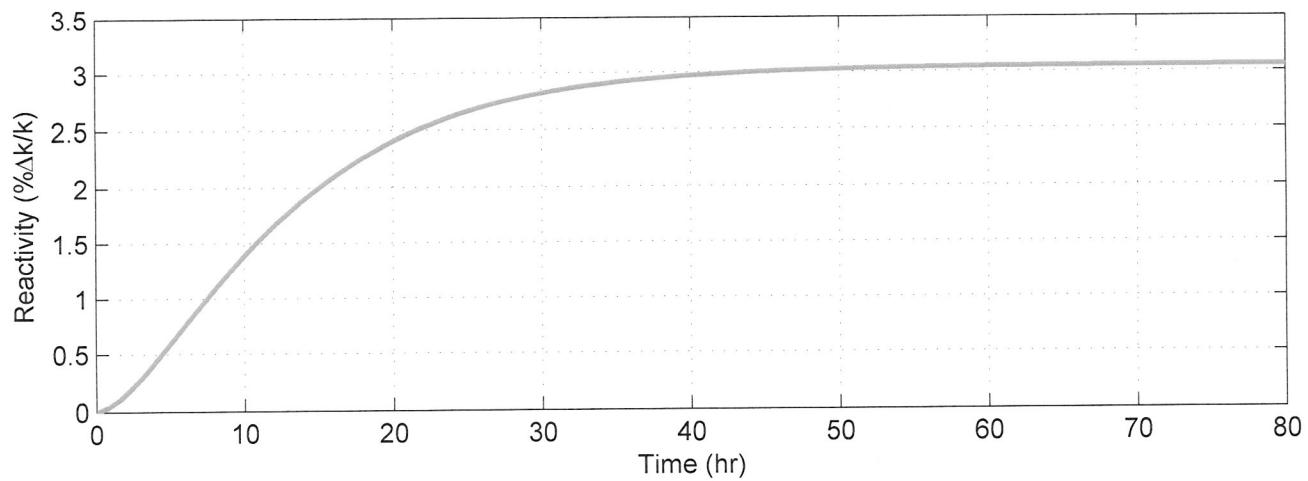
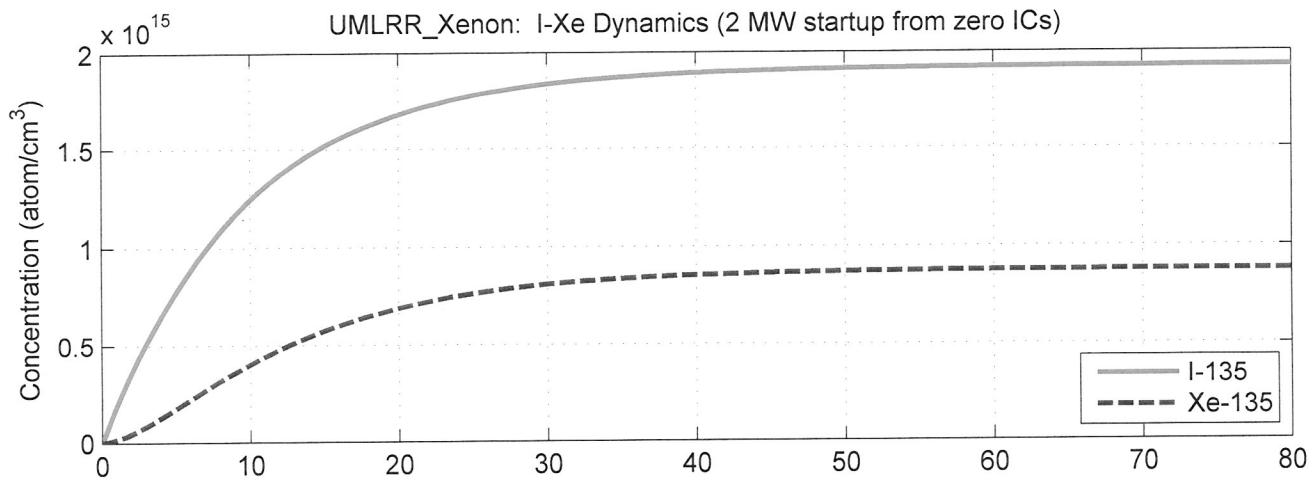
this is a lot!

20mW
is still
not quite
at neg
worth

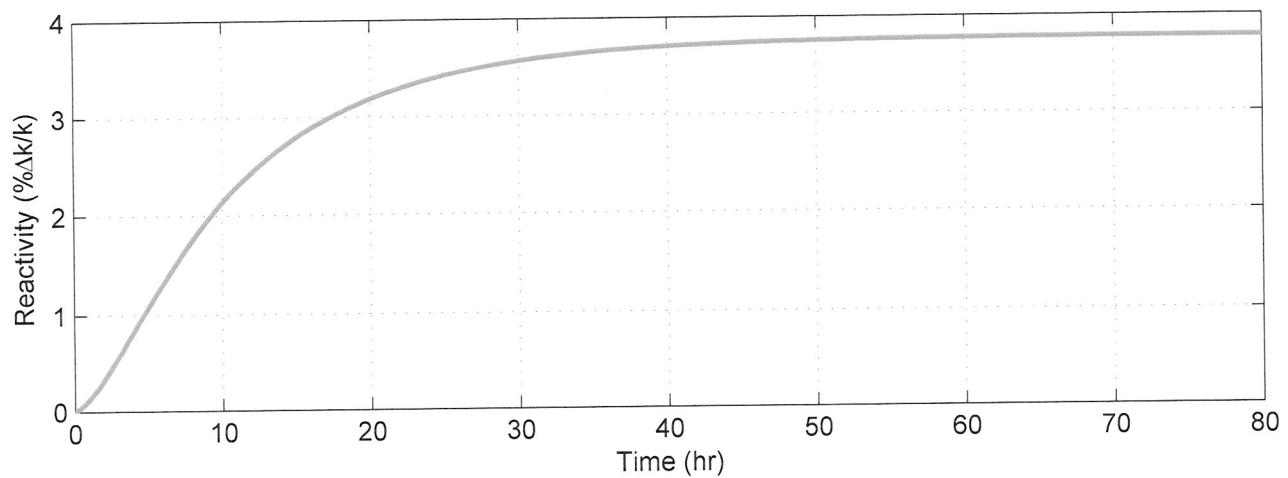
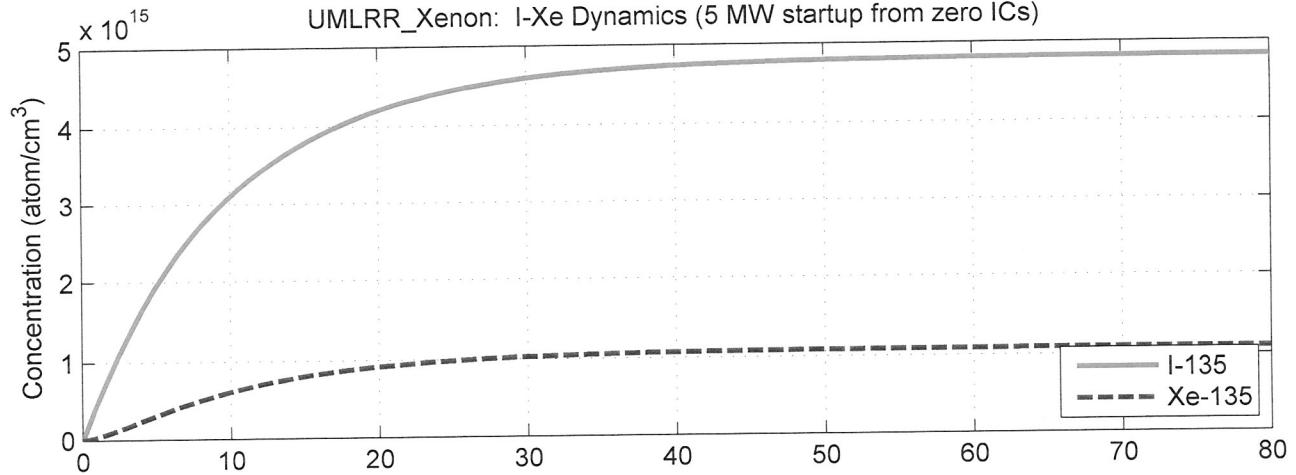
Note To overcome the additional xenon reactivity when operating at 2mW, the blades will need to be further out. Also, during operation, the operators will have to rebank more frequently. The regulating blade will still be used in auto mode, but it will move out during operation more rapidly to overcome the higher Xe worth. When the reg blade gets to about 20 inches out, a rebank will be needed (this will happen more often)

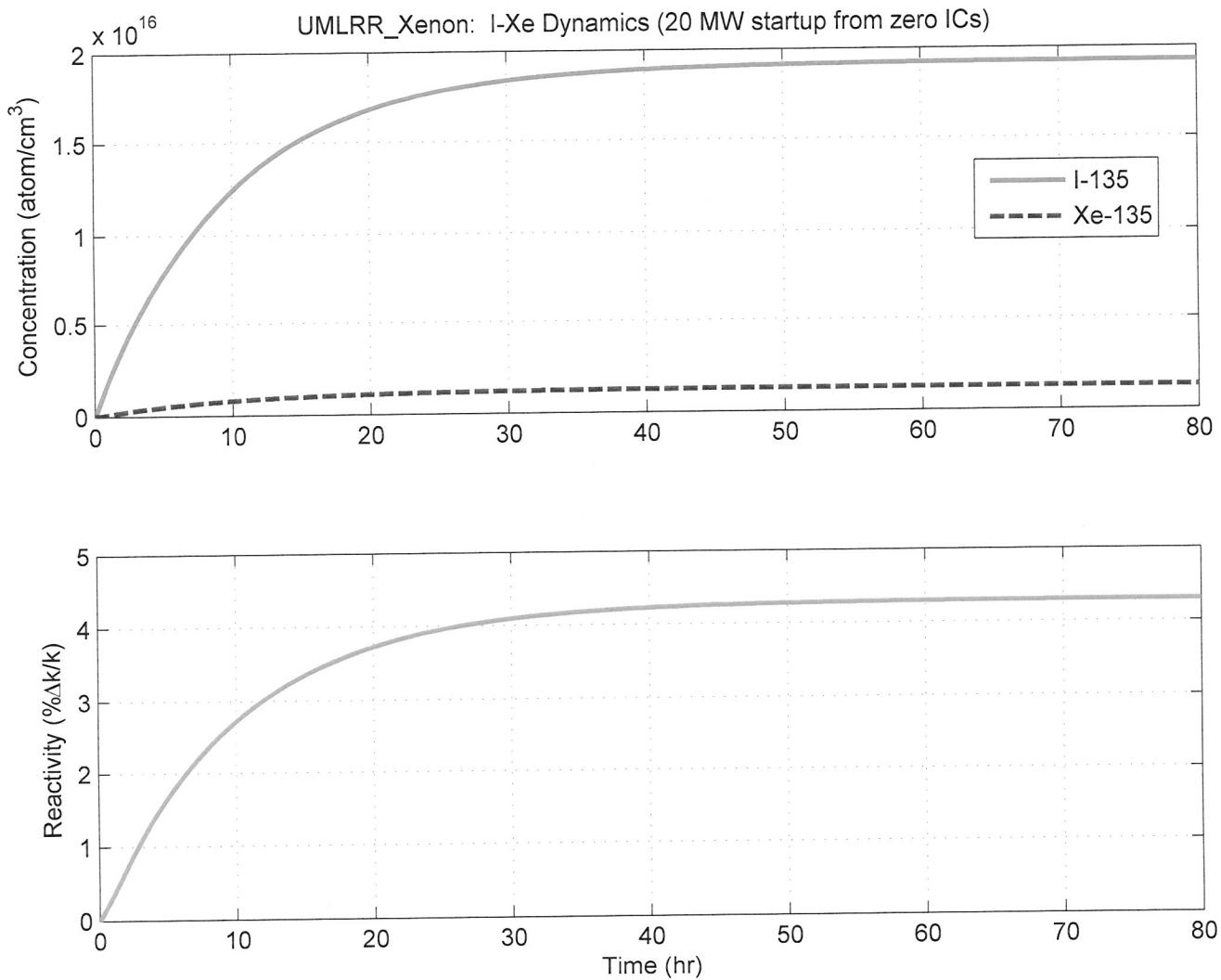
Also note that with the higher worth after shutdown, starting the next day (following an 8m fuel power run) will require the blades to be withdrawn to a much higher height. In fact, with further fuel depletion, we will have some difficulties overcoming the neg Xe worth...

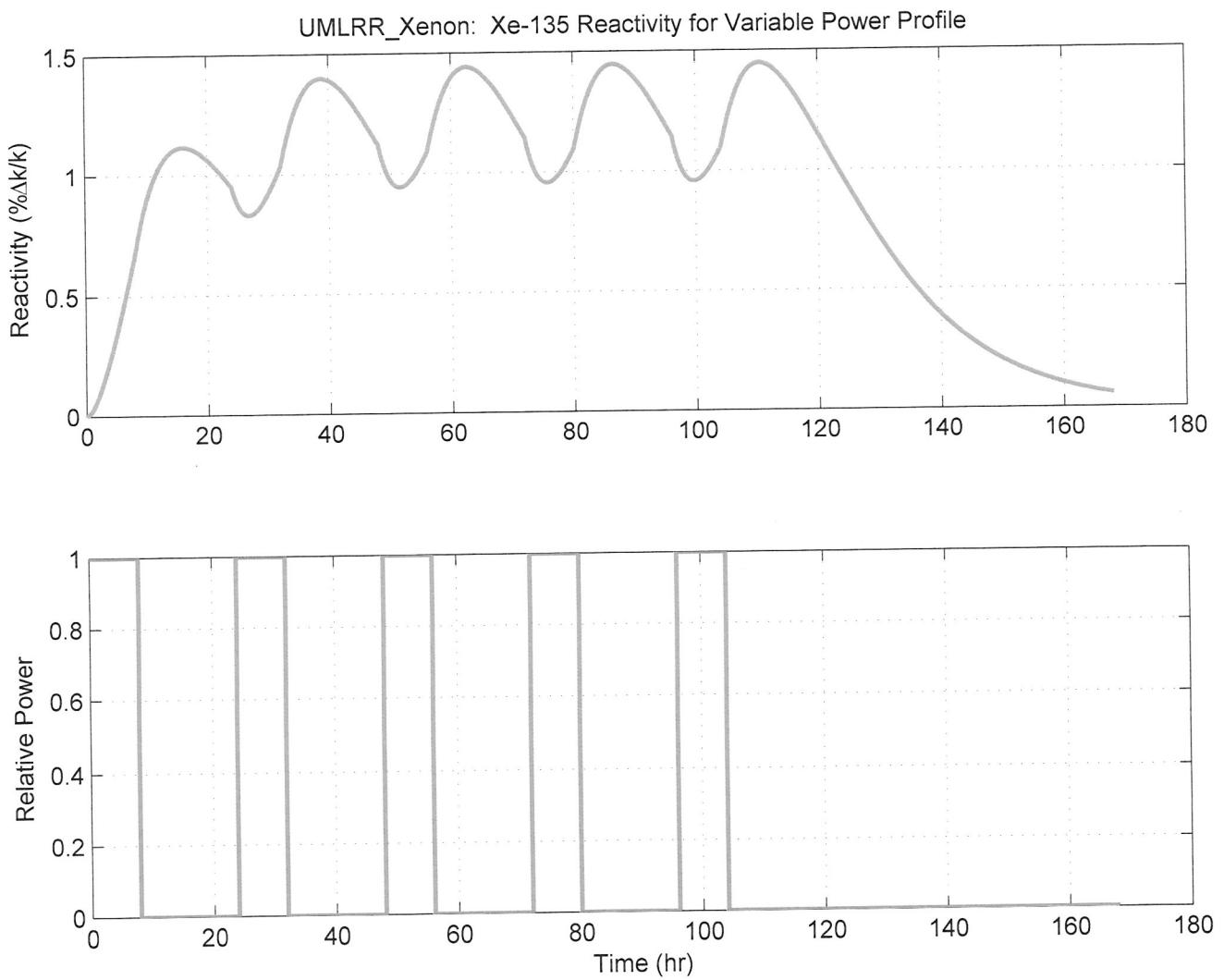




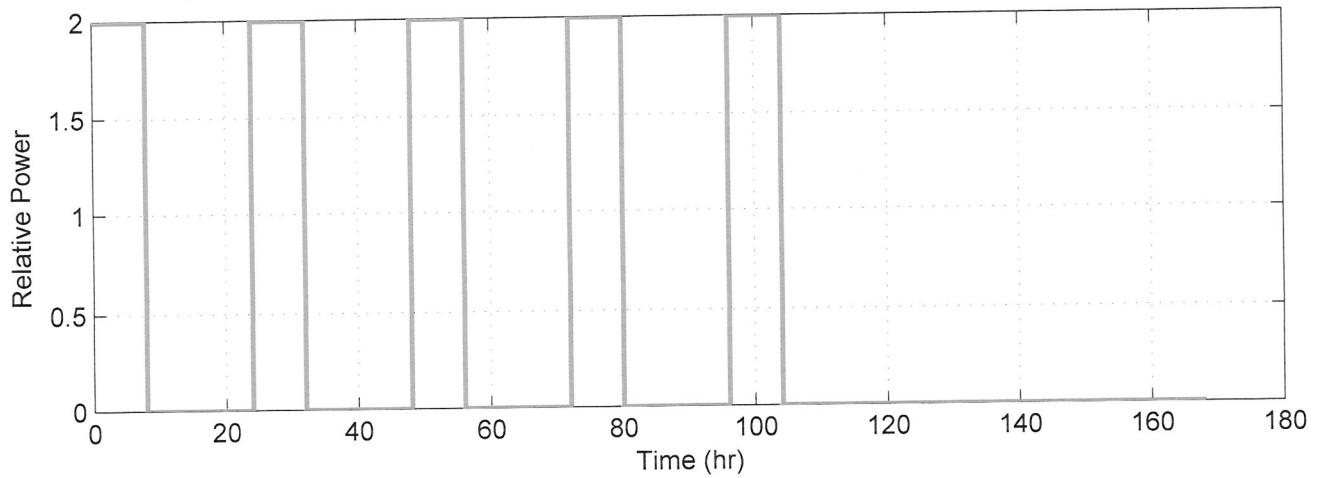
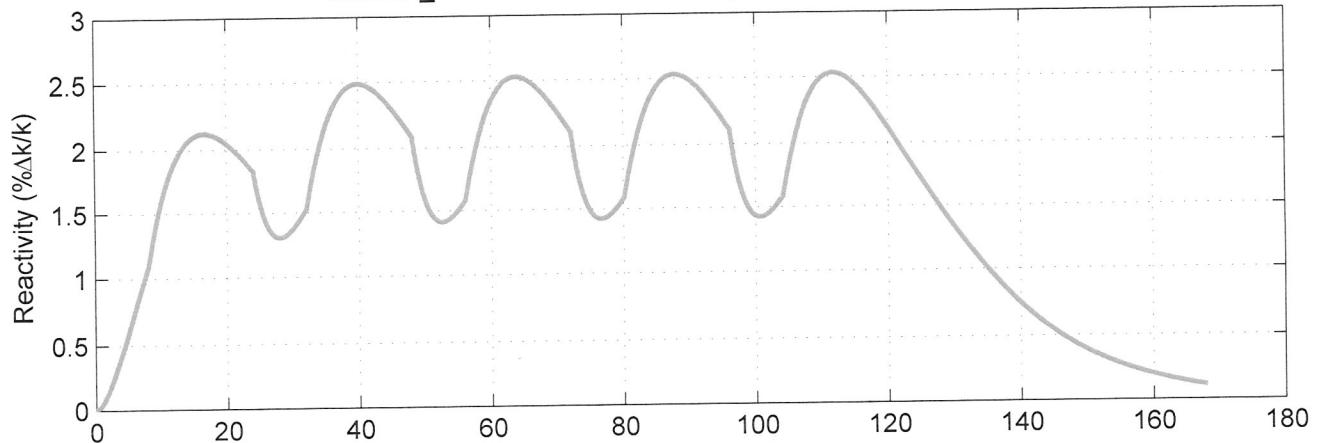
UMLRR_Xenon: I-Xe Dynamics (5 MW startup from zero ICs)

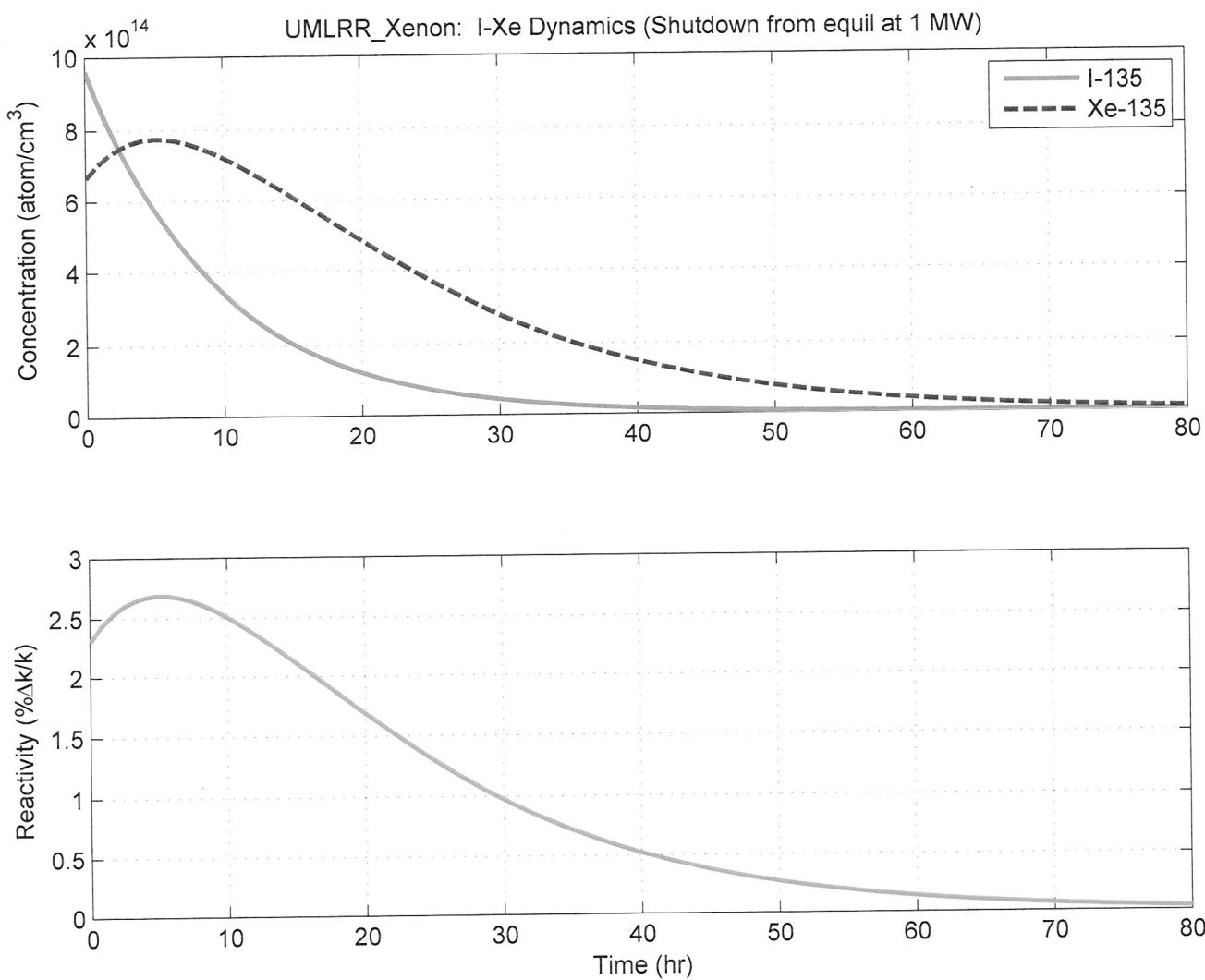


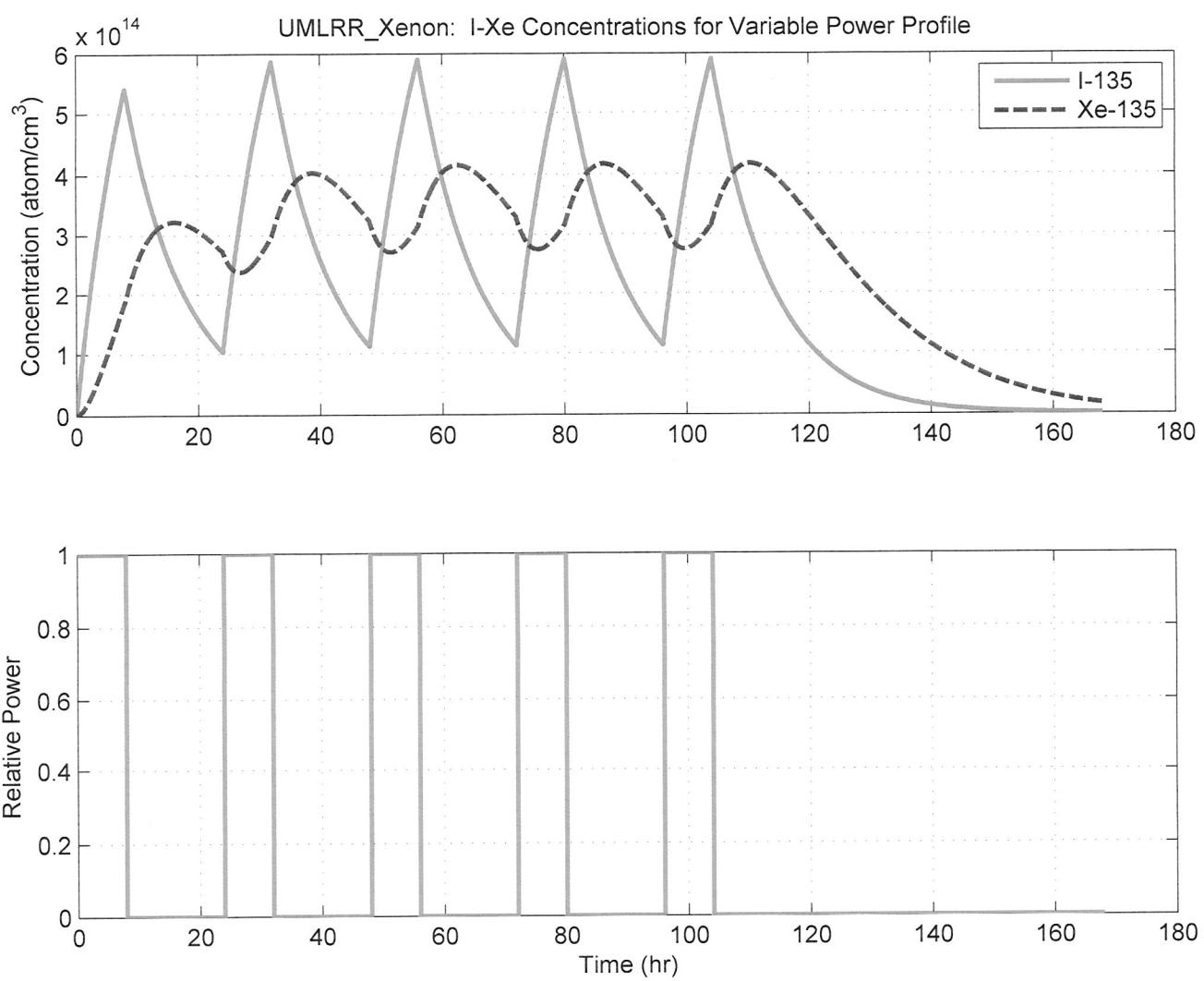


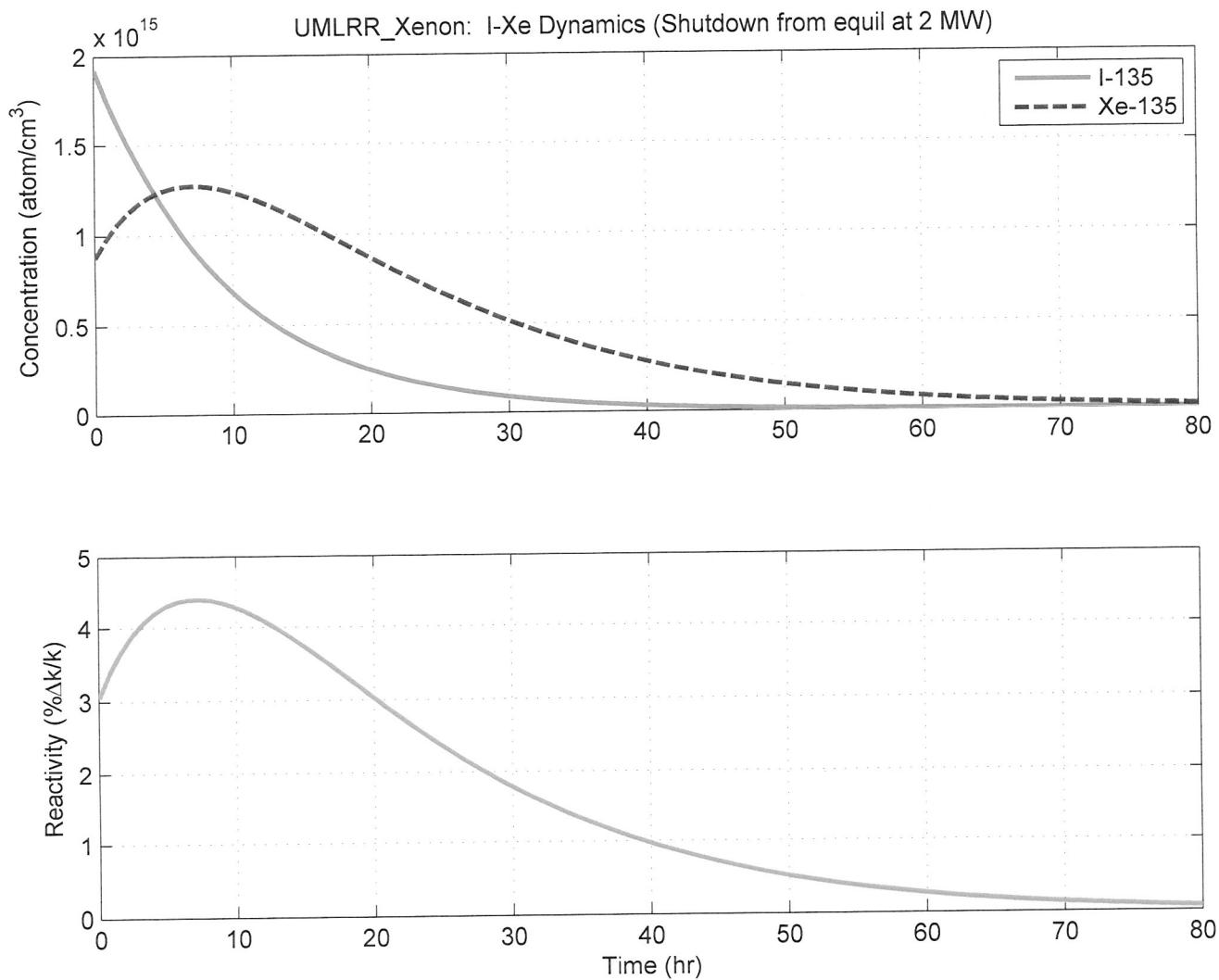


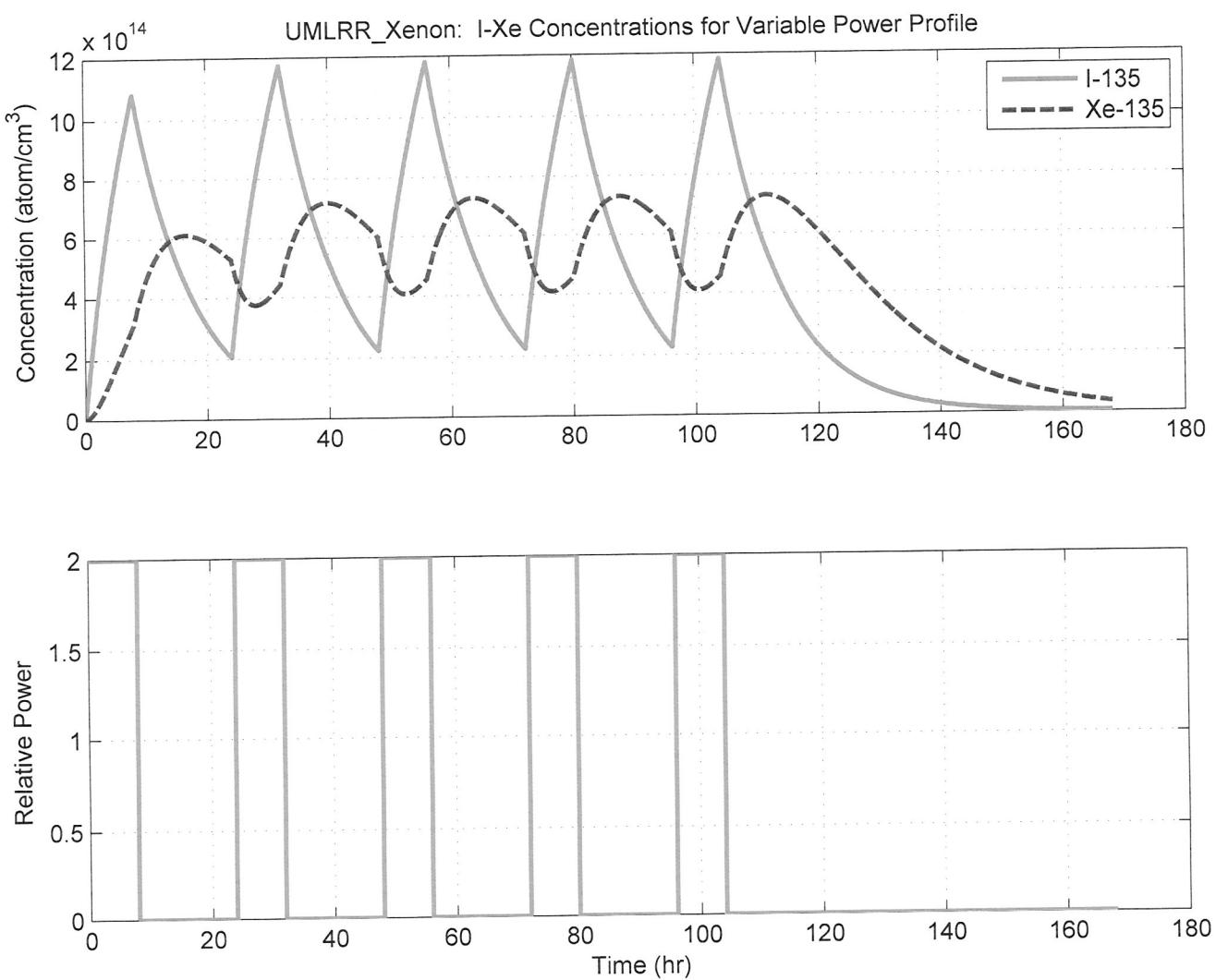
UMLRR_Xenon: Xe-135 Reactivity for Variable Power Profile



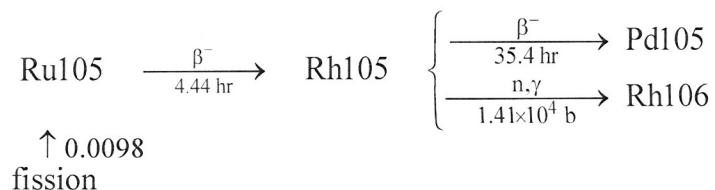








Consider the following fission product chain involving Ru105 and Rh105. In particular, Ru105 has an equilibrium yield of about 0.0098 atoms/fission and it decays to Rh105 with a half-life of 4.44 hr. Rh105 can decay to Pd105 with a half-life of 35.4 hr and it also has a large thermal absorption cross section of approximately 14,100 barns.



With this information, perform the following analyses:

- Develop an expression for the equilibrium Rh105 density and worth, and evaluate this for the case of a large high-power thermal system with negligible leakage. Assume a U235 fuelled system.
- Assuming that the reactor is shutdown quickly after it has been operating at constant power for a long time, develop an expression for the Rh105 density and reactivity worth versus time after shutdown. Carefully sketch the expected profile for $\rho(t)$ for this situation.

let $A = \text{Ru105}$ and $B = \text{Rh105}$

then the balance eqns become

$$\frac{dA}{dt} = \gamma \Sigma_f \phi - \gamma_A A$$

$$\frac{dB}{dt} = \gamma_A A - (\sigma_a^\theta \phi + \gamma_B) B$$

@ for constant Power, $\phi = \text{constant}$ and at equilibrium, $\frac{dA}{dt} = \frac{dB}{dt} = 0$

$$\therefore A_{\infty} = \frac{\gamma \Sigma_f \phi_{\infty}}{\gamma_A}$$

$$\text{and } B_{\infty} = \frac{\gamma_A A_{\infty}}{(\sigma_a^\theta \phi_{\infty} + \gamma_B)} =$$

$$\boxed{\frac{\gamma \Sigma_f \phi_{\infty}}{\sigma_a^\theta \phi_{\infty} + \gamma_B}}$$

$B_{\infty} = N_{\infty}$
for Rh105

$$\text{and } \rho_{\infty} = \frac{B_{\infty} \sigma_a^\theta / \Sigma_{f_z}}{\nu P_E P_F P_T} =$$

$$= \boxed{\frac{\nu \gamma}{\nu P_E P_F P_T} \frac{\Sigma_{f_z}}{\Sigma_{f_z}} \frac{\phi_{\infty}}{\phi_{\infty} + \gamma_B / \sigma_a^\theta}}$$

↑
directly from
lecture notes

↑
 ρ_{∞}

To get an estimate of $\rho_{\text{oo}}|_{\text{max}}$

let $P_F P_T = 1.0$ (no leakage) and $\frac{\Sigma_f}{\Sigma_{f+}} = 1.0$

$P_E \approx 1.0$ (dilute homo system)

$$\Phi_{\infty} \rightarrow \text{large} \quad \frac{\Phi_{\infty}}{\Phi_{\infty} + \frac{\lambda_B}{\sigma_{\alpha}^B}} \rightarrow 1.0$$

thus $\rho_{\text{oo}}|_{\text{max}} = \frac{\gamma}{\nu} = \frac{0.0098}{2.43} \approx \text{U235 fuel}$

$$= 0.00403 \Delta E/K$$

$$= 0.403 \text{ MeV/K}$$

this is NOT negligible!

$$\rho_{\text{oo}}|_{\text{max}} = \frac{0.00403}{0.00685} = 0.588 \text{ dollar}$$

$\leftarrow \beta \text{ for U235}$

or $\approx 59 \text{ ¢}$

check

$$\lambda_B = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{35.4 \text{ min}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = 5.439 \times 10^{-6} \text{ sec}^{-1}$$

$$\sigma_{\alpha}^B = 14,100 \text{ b} \times 10^{-24} \frac{\text{cm}^2}{\text{b}} = 1.41 \times 10^{-20} \text{ cm}^2$$

$$\therefore \frac{\lambda_B}{\sigma_{\alpha}^B} = \frac{5.439 \times 10^{-6}}{1.41 \times 10^{-20}} = 3.86 \times 10^{14} \text{ cm}^{-2} \text{ sec}^{-1}$$

This is quite large!!!

So our assumption

that $\frac{\Phi_{\infty}}{\Phi_{\infty} + \frac{\lambda_B}{\sigma_{\alpha}^B}} \rightarrow 1$ is pretty poor

and the estimate of $\rho_{\text{oo}}|_{\text{max}}$ is high!!!

(b) At shutdown, $\phi \rightarrow 0$ very quickly, so the balance eqns become

$$\frac{dA}{dt} = -\lambda_A A \quad A(0) = A_\infty$$

$$\frac{dB}{dt} = \lambda_A A - \lambda_B B \quad B(0) = B_\infty$$

This is just a simple radioactive decay problem. We developed analytical solns to this problem last semester.

$$A(t) = A_\infty e^{-\lambda_A t}$$

$$B(t) = B_\infty e^{-\lambda_B t} + \frac{\lambda_A A_\infty}{\lambda_B - \lambda_A} [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

and $\rho(t) = \frac{\sigma_a^B / \varepsilon_{fz}}{\gamma P_e P_F P_t} B(t) = \text{const} * B(t)$

