Sphere mm

Note that 1 ft of moderating material is approximately 30 cm. Using the Matlab ***spheremm_gui*** program we want to explore neutron diffusion in two different moderating materials, water and graphite, in spherical geometry. For consistency, select R = 30 cm and generate/record the flux plots and balance table information for these two cases (explore both the linear and logarithmic plot scales). Make sure you understand what this code is doing…

Based on the data generated, answer the following questions:

1. What is the appropriate thermal diffusion length for water? How about graphite? What is the physical interpretation of the diffusion length, L? Based on this, what can you say about the difference in thermal neutron diffusion in water versus graphite?

$\boxed{L = 2.8 \text{ cm for water}}$ and $\boxed{L = 59.2 \text{ cm for graphite}}$

$L = \sqrt{L^2}$ and $L^2 = \frac{1}{6}\bar{r}^2$ $\quad L^2$ = diffusion area, L = diffusion length

where $\bar{r}^2$ is the average of the square of the crow flight distance from birth to death of a neutron.

Thus, L is related to the average distance travelled between birth and death.

Since $L_{graphite} >> L_{water}$, neutrons will travel farther in graphite than in water

Note that $L^2 = D/\Sigma_a$. Thus, the above result is primarily due to the fact that the thermal absorption cross section, Σ_a, in water is much larger than in graphite

See table 5.2 in Lamarsh

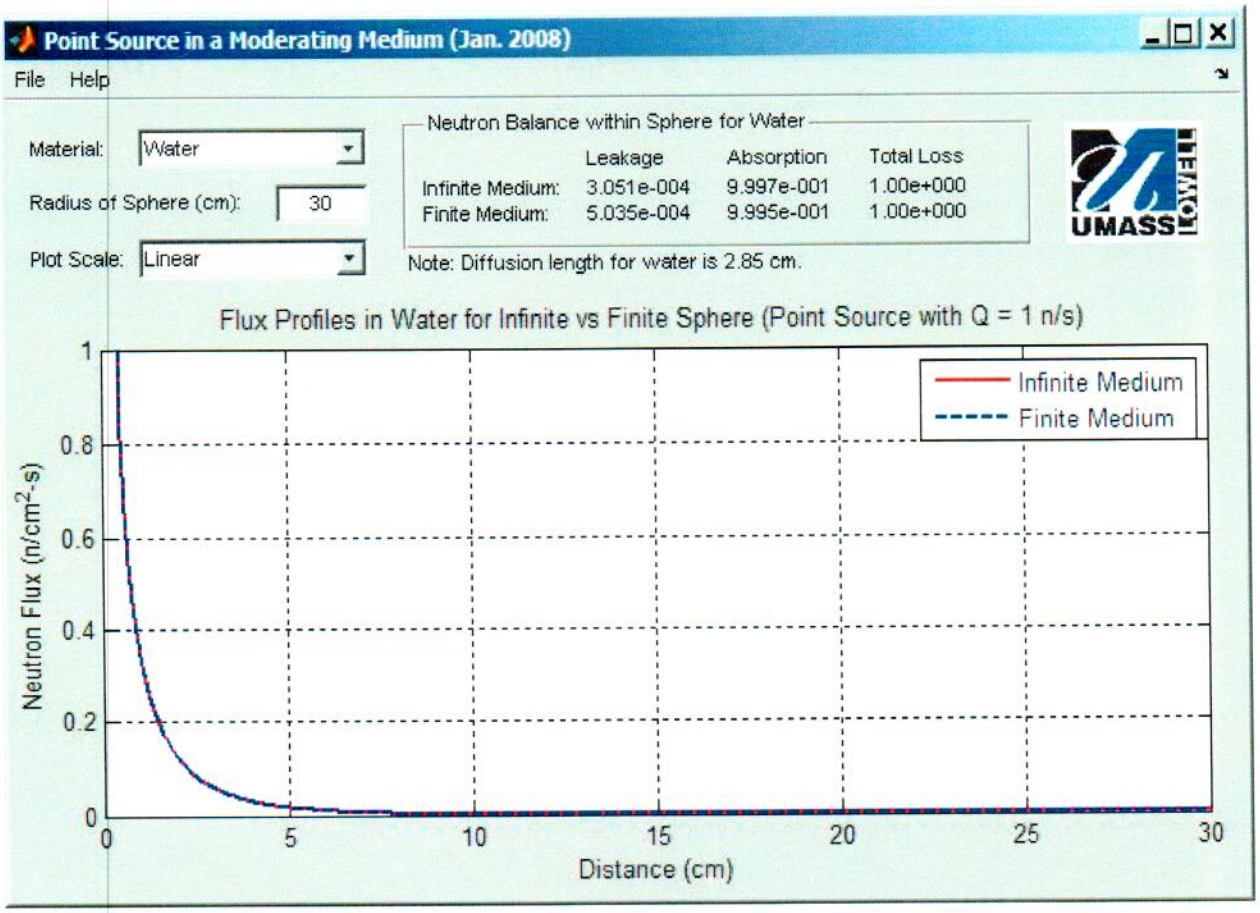
$\bar{\Sigma}_{a\,water} = 0.0197\,b$ vs $\bar{\Sigma}_{a\,graphite} = 0.00024\,b$

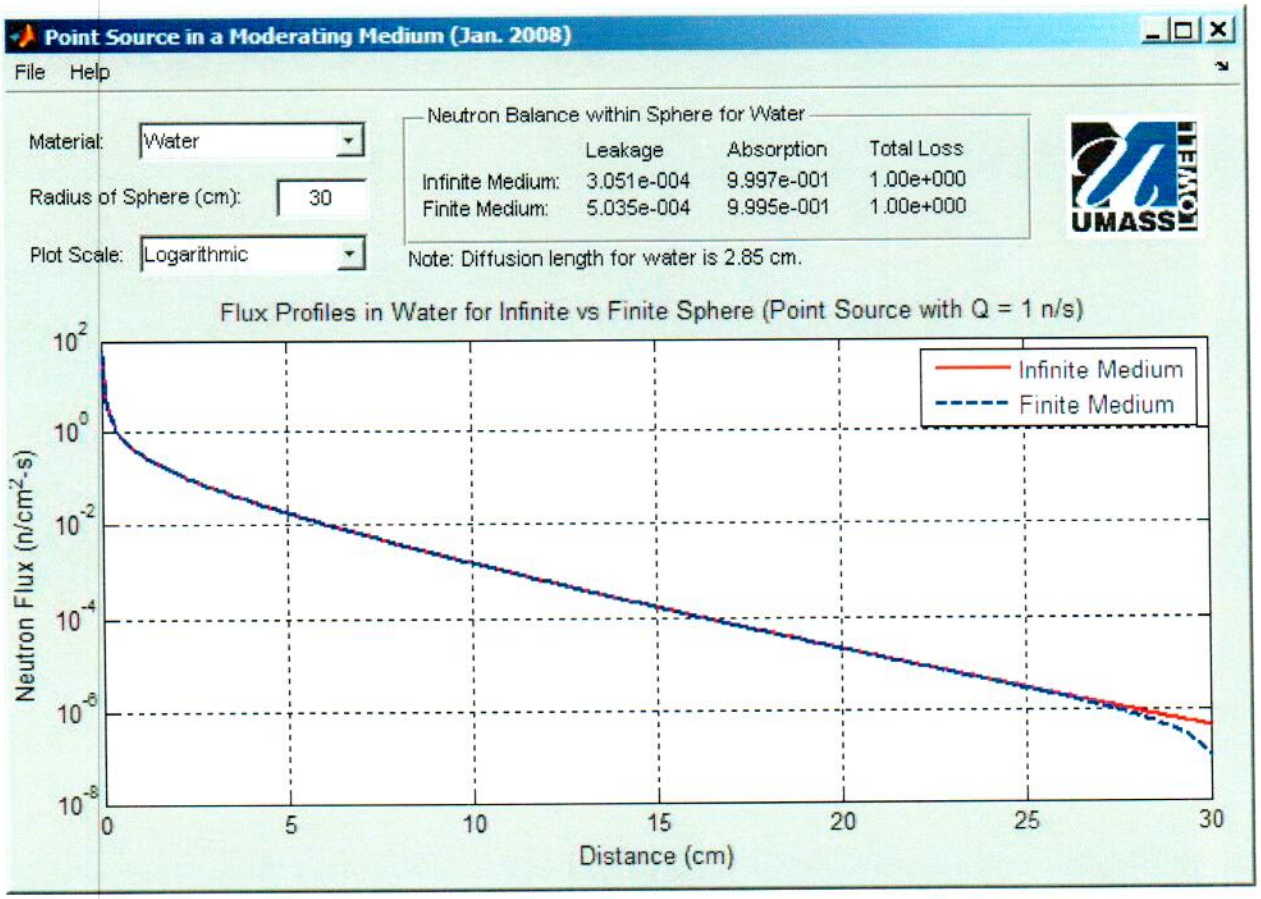
about a factor of 80 different

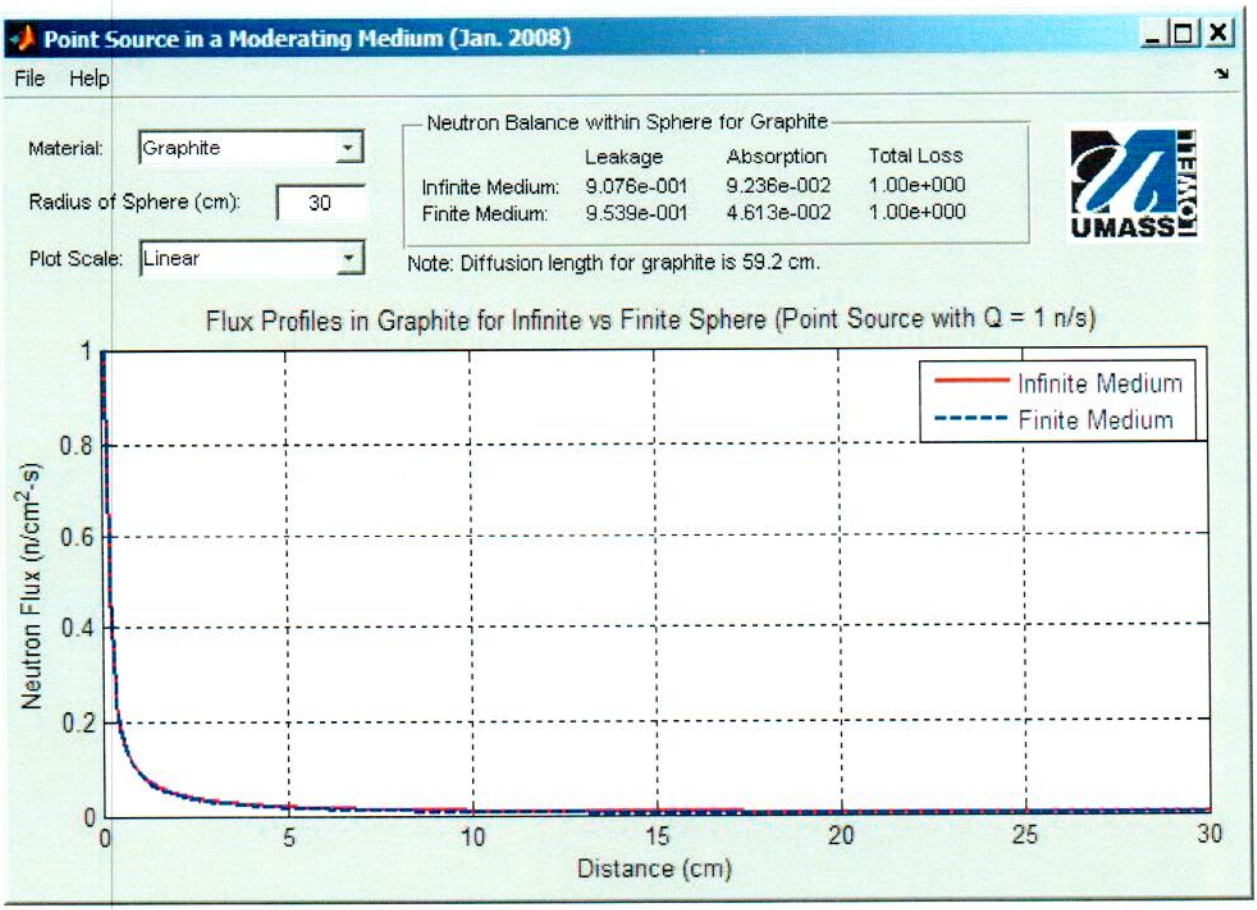
but $\bar{D}_{water} = 0.16$ cm vs $\bar{D}_{graphite} = 0.84$

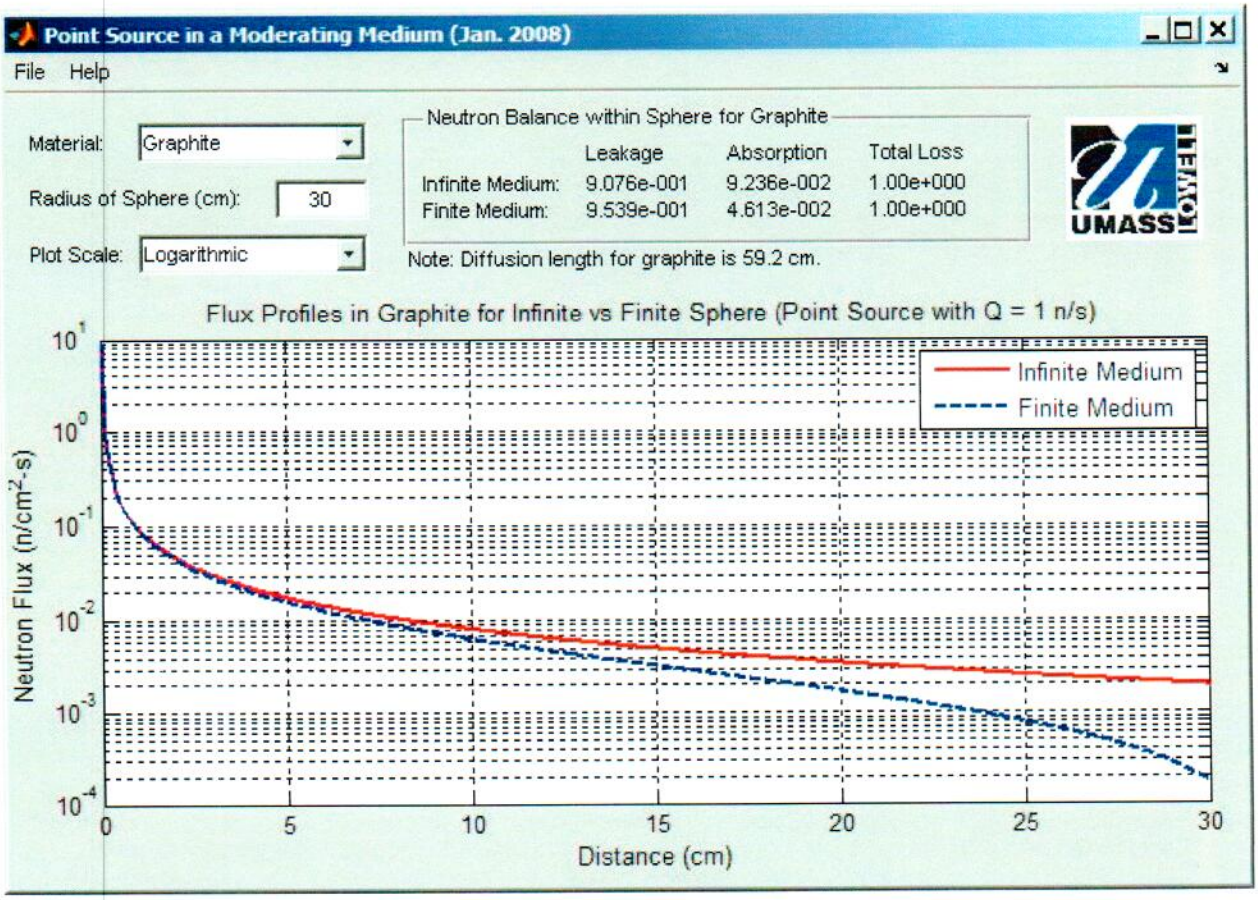
only about a factor of 5 different

$\therefore$ Σ_a is the dominant factor...


Point Source in a Moderating Medium (Jan. 2008)
File Help
Material: Water
Radius of Sphere (cm): 30
Plot Scale: Linear
Neutron Balance within Sphere for Water
Leakage Absorption Total Loss
Infinite Medium: 3.051e-004 9.997e-001 1.00e+000
Finite Medium: 5.035e-004 9.995e-001 1.00e+000
Note: Diffusion length for water is 2.85 cm.
UMASS LOWELL
Flux Profiles in Water for Infinite vs Finite Sphere (Point Source with Q = 1 n/s)
Infinite Medium
Finite Medium
Neutron Flux (n/cm²-s)
Distance (cm)


Point Source in a Moderating Medium (Jan. 2008)
File Help
Material: Water
Radius of Sphere (cm): 30
Plot Scale: Logarithmic
Neutron Balance within Sphere for Water
Leakage Absorption Total Loss
Infinite Medium: 3.051e-004 9.997e-001 1.00e+000
Finite Medium: 5.035e-004 9.995e-001 1.00e+000
Note: Diffusion length for water is 2.85 cm.
UMASS LOWELL
Flux Profiles in Water for Infinite vs Finite Sphere (Point Source with Q = 1 n/s)
Infinite Medium
Finite Medium
Neutron Flux (n/cm²-s)
Distance (cm)


Point Source in a Moderating Medium (Jan. 2008)
File Help
Material: Graphite
Radius of Sphere (cm): 30
Plot Scale: Linear
Neutron Balance within Sphere for Graphite
Leakage Absorption Total Loss
Infinite Medium: 9.076e-001 9.236e-002 1.00e+000
Finite Medium: 9.539e-001 4.613e-002 1.00e+000
Note: Diffusion length for graphite is 59.2 cm.
UMASS LOWELL
Flux Profiles in Graphite for Infinite vs Finite Sphere (Point Source with Q = 1 n/s)
Infinite Medium
Finite Medium
Neutron Flux (n/cm2-s)
Distance (cm)


Point Source in a Moderating Medium (Jan. 2008)
File Help
Material: Graphite
Radius of Sphere (cm): 30
Plot Scale: Logarithmic
Neutron Balance within Sphere for Graphite
Leakage Absorption Total Loss
Infinite Medium: 9.076e-001 9.236e-002 1.00e+000
Finite Medium: 9.539e-001 4.613e-002 1.00e+000
Note: Diffusion length for graphite is 59.2 cm.
UMASS LOWELL
Flux Profiles in Graphite for Infinite vs Finite Sphere (Point Source with Q = 1 n/s)
Infinite Medium
Finite Medium
Neutron Flux (n/cm2-s)
Distance (cm)

2. Using the ***spheremm_gui*** simulation tool, find a value of R where leakage accounts for about 25% of the overall neutron balance for the case of the infinite media problem (i.e. 25% of the neutron's emitted into the infinite sphere pass the r = R location). Do this for both water and graphite. Now convert these values into number of diffusion lengths by dividing by the appropriate value of L (i.e., the number of diffusion lengths from the source is simply distance/diffusion length). Compare this result for the water and graphite media for the case where the leakage is 25% of the overall neutron balance. Did you expect this result? Explain...

→ for 25% leakage

for water $R = 7.664$ cm

and # diffusion lengths $= \frac{7.664}{2.85} = 2.689$

these are the same

for graphite $R = 159.3$

and # diffusion lengths $= \frac{159.3}{59.2} = 2.691$

Note also that (from spheremm report)

leakage $= Q\left(\frac{R}{L}+1\right)e^{-R/L}$

but for $Q = 1$ n/s and $\frac{R}{L} = 2.690$ ← # of diffusion lengths

we have leakage $= (1)(3.690)\,e^{-2.690}$

$= 0.250$ as requested

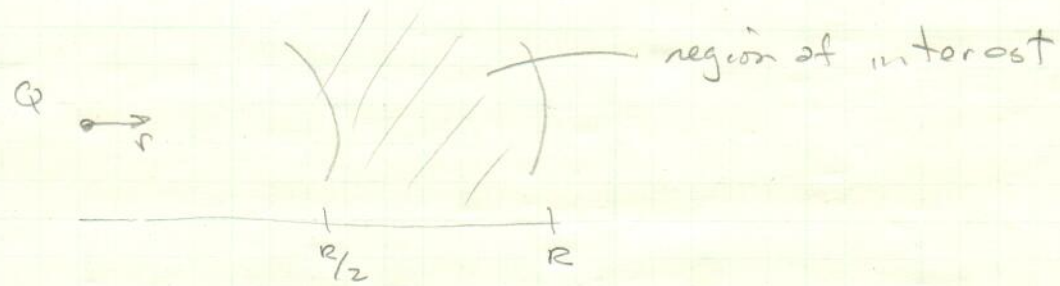
Thus, when the distance is expressed in terms of the number of diffusion lengths within the material, we should indeed expect the results given here...

The flux in an infinite moderating medium with an isotropic point source with strength Q at the origin of the system (i.e. at r = 0) was formally derived in the Lecture Notes and is given by

$$\phi(r) = \frac{Q}{4\pi D}\frac{1}{r}e^{-r/L}$$

Formally demonstrate that the neutron balance in a region defined by $R/2 \le r \le R$ is satisfied -- that is, show that **leakage + absorption = source** within the region of interest, where R is simply some given radius. Show all your work…

Hint: In doing this formal balance, note that, for the case of a point source at r = 0, there is no source in the spherical shell region between R/2 and R.



Balance: leakage at R/2 + leakage at R + absorption within = 0

since no source in region

$$\text{leakage at } R/2 = \int_A \vec{J}\cdot\hat{n}\,dA = -J\left(\frac{R}{2}\right)4\pi\left(\frac{R}{2}\right)^2$$

$-\hat{a}_r$

$$\text{leakage at } R = \int_A \vec{J}\cdot\hat{n}\,dA = +J(R)\,4\pi R^2$$

$+\hat{a}_r$

∴ total leakage out of region $\frac{R}{2} \le r \le R$ is given by

$$-J\left(\frac{R}{2}\right)\pi R^2 + J(R)\,4\pi R^2$$

by $\vec{J} = -D\frac{d}{dr}\phi\,\hat{a}_r = -\frac{Q}{4\pi}\left[-\frac{1}{rL}e^{-r/L} - \frac{1}{r^2}e^{-r/L}\right]\hat{a}_r$

$$\therefore \boxed{J(r) = \frac{Q}{4\pi}\left[\frac{1}{rL} + \frac{1}{r^2}\right]e^{-r/L}}$$

$$\therefore \text{total leakage} = -\frac{QR^2}{4}\left[\frac{2}{RL} + \frac{4}{R^2}\right]e^{-R/2L} + QR^2\left[\frac{1}{RL} + \frac{1}{R^2}\right]e^{-R/L}$$

$$\boxed{\text{total leakage} = -Q\left[\frac{R}{2L} + 1\right]e^{-R/2L} + Q\left[\frac{R}{L} + 1\right]e^{-R/L}}$$

$$\phi = \frac{Q}{4\pi D}\frac{1}{r}e^{-r/L}$$

$$\text{abs rate} = \int_{R/2}^{R} \Sigma_a \phi(r) 4\pi r^2 dr$$

$$= \frac{Q\Sigma_a}{D}\int_{R/2}^{R} r e^{-r/L} dr \qquad \text{but } \frac{\Sigma_a}{D} = \frac{1}{L^2}$$

$\int u\,dv = uv - \int v\,du$

let $u = r \quad du = dr \quad dv = e^{-r/L}dr \quad v = -Le^{-r/L}$

$$\therefore \text{abs rate} = \frac{Q}{L^2}\left[-rLe^{-r/L} - \int -Le^{-r/L}dr\right]_{R/2}^{R}$$

$$= \frac{Q}{L}\left(-Re^{-R/L} + \frac{R}{2}e^{-R/2L}\right)$$

$$+ \frac{Q}{L}\left[-Le^{-r/L}\right]_{R/2}^{R}$$

$$\boxed{\text{abs rate} = -\frac{QR}{L}e^{-R/L} + \frac{QR}{2L}e^{-R/2L} + Q\left[e^{-R/2L} - e^{-R/L}\right]}$$

now adding the leakage and absorption rates, we get

leakage $\quad -Q\left(\frac{R}{2L}+1\right)e^{-R/2L} + Q\left(\frac{R}{L}+1\right]e^{-R/L}$

+ abs $\quad Q\left(\frac{R}{2L}+1\right)e^{-R/2L} - Q\left(\frac{R}{L}+1\right)e^{-R/L}$

source

$$0 = 0$$

OK

→ This does indeed balance!!!

→ It was a little tedious, but it shows that, with care, the neutron balance eqn gives $0 = 0$ as it should for the case where there is no source within the region of interest...

Uniform Source in a Finite Sphere

Consider a bare sphere of moderator of radius R that contains a uniformly distributed source emitting Q n's/cm³-s.

(a) For $\phi(R+d) = 0$ where d is the extrapolation distance, determine $\phi(r)$ throughout the sphere.

As before, for 1-D spherical geometry, we have

$$\nabla^2\phi - \frac{1}{L^2}\phi = -\frac{Q}{D} \qquad \text{(where } Q = \text{constant)}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{1}{L^2}\phi = -\frac{Q}{D}$$

Now, $\phi(r) = \phi_H(r) + \phi_p(r)$

where $\phi_H(r)$ is solution to $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{1}{L^2}\phi = 0$

but we have already done this problem

$$\therefore \boxed{\phi_H(r) = A_1\frac{\sinh r/L}{r} + A_2\frac{\cosh r/L}{r}} \quad \text{homo soln}$$

see details in attachment

For $\phi_p(r)$, let's assume $\phi_p = C$ { form of forcing function and its derivatives

Then $0 - \frac{1}{L^2}C = -\frac{Q}{D}$ (here the choice of ϕ_p is obvious)

or $\boxed{C = \frac{QL^2}{D}}$ $\quad \therefore \boxed{\phi_p = \frac{QL^2}{D}}$ part. soln

Now $$\boxed{\phi(r) = A_1\frac{\sinh r/L}{r} + A_2\frac{\cosh r/L}{r} + \frac{QL^2}{D}} \quad \text{general soln}$$

BC#1 flux must be finite as $r \to 0 \Rightarrow A_2 = 0$

$$\therefore \phi(r) = A_1\frac{\sinh r/L}{r} + \frac{QL^2}{D}$$

BC#2 $$\phi(R+d) = 0 = A_1\frac{\sinh\left(\frac{R+d}{L}\right)}{R+d} + \frac{QL^2}{D}$$

or $$\boxed{A_1 = -\frac{QL^2}{D}(R+d)\frac{1}{\sinh\left(\frac{R+d}{L}\right)}}$$

also $L^2 = \frac{D}{\Sigma_a}$

$\therefore \frac{L^2}{D} = \frac{1}{\Sigma_a}$

and
$$\boxed{\phi(r) = \frac{Q}{\Sigma_a}\left[1 - \frac{R+d}{r}\,\frac{\sinh r/L}{\sinh\left(\frac{R+d}{L}\right)}\right]}$$
flux due to uniform source in finite sphere

$L^2 = \frac{D}{\Sigma_a}$
$\Sigma_a = \frac{D}{L^2}$

and
$$J(r) = -D\frac{d\phi}{dr} = +\frac{QD}{\Sigma_a}\,\frac{R+d}{\sinh\frac{R+d}{L}}\,\frac{d}{dr}\left[\frac{\sinh r/L}{r}\right]$$

$$\boxed{J(r) = QL^2\,\frac{R+d}{\sinh\frac{R+d}{L}}\left[\frac{1}{rL}\cosh r/L - \frac{1}{r^2}\sinh r/L\right]}$$

→ leakage out of sphere

$$\text{leakage} = \int_A \vec{J}\cdot\hat{n}\,dA = J(R)\,4\pi R^2$$

$$\boxed{\text{leakage} = 4\pi\,\frac{QL^2(R+d)}{\sinh\frac{R+d}{L}}\left[\frac{R}{L}\cosh\frac{R}{L} - \sinh\frac{R}{L}\right]}$$

→ Absorption rate within sphere

$$\text{abs. rate} = \int_V \Sigma_a\phi\,d\vec{r} = \int_0^R \Sigma_a\phi\,4\pi r^2\,dr$$

$$= 4\pi Q\int_0^R\left(r^2 - \frac{R+d}{\sinh\frac{R+d}{L}}\,r\sinh\frac{r}{L}\right)dr$$

$$= \frac{4}{3}\pi R^3 Q - \frac{4\pi(R+d)Q}{\sinh\frac{R+d}{L}}\left(RL\cosh\frac{R}{L} - L^2\sinh\frac{R}{L}\right)$$

$\int u\,dv = uv - \int v\,du$

$u = r \quad dv = \sinh\frac{r}{L}\,dr$

$du = dr \quad v = L\cosh\frac{r}{L}$

$\therefore \int r\sinh r/L\,dr =$

$= rL\cosh\frac{r}{L}\Big|_0^R - \int_0^R L\cosh\frac{r}{L}\,dr$

$= RL\cosh\frac{R}{L} - L^2\sinh\frac{R}{L}$

or
$$\boxed{\text{abs rate} = \frac{4}{3}\pi R^3 Q - \frac{4\pi QL^2(R+d)}{\sinh\frac{R+d}{L}}\left[\frac{R}{L}\cosh\frac{R}{L} - \sinh\frac{R}{L}\right]}$$

→ Source generation rate within sphere.

$$\text{Total Source} = \int_0^R \underbrace{Q}_{\text{const.}} 4\pi r^2 dr = Q \int_0^R 4\pi r^2 dr$$

$$\boxed{\text{Total Source} = \frac{4}{3}\pi R^3 Q}$$

→ Now the balance eqn becomes

leakage + absorption = source

and, upon substitution, this gives

$$\frac{4}{3}\pi R^3 Q = \frac{4}{3}\pi R^3 Q \quad \text{OK}$$

→ let's implement this into sphere_1region_uniformsrc.m

Try R = 100, 400, 800 cm

Notice how the influence of the external boundary is seen for about 4-5 diffusion lengths back into the sphere (after ~ 5L, the flux profile is essentially constant).

for water $L^2 = 8.1\ cm^2$ $L = 2.85$ cm

and $5L \simeq 14.2$ cm

for graphite $L^2 = 3500\ cm^2$ $L \simeq 59.2$ cm

and $5L \simeq 296$ cm

ok, this all makes sense

ODE: $$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{1}{L^2}\phi = -\frac{Q}{D}}$$ ← const. (1)

let $\boxed{\omega = r\phi}$ (2)

$\therefore$ $\boxed{\phi = \frac{\omega}{r}}$

$$\frac{d\phi}{dr} = -\frac{1}{r^2}\omega + \frac{1}{r}\frac{d\omega}{dr}$$

$$r^2\frac{d\phi}{dr} = -\omega + r\frac{d\omega}{dr}$$

$$\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \cancel{-\frac{d\omega}{dr}} + r\frac{d^2\omega}{dr^2} + \cancel{\frac{d\omega}{dr}} = r\frac{d^2\omega}{dr^2}$$

$\therefore$ eqn(1) becomes

$$\frac{1}{r}\frac{d^2\omega}{dr^2} - \frac{1}{L^2}\frac{\omega}{r} = -\frac{Q}{D}$$

or $$\boxed{\frac{d^2\omega}{dr^2} - \frac{1}{L^2}\omega = -\frac{Qr}{D}}$$ (3)

homo soln

$$\boxed{\omega_H(r) = A_1 \sinh\frac{r}{L} + A_2\cosh\frac{r}{L}}$$ (4) } from cartesian geometry problem done previously

part soln

$$\omega_p(r) = c_1 r + c_2$$

$$\omega_p' = c_1$$

$$\omega_p'' = 0$$

$\therefore$ upon subst. $$0 - \frac{1}{L^2}(c_1 r + c_2) = -\frac{Qr}{D}$$

$$\therefore \frac{c_1}{L^2} = \frac{Q}{D} \qquad \frac{c_2}{L^2} = 0$$

$$\therefore c_1 = \frac{QL^2}{D} \qquad c_2 = 0$$

$$= \frac{Q}{\Sigma_a}$$

$$\therefore \boxed{\omega_p(r) = \frac{Q}{\Sigma_a} r}$$ (5)

general soln

$$w(r) = w_H + w_p$$

$$w(r) = A_1 \sinh\frac{r}{L} + A_2 \cosh\frac{r}{L} + \frac{Q}{\Sigma_a} r$$

can be written as $\frac{Q}{D}L^2$

but

$$\boxed{\phi(r) = \frac{w(r)}{r} = A_1 \frac{1}{r}\sinh\frac{r}{L} + A_2 \frac{1}{r}\cosh\frac{r}{L} + \frac{Q}{\Sigma_a}}$$

general soln for sphere with uniform source

{ 1-group diffusion theory in 1-D spherical geometry (finite region form)

→ Note that there are two arbitrary constants

→ thus, two independent boundary conditions are needed to obtain a unique soln...

Consider a 2-region problem that involves concentric spheres. The inner sphere has a radius R and it has a uniformly distributed source emitting Q neutrons / cm^3-s. Surrounding the inner moderating medium is a very large region filled with a second moderating medium (this can be considered as an infinite region). There is no neutron source in the outer layer.

For consistency, use the subscript 1 to denote the material properties for the inner region (i.e. L_1, D_1, Σ_{a1}, etc) and subscript 2 to represent the outer (infinite) region.

Using 1-group theory, formally develop the flux profile in both regions 1 and 2.

$\boxed{r \le R}$ Region 1

$$\nabla^2 \phi_1 - \frac{1}{L_1^2}\phi_1 = -\frac{Q}{D_1} \quad \leftarrow \text{constant}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi_1}{dr}\right) - \frac{1}{L_1^2}\phi_1 = -\frac{Q}{D_1}$$

$$\phi_1(r) = \phi_{H_1}(r) + \phi_{P_1}(r)$$

$\boxed{R \le r \le \infty}$ Region 2

$$\nabla^2 \phi_2 - \frac{1}{L_2^2}\phi_2 = 0 \quad \leftarrow \text{no source}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi_2}{dr}\right) - \frac{1}{L_2^2}\phi_2 = 0$$

$$\phi_2(r) = \phi_{H_2}(r) + \cancel{\phi_{P_2}(r)} \to 0$$

no source

For the homo soln, we let $w = r\phi$ to get the following form $\frac{d^2}{dr^2}w - \frac{1}{L^2}w = 0$

with the soln (we have done this several times now...)

$$\boxed{\phi_{H_1}(r) = \frac{A_1}{r}\sinh\frac{r}{L_1} + \frac{A_2}{r}\cosh\frac{r}{L_1}}$$

finite region form

$$\boxed{\phi_{H_2}(r) = \frac{C_1 e^{-r/L_2}}{r} + \frac{C_2 e^{r/L_2}}{r}}$$

infinite region form.

For the particular soln,

let $\boxed{\phi_{P_1} = A_3}$ { uniform source

Then $-\frac{1}{L_1^2}A_3 = \frac{-Q}{D_1}$

$$\boxed{A_3 = \frac{QL_1^2}{D_1}}$$

$\phi_{P_2} = 0$ { no fixed source

$\therefore$ We have the following general soln

$$\phi_1(r) = \frac{A_1 \sinh \frac{r}{L_1}}{r} + \frac{A_2 \cosh \frac{r}{L_1}}{r} + \frac{Q L_1^2}{D_1} \qquad r \le R$$

$$\phi_2(r) = \frac{C_1 e^{-r/L_2}}{r} + \frac{C_2 e^{r/L_2}}{r} \qquad R \le r \le \infty$$

This has four arbitrary constants. — Thus, we need four BCs to obtain a unique soln.

<u>BC #1</u> flux must remain finite as $r \to 0$

$$\therefore \boxed{A_2 = 0} \quad \text{since} \lim_{r \to 0} \frac{\cosh r/L_1}{r} \to \infty$$

<u>BC #2</u> flux must be finite as $r \to \infty$

$$\therefore \boxed{C_2 = 0} \quad \text{since} \lim_{r \to \infty} \frac{e^{r/L_2}}{r} \to \infty$$

After these two conditions, we have

$$\boxed{\begin{aligned} \phi_1(r) &= \frac{A_1 \sinh r/L_1}{r} + \frac{Q L_1^2}{D_1} & 0 \le r \le R \\ \phi_2(r) &= \frac{C_1 e^{-r/L_2}}{r} & r \ge R \end{aligned}}$$

and the remaining two BCs are the continuity of flux and current conditions at $r = R$

<u>BC #3</u> $$\frac{A_1 \sinh(R/L_1)}{R} + \frac{S L_1^2}{D_1} = \frac{C_1 e^{-R/L_2}}{R}$$

or

$$\boxed{\underbrace{\frac{\sinh(R/L_1)}{R}}_{a_{11}} A_1 - \underbrace{\frac{e^{-R/L_2}}{R}}_{a_{12}} C_1 = \underbrace{-\frac{Q L_1^2}{D_1}}_{b_1}}$$

<u>BC #4</u> $$J_1(r) = -D_1 \frac{d\phi_1}{dr} = -A_1 D_1 \left[\frac{1}{r L_1} \cosh r/L_1 - \frac{1}{r^2} \sinh \frac{r}{L_1} \right]$$

$$J_2(r) = -D_2 \frac{d\phi_2}{dr} = -C_1 D_2 \left[-\frac{1}{r L_2} e^{-r/L_2} - \frac{1}{r^2} e^{-r/L_2} \right]$$

with the condition

$$J_1(r)\Big|_{r=R} = J_2(r)\Big|_{r=R}$$

writing this as $J_1(R) - J_2(r) = 0$ gives

$$\boxed{\underbrace{D_1\left[\frac{1}{R^2}\sinh\frac{R}{L_1} - \frac{1}{RL_1}\cosh\frac{R}{L_1}\right]}_{a_{21}} A_1 - \underbrace{D_2\left[\frac{1}{R^2}e^{-R/L_2} + \frac{1}{RL_2}e^{-R/L_2}\right]}_{a_{22}} C_1 = 0}$$

The two boxed eqns can be solved explicitly (or in Matlab) for the unknown A_1 and C_1 coeffs.

$$a_{11} A_1 + a_{12} C_1 = b_1$$

$$a_{21} A_1 + a_{22} C_1 = 0 \Rightarrow \boxed{A_1 = \frac{-a_{22} C_1}{a_{21}}}$$

$$-\frac{a_{11} a_{22}}{a_{21}} C_1 + a_{12} C_1 = b_1$$

$$\left(\frac{a_{12} a_{21} - a_{11} a_{22}}{a_{21}}\right) C_1 = b_1 \Rightarrow \boxed{C_1 = \frac{b_1 a_{21}}{a_{12} a_{21} - a_{11} a_{22}}}$$

and $$\boxed{A_1 = \frac{-b_1 a_{22}}{a_{12} a_{21} - a_{11} a_{22}}}$$

OK, with these coeffs we can evaluate and plot the resultant flux profile.

Let's implement this and compare the results from the two-region problem with the one-region case with equivalent R

this effectively compares a bare and reflected system.

See <u>Sphere_2region_uniformsrc.m</u>

leakage & absorption on following page

→ Let's also compute the leakage out of the same sphere

$$\phi_1(r) = A_1 \frac{\sinh r/L_1}{r} + \frac{Q L_1^2}{D_1}$$

$$J_1(r) = -D_1 \frac{d}{dr}\phi_1 = -D_1 A_1 \left[\frac{1}{L_1 r}\cosh r/L_1 - \frac{1}{r^2}\sinh\frac{r}{L_1}\right]$$

$$= D_1 A_1 \left(\frac{1}{r^2}\sinh\frac{r}{L_1} - \frac{1}{L_1 r}\cosh\frac{r}{L_1}\right)$$

and

leakage out of sphere of radius R $= \int \vec{J}\cdot\hat{n}\,dA = J_1(R)\,4\pi R^2$

$$= \boxed{4\pi D_1 A_1 \left(\sinh\frac{R}{L_1} - \frac{R}{L_1}\cosh\frac{R}{L_1}\right)}$$

→ Absorption within sphere of radius R

$L^2 = \frac{D}{\Sigma_a}$

abs rate $= \int \Sigma_{a1}\phi_1(r)\,4\pi r^2\,dr$

$$= 4\pi A_1 \Sigma_{a1} \underbrace{\int_0^R r\sinh\frac{r}{L_1}\,dr}_{} + \underbrace{Q\frac{\Sigma_{a1}L_1^2}{D_1}\left(\frac{4}{3}\pi R^3\right)}_{Q\left(\frac{4}{3}\pi R^3\right)}$$

$$\int_0^R r\sinh\frac{r}{L_1}\,dr = RL_1\cosh\frac{R}{L_1} - L_1^2\sinh\frac{R}{L_1}$$

from previous problem

$$\therefore \text{abs rate} = 4\pi A_1 \frac{D_1}{L_1^2}\left[RL_1\cosh\frac{R}{L_1} - L_1^2\sinh\frac{R}{L_1}\right] + \frac{4\pi R^3 Q}{3}$$

since $L^2 = \frac{D}{\Sigma_a}$

we write $\Sigma_{a1} = \frac{D_1}{L_1^2}$

$$= 4\pi A_1 D_1\left(\frac{R}{L_1}\cosh\frac{R}{L_1} - \sinh\frac{R}{L_1}\right) + \frac{4\pi R^3 Q}{3}$$

→ neutron balance

note that leakage + abs rate = total source production rate

or $\quad Q\left(\frac{4}{3}\pi R^3\right) = Q\left(\frac{4}{3}\pi R^3\right)$

OK