

A particular oxide fueled fast reactor core consisting of 50% sodium, 30% fuel, and 20% stainless steel by volume was described in detail in HW #2. In that HW, you were asked to compute the material atom densities and several other quantities, including the k_{∞} of the core material.

- 4
- Using the results from HW #2 (or see the tabulated cross sections for Fast Reactor #1 in the **core_ref1g_gui** code), compute the critical size of a bare 1-D reactor using Cartesian, spherical, and cylindrical geometry. This should be done by hand (i.e. show your work).
 - Now run the **core_ref1g_gui** code, select the Reactor Type: Fast Reactor #1 and validate your hand calculations from Part a. Are your results as expected – that is, do things agree with your hand calculations and with your overall understanding of bare critical systems? In reviewing these results, tabulate the non-leakage probability (NLP) for the three geometries for the just critical case (i.e. when $k_{\infty} = 1$). Explain your results...

Summary data from previous work (Fast Reactor #1)

given:

core 50% sodium
30% fuel
20% SS

fuel 15% PuO_2
85% UO_2

computed:

$$\begin{aligned}\sqrt{\Sigma_f} &= 7.145 \times 10^{-3} \text{ cm}^{-1} \\ \Sigma_a &= 5.173 \times 10^{-3} \text{ cm}^{-1} \\ \Sigma_a^F &= 5.006 \times 10^{-3} \text{ cm}^{-1} \\ k_{\infty} &= \frac{\sqrt{\Sigma_f}}{\Sigma_a} = 1.381 \\ D &= 2.075 \text{ cm} \\ d &= 4.42 \text{ cm} \\ L^2 &= D/\Sigma_a = 401.1 \text{ cm}^2\end{aligned}$$

② Calc critical size for various 1-D bare core geometries

$$k_{\infty} = \frac{\sqrt{\Sigma_f}}{DB^2 + \Sigma_a}$$

but for crit. cal. $k_{\infty} = 1.000$

$$\therefore B^2 = \frac{\sqrt{\Sigma_f} - \Sigma_a}{D}$$

critical

using the above data for Fast Reactor #1

$$B^2 = \frac{7.145 \times 10^{-3} - 5.173 \times 10^{-3}}{2.075} = [9.504 \times 10^{-4} \text{ cm}^{-2}]$$

Slab $B^2 = \left(\frac{\pi}{a_0 + 2d} \right)^2$

and $B = 3.083 \times 10^{-2} \text{ cm}$

$\swarrow B$

$$a_0 = \frac{\pi}{B} - 2d = \frac{\pi}{3.083 \times 10^{-2}} - 2(4.42)$$

$a_0 = 93.06 \text{ cm}$

Sphere $B^2 = \left(\frac{\pi}{R_o + d} \right)^2$

$$R_o = \frac{\pi}{B} - d = \frac{\pi}{3.083 \times 10^{-2}} - 4.42$$

Sphere $R_o = 97.48 \text{ cm}$

Cylinder $B^2 = \left(\frac{2.4048}{R_o + d} \right)^2$

$$R_o = \frac{2.4048}{B} - d = \frac{2.4048}{3.083 \times 10^{-2}} - 4.42$$

Cylinder $R_o = 73.58 \text{ cm}$

- (b) Put computed values from above into core-refl1g-gui code to see if we get the same results (Fast Reactor #1 data)

for slab $a_o = 93.06 \text{ cm}$ $K_{\text{code}} = 1.060$

Sphere $R_o = 97.48 \text{ cm}$ $K_{\text{code}} = 1.000$

Cylinder $R_o = 73.58 \text{ cm}$ $K_{\text{code}} = 1.000$

OK

This works!

Note also that the non-leakage probability is the same in all cases $P_{NL} = 0.724$

However, this makes perfect sense because

$$K_{\text{eff}} = \frac{\gamma \bar{\varepsilon}_f}{DB^2 + \bar{\varepsilon}_a} = \frac{\gamma \bar{\varepsilon}_c}{\bar{\varepsilon}_a} \frac{\bar{\varepsilon}_a}{DB^2 + \bar{\varepsilon}_a} = K_{\infty} P_{NL}$$

Since $K_{\text{eff}} = 1.0$ and $K_{\infty} = 1.381$

(some for all geometries since this is a material property)

then $P_{NL} = \frac{K_{\text{eff}}}{K_{\infty}} = \frac{1}{1.381}$

$$= 0.724 \quad \text{OK} \quad \text{This makes sense...}$$

Let's again consider the fast reactor core described in HW #2.

- If the sodium-fuel-structure combination described in that problem was put into a cubical bare reactor configuration, estimate the length of one side of the cube that would be needed to give a beginning of life k_{eff} of about 1.10. Describe any assumptions.
 \approx (with no control)
- Estimate the mass of the PuO_2 fuel needed for this system. Again, describe the logic used to obtain your result.

Note: This problem deals with a 3-D system -- which is not quite consistent with the above HW title. However, the same methods can be used to solve multi-dimensional bare reactor problems as long as the correct B^2 is used. In particular, see Table 6.2 in Lamarsh for the appropriate B^2 for this system...

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$$

(a)

1-group k_{eff}

$$k_{eff} = \frac{k_{oo}}{1 + B^2 L^2}$$

we can solve this for B^2

$$1 + B^2 L^2 = \frac{k_{oo}}{k_{eff}} \quad \text{or} \quad B^2 = \frac{1}{L^2} \left(\frac{k_{oo}}{k_{eff}} - 1 \right)$$

from the given HW, we have

$$L^2 = \frac{D}{2a} = \frac{2.075}{5.173 \times 10^{-3}} = 401.1 \text{ cm}^2$$

$$k_{oo} = 1.381$$

and the desired $k_{eff} = 1.10$ (i.e. 10% excess keff of B_{oL})

$$\therefore B^2 = \frac{1}{401.1} \left(\frac{1.381}{1.10} - 1 \right) = 6.369 \times 10^{-4} \text{ cm}^{-2}$$

For a bare cubical reactor (from Table 6.2 in Lamarsh)

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = 3 \left(\frac{\pi}{a}\right)^2 \quad \text{where } a = b = c \text{ for a cube.}$$

$$\therefore a = \sqrt{\frac{3\pi^2}{B^2}} = \sqrt{\frac{3\pi^2}{6.369 \times 10^{-4}}} = \sqrt{4.649 \times 10^4}$$

$$\text{or } a = 215.6 \text{ cm}$$

but this is the extrapolated dimension...

next page

$$\therefore a_0 = a - 2d$$

$$= 215.6 - 8.84$$

$$a_0 = 206.8 \text{ cm}$$

$$\text{where } d = 2.13 \text{ D}$$

$$= 2.13(2.075)$$

$$= 4.42 \text{ cm}$$

\hookrightarrow length of one size
of critical bare
cylindrical reactor

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- (b) To compute the mass of PuO_2 needed, we proceed as follows.

1. calc core vol. $V_{\text{core}} = a_0^3 = (206.8)^3$

$$V_{\text{core}} = 8.844 \times 10^6 \text{ cm}^3$$

2. vol of fuel $V_{\text{fuel}} = 0.30 V_{\text{core}}$

$$= 2.653 \times 10^6 \text{ cm}^3$$

3. vol of PuO_2 $V_{\text{PuO}_2} = 0.15 V_{\text{fuel}}$

$$= 3.980 \times 10^5 \text{ cm}^3$$

4. mass of PuO_2 $m_{\text{PuO}_2} = \rho_{\text{PuO}_2} V_{\text{PuO}_2}$

$$= \left(10.9 \frac{\text{g}}{\text{cm}^3}\right) \left(3.980 \times 10^5 \text{ cm}^3\right)$$

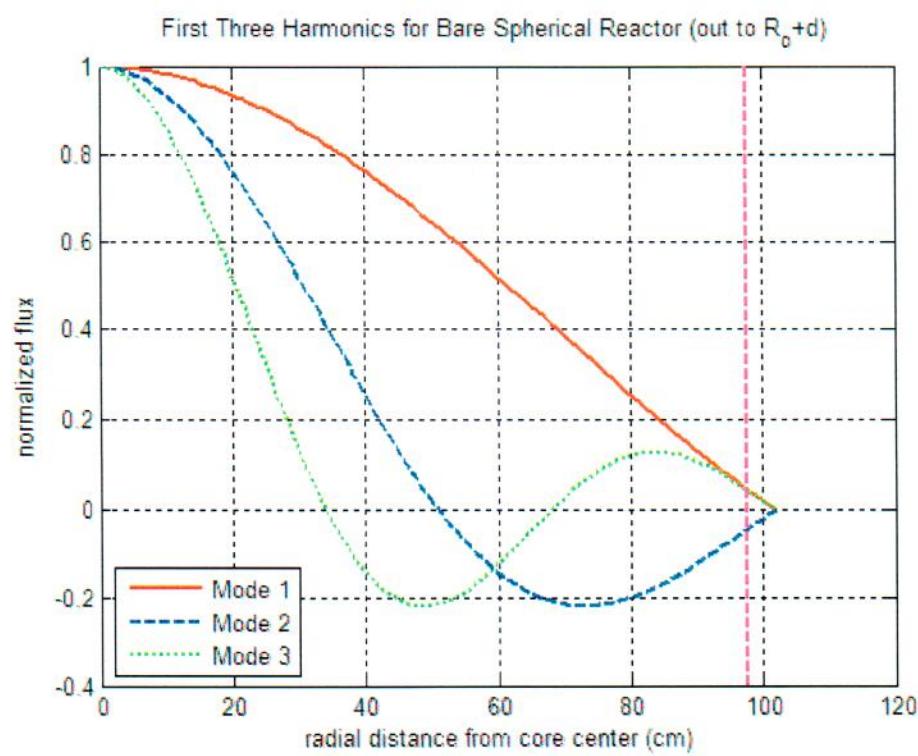
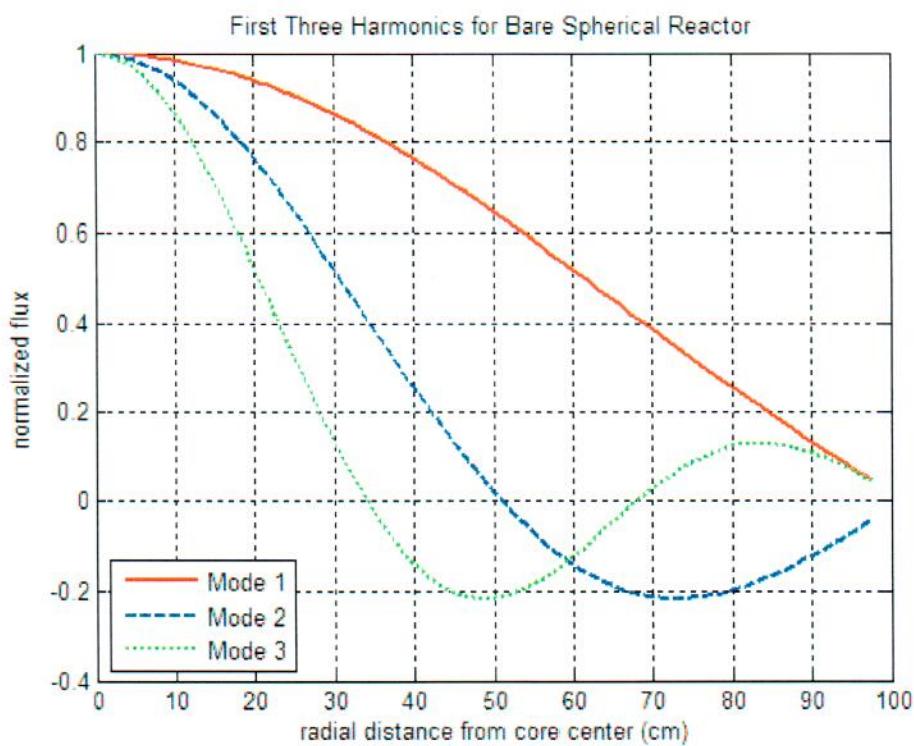
$$= 4.338 \times 10^6 \text{ g} \times \frac{1 \text{ kg}}{10^3 \text{ g}}$$

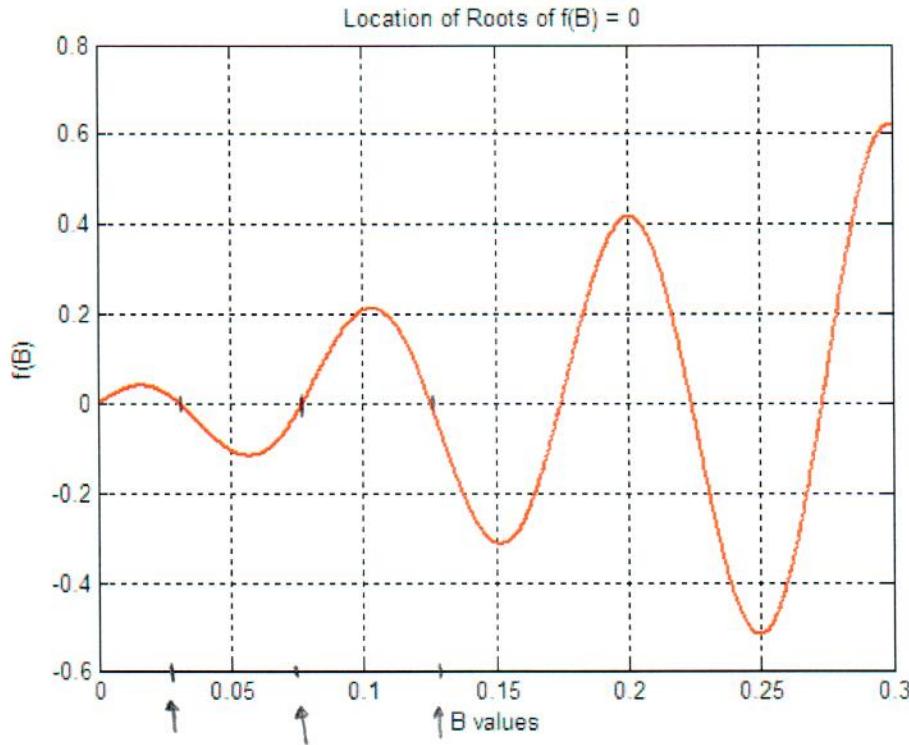
$$= 4338 \text{ kg}$$

one

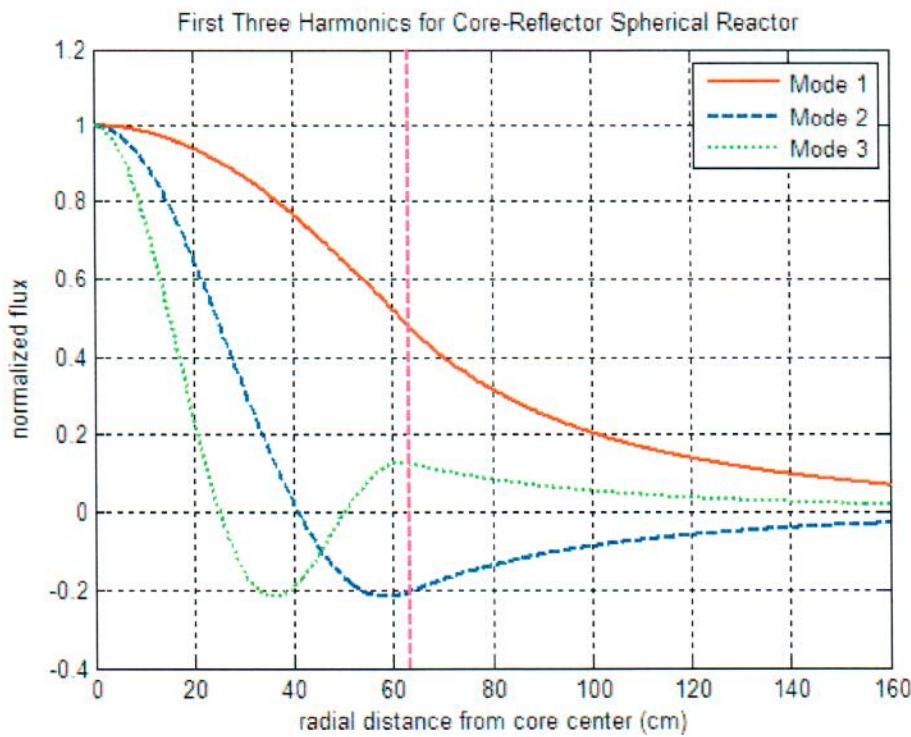
Thus, we would need approximately 4338 kg of PuO_2 to make this system crit. ab.

→ Recall that the reactor consists of 50% Na, 30% fuel, and 20% ss, where the fuel is 15% PuO_2 and 85% UO_2 (these are all volume percents...).





from fzero : $B_1 = 0.0308$ $B_3 = 0.1254$ etc. . .
 $B_2 = 0.0769$



```
% SPHERE_1G.M Plot Spatial Flux Distribution for Various Harmonics of
% the 1-Group 1-D Bare and Reflected Reactor in Spherical Geometry
%
% File prepared by J. R. White, UMass-Lowell (last update: Oct. 2016)
%

    clear all; close all; nfig = 0;

%
% Material properties (data from the core_refl1g_gui code for Fast Reactor #1)
    nusigfc = 7.145e-3; % core parameters
    sigfc = 2.421e-3;
    sigac = 5.173e-3;
    Dc = 2.075; d = 2.13*Dc;
    sigar = 2.485e-4; % reflector parameters
    Dr = 2.223;
    Lr = sqrt(Dr/sigar);

%
% Critical core size (data from the core_refl1g_gui code for Fast Reactor #1)
    R_b = 97.5; R_r = 63.4;

%
% Bare Core Case (Spherical Reactor)

%
% Determine and plot the flux profiles (first three harmonics) for bare core
    r_b = linspace(0.0001,R_b,100);
    B1_b = pi/(R_b+d); B2_b = 2*B1_b; B3_b = 3*B1_b;
    flx1_b = sin(B1_b*r_b)./(B1_b*r_b); flx2_b = sin(B2_b*r_b)./(B2_b*r_b);
    flx3_b = sin(B3_b*r_b)./(B3_b*r_b);
    nfig = nfig+1; figure(nfig)
    plot(r_b,flx1_b,'r-',r_b,flx2_b,'b--',r_b,flx3_b,'g:','LineWidth',2),grid
    title('First Three Harmonics for Bare Spherical Reactor')
    xlabel('radial distance from core center (cm)'),ylabel('normalized flux')
    legend('Mode 1','Mode 2','Mode 3','Location','SouthWest')

%
% do again but go to extrapolated distance to show that BCs match
    r_be = linspace(0.0001,R_b+d,100);
    flx1_be = sin(B1_b*r_be)./(B1_b*r_be); flx2_be = sin(B2_b*r_be)./(B2_b*r_be);
    flx3_be = sin(B3_b*r_be)./(B3_b*r_be);
    nfig = nfig+1; figure(nfig)
    plot(r_be,flx1_be,'r-',r_be,flx2_be,'b--',r_be,flx3_be,'g:','LineWidth',2),grid
    range = axis; hold on
    plot([R_b R_b],range(3:4),'m--','LineWidth',2)
    title('First Three Harmonics for Bare Spherical Reactor (out to R_o+d)')
    xlabel('radial distance from core center (cm)'),ylabel('normalized flux')
    legend('Mode 1','Mode 2','Mode 3','Location','SouthWest')
    hold off

%
% Reflected Core Case (Spherical Reactor)

%
% First plot f(B) to locate first three eigenvalues
    fB = @(B) Dc*(B.*cos(B*R_r)-sin(B*R_r)/R_r) + Dr*(1/Lr + 1/R_r)*sin(B*R_r);
    B = linspace(0.001,0.3,1000); fBB = fB(B);
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```
nfig = nfig+1; figure(nfig)
plot(B,fBB,'r-','LineWidth',2),grid
range = axis; range(2) = 0.3; axis(range);
title('Location of Roots of f(B) = 0')
xlabel('B values'),ylabel('f(B)')

%
% Note: from f(B) plot, try intervals of [0.01 0.05], [0.05 0.1], and [0.1 0.15]
r1 = linspace(0.0001,R_r,200); r2 = linspace(R_r,160,200); r_r = [r1 r2];
%
mode 1
    B1_r = fzero(fB,[0.01 0.05]);
    phi1c = sin(B1_r*r1)./(B1_r*r1);
    phi1r = sin(B1_r*R_r)/B1_r.*exp(-(r2-R_r)/Lr)./r2;
    phi1_r = [phi1c phi1r];
%
mode 2
    B2_r = fzero(fB,[0.05 0.1]);
    phi2c = sin(B2_r*r1)./(B2_r*r1);
    phi2r = sin(B2_r*R_r)/B2_r.*exp(-(r2-R_r)/Lr)./r2;
    phi2_r = [phi2c phi2r];
%
mode 3
    B3_r = fzero(fB,[0.1 0.15]);
    phi3c = sin(B3_r*r1)./(B3_r*r1);
    phi3r = sin(B3_r*R_r)/B3_r.*exp(-(r2-R_r)/Lr)./r2;
    phi3_r = [phi3c phi3r];
%
plot all three modes
    nfig = nfig+1; figure(nfig)
    plot(r_r,phi1_r,'r-',r_r,phi2_r,'b--',r_r,phi3_r,'g:','LineWidth',2),grid
    range = axis; range(3:4) = [-0.4 1.2]; axis(range); hold on
    plot([R_r R_r],range(3:4),'m--','LineWidth',2)
    title('First Three Harmonics for Core-Reflector Spherical Reactor')
    xlabel('radial distance from core center (cm)'),ylabel('normalized flux')
    legend('Mode 1','Mode 2','Mode 3','Location','NorthEast')
    hold off

%
% end of simulation
```

A bare cylindrical reactor of height 120 cm and diameter 150 cm is operating at a steady-state power of 50 MW_t. If the origin is taken at the center of the reactor, what is the power density at the point $r = 15$ cm, $z = -25$ cm?

Hint: Here you can assume that the extrapolation distance, d , is small, since no information about the core material properties is given -- thus, there is no way to accurately estimate d in this case. Note also that an explicit fission cross section is not needed since this cancels from the final expression when combining the normalization factor with the unnormalized power density term. Finally, to numerically evaluate the Bessel function expression, you should use either Matlab or some other appropriate online resource...

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The flux in a bare cylindrical reactor is given by (see Lecture Notes):

$$\phi(r, z) = A J_0 \left(\frac{2.405r}{R} \right) \cos \left(\frac{\pi z}{H} \right)$$

$$\text{where } A = \frac{3.638 P}{K \Sigma_f V}$$

$$\text{and } P(r, z) = \Sigma_f \phi(r, z)$$

$$\therefore P(r, z) = 3.638 \frac{P}{V} J_0 \left(\frac{2.405r}{R} \right) \cos \left(\frac{\pi z}{H} \right)$$

∴ all we need to do is to evaluate this expression with the data given

$$(P = 50 \text{ MW}_t)$$

$$V = \pi R^2 H = \pi (75 \text{ cm}^2)(120 \text{ cm})$$

$$V = 2.1206 \times 10^6 \text{ cm}^3$$

$$J_0 \left(\frac{(2.405)(15)}{75} \right) = J_0 (0.4810) = 0.9430$$

from
matlab

$$\cos \left(-\frac{25\pi}{120} \right) = \cos (0.6545) = 0.7934$$

radians

$$\therefore P(15, -25) = \frac{(3.638)(50 \times 10^6 \text{ W}_t)}{2.1206 \times 10^6 \text{ cm}^3} (0.9430)(0.7934)$$

$$= 64.2 \frac{\text{W}}{\text{cm}^3}$$

ans

Core Reflector model

Core Region

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \phi_c \right) + B^2 \phi_c = 0 \quad B^2 = \gamma \sqrt{\epsilon_f - \epsilon_a}$$

$$\phi_c(r) = A_1 J_0(Br) + A_2 Y_0(Br)$$

but $Y_0(Br) \rightarrow -\infty$ as $r \rightarrow 0$, thus $A_2 = 0$. which gives

$$\phi_c(r) = A_1 J_0(Br)$$

Reflector Region

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \phi_r \right) - \frac{1}{L_r} \phi_r = 0$$

$$r^2 \frac{d^2 \phi_r}{dr^2} + r \frac{d}{dr} \phi_r - r^2 \left(\frac{1}{L_r} \right) \phi_r = 0$$

This is a modified Bessel eqn of order $\nu = 0$. Thus, the general solution is given by

$$\phi_r(r) = A_3 I_0 \left(\frac{r}{L_r} \right) + A_4 K_0 \left(\frac{r}{L_r} \right)$$

but $I_0(r/L_r) \rightarrow \infty$ as $r \rightarrow \infty$, thus $A_3 = 0$ gives

$$\phi_r(r) = A_4 K_0 \left(\frac{r}{L_r} \right)$$

At this point, we force continuity of flux and current at the core-reflector interface ($r = R_0$),

$$\phi_c(r)|_{r=R_0} = \phi_r(r)|_{r=R_0} \quad \leftarrow \text{continuity of flux}$$

$$A_1 J_0(BR_0) - A_4 K_0(R_0/L_r) = 0$$

$$\text{and } J_c(r)|_{r=R_0} = J_r(r)|_{r=R_0} \quad \leftarrow \text{continuity of current}$$

$$\text{but } \frac{d}{dr} J_0(Br) = -B J_1(Br)$$

$$\frac{d}{dr} K_0(r/L_r) = -\frac{1}{L_r} K_1(r/L_r)$$

thus

$$-D_c (-BA_1 J_1(BR_0)) + D_r \left(-\frac{A_4}{L_r} K_1(R_0/L_r) \right) = 0$$

$$\text{or } D_c B A_1 J_1(BR_0) - \frac{D_r}{L_r} A_4 K_1(R_0/L_r) = 0$$

putting these last two eqns in matrix form gives

$$\begin{bmatrix} J_0(BR_0) & -k_0(R_0/L_r) \\ D_c B J_1(BR_0) & -\frac{D_r}{L_r} k_1(R_0/L_r) \end{bmatrix} \begin{bmatrix} A_1 \\ A_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For a non-trivial soln, the determinant of the coeff matrix must be zero. Applying this criticality condition gives

$$-J_0(BR_0) \frac{D_r}{L_r} k_1(R_0/L_r) + k_0(R_0/L_r) D_c B J_1(BR_0) = 0$$

or $f(B) = D_c B J_1(BR_0) k_0(R_0/L_r) - \frac{D_r}{L_r} J_0(BR_0) k_1(R_0/L_r)$

$$f(B) = 0$$

This is a classical root finding problem, "what is the value of B such that $f(B) = 0$ "

There will be an infinite number of roots, where the lowest value of B leads to the only non-negative $\phi_c(r)$ profile. \leftarrow fundamental mode solution

$\left. \begin{array}{l} \text{given core size } R_0 \\ \text{and material composition} \\ D_c, D_r \text{ and } L_r \end{array} \right\}$

Once B is known, we can write the formal flux solution as

$$\phi_c(r) = A_1 J_0(Br) \quad 0 \leq r \leq R_0$$

$$\phi_r(r) = A_1 \frac{J_0(BR_0)}{k_0(R_0/L_r)} k_0(r/L_r) \quad r \geq R_0$$

where we obtained the relationships for A_1 in terms of A_4 from the continuity of flux expression.

Now since there is no power production in the reflector, the normalization factor A_1 is determined exactly as for the bare cylindrical reactor, with the same result

$$A_1 = \frac{P_B}{2\pi k_{ef} R_0 J_1(BR_0)}$$

where the buckling is, of course, associated with the curv