The Lifetime Formulation

For this representation of point kinetics, one first defines the prompt neutron lifetime. This can be done in an intuitive fashion by arguing that, at steady state, the total neutron loss rate is given by the total neutron population divided by the neutron lifetime. In equation form, this can be written as

$$\text{loss rate} = \frac{\text{neutron population}}{\text{lifetime}} \qquad \text{or} \qquad \text{lifetime} = \frac{\text{neutron population}}{\text{loss rate}}$$

Defining l_p as the prompt neutron lifetime, the one-speed approximation gives

$$l_p = \frac{\frac{1}{v}\langle \psi_o \rangle}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle} \tag{1}$$

where the numerator is just the total neutron population at steady state conditions (recall that $\phi = nv$ or $n = \phi/v$).

Note also that within the context of the 1-group diffusion equation, the multiplication factor, k, can be written as:

$$k = \frac{\text{neutron production rate from fission}}{\text{loss rate}} = \frac{\text{production}}{\text{loss}} = \frac{\langle \nu \Sigma_f \psi_o \rangle}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle} \tag{2}$$

We now divide every term in the neutron balance equation by the neutron loss rate to give

$$\frac{\frac{1}{v}\langle \psi_o \rangle}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}\frac{dT}{dt} = (1-\beta)\frac{\langle \nu \Sigma_f \psi_o \rangle}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle} T + \sum_i \lambda_i \frac{1}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}\langle C_i \rangle$$
$$+ \frac{1}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}\langle Q \rangle - \frac{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle} T$$

and use the definitions of k and l_p to simplify to

$$l_p \frac{dT}{dt} = \left[(1-\beta)k - 1\right] T + \sum_i \lambda_i \frac{1}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}\langle C_i \rangle + \frac{1}{\langle -\nabla \bullet D \nabla \psi_o \rangle + \langle \Sigma_a \psi_o \rangle}\langle Q \rangle$$

Dividing by l_p gives

$$\frac{dT}{dt} = \frac{\left[(1-\beta)k - 1\right]}{l_p} T + \sum_i \lambda_i \frac{1}{\frac{1}{v}\langle \psi_o \rangle}\langle C_i \rangle + \frac{1}{\frac{1}{v}\langle \psi_o \rangle}\langle Q \rangle \tag{3}$$

Now, if we define the time dependent ***normalized*** precursor and external source amplitudes as

$$c_i(t) = \frac{1}{\frac{1}{v}\langle \psi_o \rangle}\langle C_i(t) \rangle \tag{4}$$

$$q(t) = \frac{1}{\frac{1}{v}\langle \psi_o \rangle} \langle Q(t) \rangle \tag{5}$$

we get the standard form of the neutron balance equation for the **lifetime formulation of point kinetics** , or

$$\frac{dT}{dt} = \frac{[(1-\beta)k-1]}{l_p} T \quad + \quad \sum_i \lambda_i c_i \quad + \quad q \tag{6}$$

To complete the derivation, we need to manipulate the precursor balance equation in a similar fashion. In particular, dividing the precursor equations by the total neutron population, $\langle \psi_o \rangle / v$, gives

$$\frac{1}{\frac{1}{v}\langle \psi_o \rangle} \frac{d}{dt} \langle C_i \rangle = \beta_i \frac{\langle \nu \Sigma_f \psi_o \rangle}{\frac{1}{v}\langle \psi_o \rangle} T \quad - \quad \lambda_i \frac{1}{\frac{1}{v}\langle \psi_o \rangle} \langle C_i \rangle \qquad \text{for } i = 1, 2, \cdots 6$$

Using the definitions in eqns. (1), (2), (4), and (5), this expression for the precursor balance can be converted into standard form, giving

$$\frac{dc_i}{dt} = \beta_i \frac{k}{l_p} T \quad - \quad \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6 \tag{7}$$

Equations (6) and (7) represent the **Lifetime Formulation** of the point kinetics equations.

The solution of the **Lifetime Formulation** of the point kinetics equation with no reactivity feedback and no external source for a **step change** in external reactivity (step change in k) is typically given in terms of the so-called **reactivity equation**. For six delayed neutron groups using the **lifetime formulation**, the reactivity equation can be written as

$$\rho = \frac{\omega \ell_p}{1+\omega \ell_p} + \frac{\omega}{1+\omega \ell_p} \sum_i \frac{\beta_i}{\omega + \lambda_i}$$

Now, your job for this problem is to explain, in detail, your understanding of point kinetics via a thorough discussion of this particular form of the **reactivity equation**. In your discussion you should address such things as:

What does this expression mean and where does this come from (a formal derivation is not required here since this is done in a previous HW problem -- but a good explanation of the general process will suffice in making this problem stand on its own)?

Discuss its interpretation in terms of the actual time dependent behavior of the neutron density for both positive and negative reactivity insertions.

How many roots are there? What is the sign of the roots for both positive and negative reactivity? What is the significance of the dominant root? How about the most negative root?

Be sure to introduce the concepts of reactor period and prompt jump/drop in your discussions and also use appropriate sketches as needed!!!

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For 6 delayed neutron groups, the solution to the point kinetics eqns for a step change in ρ gives a solution of the form

$$P(t) = T(t) = \sum_{k=1}^{7} A_k e^{\omega_k t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \cdots + A_7 e^{\omega_7 t}$$

This comes about because we have 7 coupled linear constant coeff. ODEs — and the general solution is the linear combination of the seven linearly independent individual solutions. Also, since the system is linear and constant coeffs, each of the solutions is just a simple exponential $\sim e^{\omega t}$

When the assumed soln of the form $A e^{\omega t}$ is substituted into the pt. kinetics eqn., we get the characteristic eqn — which here we call the reactivity equation.

The roots of the reactivity eqn are the desired ω_k values in the above soln for $T(t)$ or $P(t)$

A sketch of the RHS of the reactivity eqn can help visualize the location of these values

(see next page)

from Duderstadt and Hamilton

ω_7 is a much larger negative number than all the other ω_k values

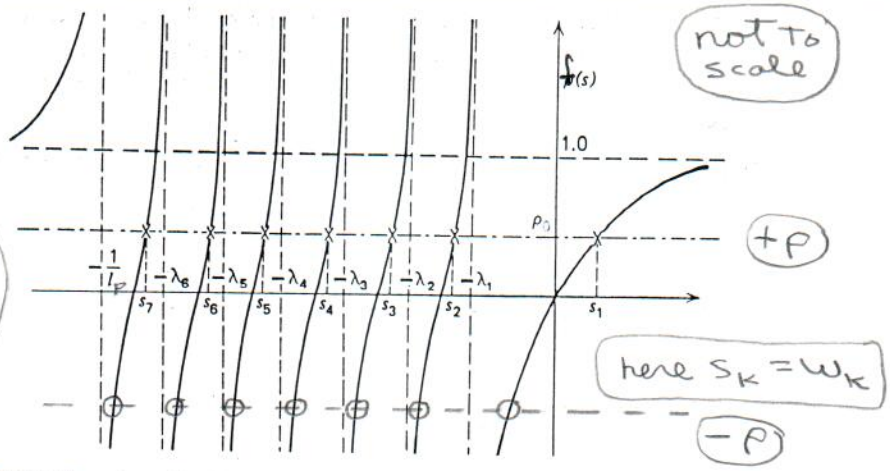


FIGURE 6-2. A graphical determination of the roots to the inhour equation

Here we see that $f(s) = f(\omega) \to \pm\infty$ at the six values of $-\lambda_k$ and at $-1/l_p$. Thus, there are seven asymptotes and there is a root to the reactivity eqn for either positive or negative ρ

the most positive root is for $\omega > -\lambda_1$ and this root is denoted ω_1 and its reciprocal is referred to as the reactor period

$$\tau = \frac{1}{\omega_1}$$

ω_1 is the dominant root in that all the others decay away more rapidly

Note that

$$+\rho \Rightarrow \begin{cases} \omega_1 = \text{pos} \\ \omega_2 - \omega_7 = \text{neg} \end{cases} \Rightarrow \text{growing exponential after initial transient}$$

$$-\rho \Rightarrow \begin{cases} \omega_1 = \text{neg} \\ \omega_2 - \omega_7 = \text{neg} \end{cases} \Rightarrow \text{decaying exponential after initial transient}$$

$$P(t) = P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau}$$

where P_1 is the normalized amplitude (or power) after the initial transient and τ is the reactor period (+ for pos ρ and − for neg ρ)

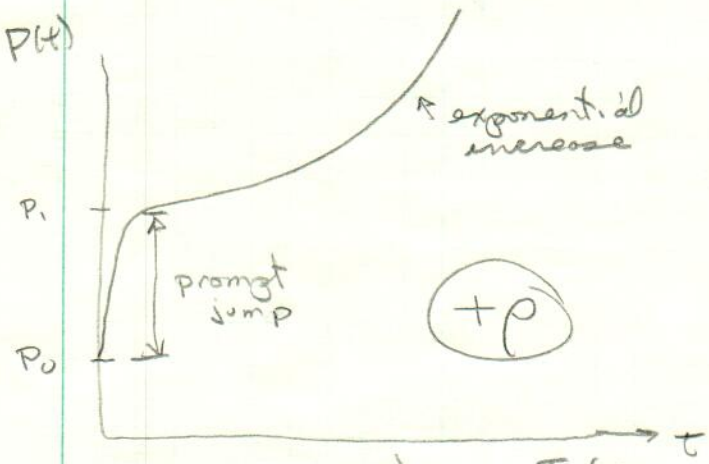
Note that, because ω_1 is such a large negative value, the term containing ω_1 (i.e. $e^{\omega_1 t}$) decays to zero very rapidly. This term is associated with the prompt neutrons and this very fast transient is often referred to as the prompt jump / drop.

$+\rho$ $\quad$ $-\rho$

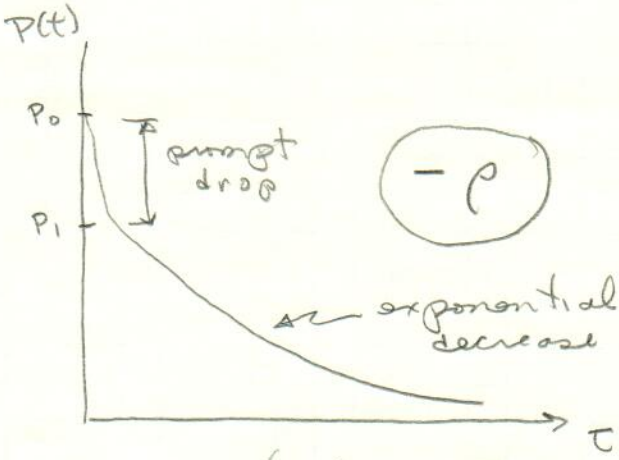
the typical $P(t)/P_0$ or $T(t)/T_0$ profiles for both positive and negative step changes in reactivity are sketched below

→ note the prompt jump / drop
→ exponential increase / decay

} depending on the sign of ρ



$$P(t) = \left(\frac{P_1}{P_0}\right) P_0 \, e^{t/\tau}$$



$$P(t) = \left(\frac{P_1}{P_0}\right) P_0 \, e^{-t/\tau}$$

Lamarsh 3E based on Prob 7.4

Reactivity Units

Express the following reactivities for both Pu239 and U235 fueled thermal reactor in % Δk/k.

a. 0.001 b. \$2 c. −50¢

Note also calc k-eff

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from Table 7.2 in Lamarsh (for a thermal system)

$\beta_{U235} = 0.0065$

$\beta_{Pu239} = 0.0021$

$k = \frac{1}{1-\rho}$

given ρ	ρ in % Δk/k		k_{eff}	
	U235	Pu239	U235	Pu239
0.001	0.10	0.10	1.0010	1.0010
\$2	1.30	0.42	1.0132	1.0042
−50¢	−0.325	−0.105	0.9967	0.9990

Note that when ρ is given in dollars and cents, then the absolute reactivity is different for U235 vs Pu239 and keff

A U235 fueled thermal reactor has an initial power of 10 MW and it is put on a reactor period of about 5 minutes. Estimate how long it takes to increase the power level by a factor of 50.

→ For a reactor period of $\tau = 5\,\text{min} = 300\,\text{sec}$ Fig 7.2 in Lamarsh gives an approximate reactivity of about $\rho = 4¢$ (also see kinetics.gui)

→ For this small value of $\rho = 0.04\beta = 0.04(.0065)$
$= 2.6\times10^{-4}\ \frac{\Delta k}{k}$

the prompt jump will be very small

$$\frac{P_1}{P_0} = \frac{\beta}{\beta-\rho} = \frac{1}{1-\rho'} \leftarrow \rho \text{ in } \$ = \frac{1}{0.96} = 1.04$$

→ Thus with only a roughly 4% change for the prompt jump we can ignore this completely.

thus
$$\frac{P(t)}{P_0} = e^{+t/\tau}$$

or $50 = e^{t/5\,\text{min}}$

$\ln 50 = t/5\,\text{min}$

or $t = (\ln 50)(5\,\text{min}) = \boxed{19.56\,\text{min}}$

$\boxed{\sim 20\,\text{min}}$ ans

(Note) If we don't ignore the prompt jump, then

$$\frac{P(t)}{P_0} = \left(\frac{P_1}{P_0}\right) e^{t/\tau}$$

not much difference

$$\frac{50}{1.04} = 48.1 = e^{t/5\,\text{min}}$$

or $t = (\ln 48.1)(5\,\text{min}) = \boxed{19.37\,\text{min}}$ ans

or still $\boxed{\sim 20\,\text{min}}$

Lamarsh 3E
Prob 7.12
pg 348

Reactor Kinetics

A U235 fueled reactor is scrammed by the instantaneous insertion of 5 dollars of negative reactivity after having operated at a constant power of 1 megawatt for several hours. Ignoring all fission product effects (decay heat and xenon reactivity effects), estimate how long it takes for the fission power to drop to 1 milliwatt.

from Fig 7.2 in Lamarsh (or <u>kinetics-gui</u>) | also table 7.2

$\rho = -5 \Rightarrow \tau = 80$ sec | $\beta = .0065$

prompt drop $\quad \dfrac{P_1}{P_0} = \dfrac{\beta(1-\rho)}{\beta-\rho}$ ← lifetime model

$$\Rightarrow \boxed{\frac{P(t)}{P_0} = \frac{\beta(1-\rho)}{\beta-\rho}\, e^{-t/\tau}}$$

then $\quad \dfrac{P(t)}{P_1} = e^{-t/\tau}$

for $\beta = 0.0065 \qquad \rho = -5\beta = -5(.0065)$
$= -0.0325$

lifetime model

$$\therefore \frac{P_1}{P_0} = \frac{.0065(1+.0325)}{.0065+.0325} = \frac{.00671}{.039}$$

$$\frac{P_1}{P_0} = 0.172$$

very similar both about 0.17

also $\boxed{\dfrac{P_1}{P_0} = \dfrac{\beta}{\beta-\rho} = 0.167}$ generation time model

$$\therefore \frac{P(t)}{P_0} = 0.172\, e^{-t/80s}$$

↳ given as $\dfrac{10^{-3}\,W}{10^{6}\,W} = 10^{-9}$

$$\frac{10^{-9}}{0.172} = e^{-t/80s}$$

$$\ln 5.814\times10^{-9} = -\frac{\tau}{80s}$$

$$\text{or } t = -80\text{ sec}\,(\ln 5.814\times10^{-9}) = \boxed{1517\text{ sec}}$$

$$\simeq \boxed{25.3\text{ min}}$$

Lamarsh 3E
Prob
7.17

Small Reactivities

generation time formulation

Pu239 fueled thermal system is at 1 MW. The power is then to be increased to 100 MW in 8 hrs.

a. What stable period is required?

b. What value of reactivity is needed to produce the desired period?

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for Pu239 $\beta = 0.0021$

→ Note that a change to only a factor of 100 in 8 hrs is pretty slow so the reactor period will be quite large

→ Also, for large τ, $\omega_1 \simeq 1/\tau$, is small which implies that a small reactivity is needed here

→ Thus, these arguments imply that the prompt jump will be negligible

$$\therefore \frac{P(t)}{P_0} = e^{+t/\tau}$$

P_0 ← where this is 1 MW

(a) Now for $P = 100$ MW at $t = 8$ hr, we have

$$\frac{100}{1} = e^{8/\tau}$$

or $\ln 100 = 8/\tau$ $\quad \tau = \frac{8}{\ln 100} = \boxed{1.737 \text{ hr}}$

$= \boxed{6254 \text{ sec}}$ ans

(b) Now, for small ρ (and large τ), we have

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i} \simeq \Lambda\omega + \omega \sum_i \frac{\beta_i}{\lambda_i}$$

$(\omega \ll \lambda_i)$

$$= \frac{1}{\tau}\left(\Lambda + \sum_i \frac{\beta_i}{\lambda_i}\right)$$

since $\Lambda \ll \sum_i \frac{\beta_i}{\lambda_i}$ $\quad \simeq \frac{1}{\tau}\sum_i \frac{\beta_i}{\lambda_i} = \frac{t_d \beta}{\tau}$ $\quad t_d$ = mean lifetime of delayed neutrons

from table 7.3 in Lamarsh $\beta t_d|_{Pu239} = (0.0021)(15.4 s) = 0.0324$ sec

$$\therefore \rho = \frac{0.0324 \text{ sec}}{6254 \text{ sec}} = \boxed{5.18 \times 10^{-6}} \quad \text{or} \quad \boxed{\frac{\rho}{\beta} = 2.47 \times 10^{-3} = 0.25\,¢}$$

⇒ from kinetics-gui $\rho = 1.888 \times 10^{-3}\$ = 0.19¢$ ← different λ_i → ans

close enough