

Consider an oxide fueled fast reactor core consisting of 50% sodium, 30% fuel, and 20% stainless steel by volume. The fuel material consists of 15 % PuO<sub>2</sub> and 85 % UO<sub>2</sub> by volume. The spectrum-averaged 1-group microscopic cross sections (in units of barns) are tabulated below:

isotope	average neutrons per fission, $\bar{v}$	fission cross section $\sigma_f$	absorption cross section $\sigma_a$	transport cross section $\sigma_{tr}$
U238	2.6	0.05	0.404	8.2
Pu239	3.0	1.95	2.4	8.6
Na	--	--	0.0018	3.7
Fe	--	--	0.0087	3.6

Assuming that all the plutonium in the PuO<sub>2</sub> is Pu239, the uranium in UO<sub>2</sub> is U238, the stainless steel is predominantly iron, and the cross sections for oxygen are negligible, compute the following core-averaged macroscopic cross sections and the extrapolation distance:  $v\Sigma_f$ ,  $\Sigma_f$ ,  $\Sigma_a$ , D and d = 2.13D.

The required material density information is as follows:

$$\text{UO}_2 \rightarrow 10.4 \text{ g/cm}^3 \quad \text{PuO}_2 \rightarrow 10.9 \text{ g/cm}^3 \quad \text{Na} \rightarrow 0.87 \text{ g/cm}^3 \quad \text{Fe} \rightarrow 7.80 \text{ g/cm}^3$$

atom densities

$$N_{U238} = \left( \frac{10.4 \text{ g UO}_2}{\text{cm}^3 \text{ of UO}_2} \right) \left( \frac{238 \text{ g of U}}{270 \text{ g of UO}_2} \right) \left( \frac{0.85 \text{ cm}^3 \text{ of UO}_2}{\text{cm}^3 \text{ of fuel}} \right) \times \\ \left( \frac{0.30 \text{ cm}^3 \text{ of fuel}}{\text{cm}^3 \text{ of core}} \right) \left( \frac{0.6022 \times 10^{-24} \text{ at } U238}{238 \text{ g of U238}} \right) \left( \frac{10^{-24} \text{ cm}^2}{\text{b}} \right)$$

$$\therefore N_{U238} = (10.4) \left( \frac{238}{270} \right) (0.85) (0.30) \left( \frac{0.6022}{238} \right) = 5.915 \times 10^{-3} \frac{\text{at}}{\text{b-cm}}$$

$$N_{Pu239} = (10.9) \left( \frac{239}{271} \right) (0.15) (0.30) \left( \frac{0.6022}{239} \right) = 1.090 \times 10^{-3} \frac{\text{at}}{\text{b-cm}}$$

$$N_{Na} = \left( \frac{0.87 \text{ g Na}}{\text{cm}^3 \text{ of Na}} \right) \left( \frac{0.50 \text{ cm}^3 \text{ of Na}}{\text{cm}^3 \text{ of core}} \right) \left( \frac{0.6022}{23} \right) = 1.139 \times 10^{-2} \frac{\text{at}}{\text{b-cm}}$$

$$N_{Fe} = (7.8) (0.20) \left( \frac{0.6022}{55.8} \right) = 1.684 \times 10^{-2} \frac{\text{at}}{\text{b-cm}}$$

These are homogenized over the reactor core

### Mono Cross Sections

$$\begin{aligned}\sqrt{\Sigma_f} &= N_U \sqrt{\sigma_{f,U}} + N_{P_U} \sqrt{\sigma_{f,P_U}} \\ &= (5.915 \times 10^{-3})(2.6)(0.05) + (1.090 \times 10^{-3})(3.0)(1.95) \\ &= 7.690 \times 10^{-4} + 6.377 \times 10^{-3} \\ &= \boxed{7.145 \times 10^{-3} \text{ cm}^{-1}}\end{aligned}$$

$$\begin{aligned}\Sigma_f &= N_U \sigma_{f,U} + N_{P_U} \sigma_{f,P_U} \\ &= (5.915 \times 10^{-3})(0.05) + (1.090 \times 10^{-3})(1.95) \\ &= 2.958 \times 10^{-4} + 2.126 \times 10^{-3} \\ &= \boxed{2.421 \times 10^{-3} \text{ cm}^{-1}}\end{aligned}$$

$$\begin{aligned}\Sigma_a &= N_U \sigma_{a,U} + N_{P_U} \sigma_{a,P_U} + N_{Na} \sigma_{a,Na} + N_{Fe} \sigma_{a,Fe} \\ &= (5.915 \times 10^{-3})(0.404) + (1.090 \times 10^{-3})(2.4) \\ &\quad + (1.139 \times 10^{-2})(0.0018) + (1.684 \times 10^{-2})(0.0087) \\ &= \boxed{5.173 \times 10^{-3} \text{ cm}^{-1}}\end{aligned}$$

(Note)

$$\begin{aligned}\Sigma_a^F &= N_U \sigma_{a,U} + N_{P_U} \sigma_{a,P_U} \\ &= 5.006 \times 10^{-3} \text{ cm}^{-1}\end{aligned}$$

$$\begin{aligned}\Sigma_{tr} &= N_U \sigma_{tr,U} + N_{P_U} \sigma_{tr,P_U} + N_{Na} \sigma_{tr,Na} + N_{Fe} \sigma_{tr,Fe} \\ &= (5.915 \times 10^{-3})(8.2) + (1.090 \times 10^{-3})(8.6) \\ &\quad + (1.139 \times 10^{-2})(3.7) + (1.684 \times 10^{-2})(3.6) \\ &= \boxed{1.606 \times 10^{-1} \text{ cm}^{-1}}\end{aligned}$$

$$D = \frac{1}{3 \Sigma_{tr}} = \frac{1}{3(0.1606)} = \boxed{2.075 \text{ cm}}$$

$$L_{core}^2 = \frac{D}{\Sigma_a} = \frac{2.075}{5.173 \times 10^{-3}} = \boxed{401.1 \text{ cm}^2}$$

$$L_{core} = \boxed{20.0 \text{ cm}}$$

$$d = 2.13 D = 2.13(2.075) = \boxed{4.42 \text{ cm}}$$

Data for  
Design Calculations

For reaction #1  
(reflector)

The core region in Prob. 1 is surrounded by a reflector composed of 30% stainless steel and 70% liquid sodium by volume. Using the data given above, compute the macroscopic cross sections for a homogeneous mixture of this material. What is the diffusion length for this region?

atom densities

$$N_{Na} = (0.87)(0.70)\left(\frac{0.6022}{23}\right) = \boxed{1.595 \times 10^{-2} \frac{\text{at}}{\text{b-cm}}}$$

$$N_{Fe} = (7.80)(0.30)\left(\frac{0.6022}{55.8}\right) = \boxed{2.525 \times 10^{-2} \frac{\text{at}}{\text{b-cm}}}$$

$$\begin{aligned} \Sigma_a &= N_{Na} \bar{\sigma}_{a,Na} + N_{Fe} \bar{\sigma}_{a,Fe} \\ &= (1.595 \times 10^{-2})(0.0018) + (2.525 \times 10^{-2})(0.0087) \\ &= \boxed{2.485 \times 10^{-4} \text{ cm}^{-1}} \end{aligned}$$

$$\begin{aligned} \Sigma_{tr} &= N_{Na} \bar{\sigma}_{tr,Na} + N_{Fe} \bar{\sigma}_{tr,Fe} \\ &= (1.595 \times 10^{-2})(3.7) + (2.525 \times 10^{-2})(3.6) \\ &= \boxed{1.499 \times 10^{-1} \text{ cm}^{-1}} \end{aligned}$$

$$D = \frac{1}{3 \Sigma_{tr}} = \frac{1}{3(0.1499)} = \boxed{2.223 \text{ cm}}$$

$$L^2 = \frac{D}{\Sigma_a} = \frac{2.223}{2.485 \times 10^{-4}} = \boxed{8946 \text{ cm}^2}$$

$$L = \sqrt{L^2} = \boxed{94.6 \text{ cm}}$$

For the core material described in the above problem, compute the following quantities:

- The fuel utilization,  $f$ , which is the ratio of absorptions in the fuel to total absorptions in the core.
- The value of  $\eta$ , which is the average number of neutrons emitted per absorption in the fuel.
- The infinite multiplication factor,  $k_{\infty}$ , for the core material.

(a) Fuel utilization

$$f = \frac{\Sigma_a^F}{\Sigma_a} = \frac{\Sigma_a^U + \Sigma_a^{Pu}}{\Sigma_a}$$

$$= \frac{5.006 \times 10^{-3}}{5.173 \times 10^{-3}} = 0.968$$

data from  
previous problem

(b)  $\eta$

$$\eta = \frac{\sqrt{\Sigma_F}}{\Sigma_a} = \frac{7.145 \times 10^{-3}}{5.006 \times 10^{-3}} = 1.427$$

(c)  $k_{\infty}$

$$k_{\infty} = \frac{\sqrt{\Sigma_F}}{\Sigma_a} = \frac{7.145 \times 10^{-3}}{5.173 \times 10^{-3}} = 1.381$$

also

$$k_{\infty} = \eta f = (1.427)(0.968) = 1.381$$

OK

Calculate the Thermal diffusion coeff and diffusion length of water near the outlet of a PWR — That is, at about  $300^{\circ}\text{C}$  and density of  $0.68 \text{ g/cm}^3$ .  
Also calc neutron age,  $\tau_T$ .

Lamarche  $\rightarrow$  from table 5.2 at  $\rho = 1 \text{ g/cc}$   $\bar{D} = 0.16 \text{ cm}$   $L_T^2 = 8.1 \text{ cm}^2$   
for water  $T = 20^{\circ}\text{C}$

*AMPA'D*

$$\bar{D}(\rho, T) = \bar{D}(\rho_0, T) \left( \frac{\rho_0}{\rho} \right) \left( \frac{T}{T_0} \right)^m \quad \text{where } m = 0.470 \text{ for H}_2\text{O}$$

$$= (0.16 \text{ cm}) \left( \frac{1.0}{0.68} \right) \left( \frac{573}{293} \right)^{0.47} \quad \left. \begin{array}{l} \text{use absolute} \\ \text{temp. scale} \end{array} \right\}$$

$$= (0.16 \text{ cm})(2.016) \quad \approx \boxed{0.32 \text{ cm}}$$

$$L_T^2(\rho, T) = \frac{\bar{D}(\rho, T)}{\bar{\varepsilon}_o(\rho, T)} = L_T^2(\rho_0, T_0) \left( \frac{\rho_0}{\rho} \right)^2 \left( \frac{T}{T_0} \right)^{m+\frac{1}{2}}$$

$$= (8.1 \text{ cm}^2) \left( \frac{1.0}{0.68} \right)^2 \left( \frac{573}{293} \right)^{0.97}$$

$$= (8.1 \text{ cm}^2)(4.15) \quad \approx \boxed{33.6 \text{ cm}^2}$$

or  $\boxed{L_T = 5.79 \text{ cm}}$

from table 5.3 at  $\rho = 1 \text{ g/cc}$   $\tau_T = 27 \text{ cm}^2$   
 $T = 20^{\circ}\text{C}$

$$\tau_T(\rho) = \frac{\tau_T(\rho)}{\varepsilon_{1-\gamma_2}(\rho)} = \tau_T(\rho_0) \left( \frac{\rho_0}{\rho} \right)^2 \quad \left. \begin{array}{l} \text{insensitive} \\ \text{to temperature} \\ \text{changes} \end{array} \right\}$$

$$= (27 \text{ cm}^2) \left( \frac{1.0}{0.68} \right)^2$$

$$= (27 \text{ cm}^2)(2.16) = \boxed{58.4 \text{ cm}^2}$$

$\rightarrow$  checked with cross-sections - qui  $\text{OK}$

Calculate the thermal diffusion length at room temperature of water solutions of boric acid ( $H_3BO_3$ ) at the following concentrations.

(a) 10 g/liter    (b) 1 g/liter    (c) 0.1 g/liter

Note: Because of the small concentration of the boric acid, the diffusion coeff. for the mixture is essentially the same as pure water.

$$L_T^2 = \frac{\bar{D}}{\bar{\Sigma}_a} = \frac{\bar{D}_{H_2O}}{\bar{\Sigma}_{a\text{ mix}}} = \boxed{\frac{\bar{D}_{H_2O}}{\bar{\Sigma}_{a\text{ H}_2O} + \bar{\Sigma}_{a\text{ }H_3BO_3}}}$$

Lamarsch → from Table 5.2 at room temp.

$$\boxed{\bar{D}_{H_2O} = 0.16 \text{ cm}} \quad \boxed{\bar{\Sigma}_{a\text{ H}_2O} = 0.0197 \text{ cm}^{-1}} \quad L_T = 2.85 \text{ cm}$$

pure  $H_2O$

thus, for the different concentrations, we need to compute  $\bar{\Sigma}_{a\text{ }H_3BO_3}$

from Table II.3 2200 n/s cross section

$$\sigma_{a_H} = 0.3326 \quad \sigma_{a_O} = 0.000276$$

Note JENDL has  
 $\sigma_{a_B} = 764.6$   
used this →

$$\boxed{\sigma_{a_B} = 759 \text{ b}}$$

(thus, the hydrogen & oxygen make a negligible contribution)

$$\therefore \bar{\Sigma}_{a\text{ }H_3BO_3} = \frac{\sqrt{\pi}}{2} N_B \sigma_{a_B} (\varepsilon_0)$$

$$\begin{aligned} M_{H_3BO_3} &= 3 + 10.8 + 3(16) \\ &= 61.8 \text{ g/g mole} \end{aligned}$$

$$N_B = \frac{X \text{ g of } H_3BO_3}{1000 \text{ cm}^3} \times \frac{10.8 \text{ g of B}}{61.8 \text{ g of } H_3BO_3} \times \frac{0.6022 \text{ at } 16 \text{ B}}{10.8 \text{ g of B}} \frac{\text{cm}^2}{\text{b}}$$

$$\boxed{N_B = \frac{0.6022 X}{61.8 \times 10^3} \text{ at } 16 \text{-cm}}$$

where  $X$  = concentration in  
g/liter

### Table of Results

Conc	$X \left(\frac{\text{g}}{\text{liter}}\right)$	$N_B \left(\frac{\text{at}}{\text{b} \cdot \text{cm}}\right)$	$\bar{\Sigma}_{aB} \left(\text{cm}^{-1}\right)$	$\bar{\Sigma}_{a\text{ mix}} \left(\text{cm}^{-1}\right)$	$L_T^2 \left(\text{cm}^2\right)$	$L_T \text{ (cm)}$
a	10	$9.75 \times 10^{-5}$	$6.56 \times 10^{-2}$	0.0253	1.88	1.37
b	1	$9.75 \times 10^{-6}$	$6.56 \times 10^{-3}$	0.0263	6.08	2.47
c	0.1	$9.75 \times 10^{-7}$	$6.56 \times 10^{-4}$	0.0204	7.84	2.80

Ans

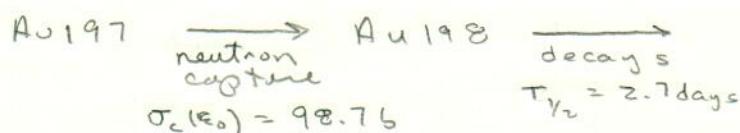
The thermal (0.0253 eV) capture cross section for Au-197 is about 98.7 b. If a thin 0.12 g gold foil is placed in a thermal system where the temperature is about 150 °C and the thermal flux is  $\phi_T = 2.0 \times 10^{13}$  neutrons/cm<sup>2</sup>-sec, compute the following quantities:

- the thermal absorption rate in Au-197
- the activity of Au-198 after a 3-hour irradiation

Note that Au-197 has an isotopic abundance of 100 a/o and that Au-198 has a 2.7 day half-life and a negligible thermal absorption cross section. Explain the solution logic and clearly state any assumptions needed to solve this problem.

**Note:** Gold actually has fairly significant resonance cross sections so, if we only use the thermal reaction rate, the above Au-198 activity computations will be under-predicted. However, here we will ignore the production of Au-198 by capture in Au-197 in the epithermal and fast energy ranges by arguing that the flux above about 1 eV is low compared to the thermal flux. Thus, here we are assuming that the total capture rate per second in Au-197 can be computed by considering only thermal neutrons below about 1 eV.

The reaction of interest here is



see derivation  
in lecture notes  
Lamarsh chapt 2  
Shultz + Faw chapt 5

Assuming that the Au-197 remains essentially constant, the activity of Au-198 is given by

$$A(t) = R (1 - e^{-\lambda t}) \quad \text{assumes } A_0 = 0$$

where  $t$  = irradiation time

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(2.7 \text{ days})(\frac{24 \text{ hr}}{\text{day}})} = 0.010697 \text{ hr}^{-1}$$

capture rate  
in Au-197

$$R = N \bar{\sigma}_c \phi_T V = \sum_c \phi_T V \quad \begin{matrix} \leftarrow \text{vol of foil} \\ \leftarrow \text{reactions/sec} \end{matrix}$$

$$\text{with } \bar{\sigma}_c = \frac{\sqrt{\pi}}{2} q_c(\tau) \sigma_c(\epsilon_0) \left(\frac{T_0}{T}\right)^{1/2}$$

$$\rightarrow \frac{\sqrt{\pi}}{2} (1.00) (98.7 \text{ b}) \left(\frac{293}{423}\right)^{1/2}$$

$\leftarrow$  pure  $\frac{1}{2}$   
absorber

$\leftarrow$  abs temps

$$\boxed{\bar{\sigma}_c = 72.8 \text{ b}}$$

$\leftarrow$  average capture xsc  
for Au-197 over thermal region

$$N_V = \frac{m_{Au} N_A}{M_{Au}}$$

where  $m_{Au} = \rho V$  = mass of Au197  
 $N_A = 0.6022 \times 10^{24}$  at/gmole  
 $M_{Au} = 197$  g/gmole

$$= \frac{(0.12g)(0.6022 \times 10^{24} \text{ at/gmole})}{197 \text{ g/gmole}}$$

$$N_V = 3.668 \times 10^{20} \text{ atoms of Au197}$$

$$\therefore R = (N_V) \bar{\sigma}_c \phi_T$$

$$= (3.668 \times 10^{20} \text{ atoms}) (72.8 \times 10^{-24} \text{ cm}^2) \left( 2 \times 10^3 \frac{\text{neuts}}{\text{cm}^2 \cdot \text{s}} \right)$$

$$R = 5.341 \times 10^{11} \frac{\text{captures}}{\text{sec}}$$

ans to  
Part a

Now, the activity of Au-198 after a 3-hr irradiation is given as

$$A_{3\text{hr}} = \left( 5.341 \times 10^{11} \frac{\text{decays}}{\text{sec}} \right) \left( 1 - e^{-(0.010697)(3)} \right)$$

$$= (5.341 \times 10^{11}) (0.03158)$$

$$= 1.687 \times 10^{10} \text{ dps} + \frac{-1 \text{ Ci}}{3.7 \times 10^{16} \text{ dps}}$$

$$A_{3\text{hr}} = 0.456 \text{ Ci}$$

ans to  
Part b