

Two beams of 2 eV neutrons intersect at an angle of 60° . The density of neutrons in both monoenergetic beams is 5×10^7 neutrons/cm³.

- Compute the neutron intensity (i.e. flux) of each beam.
- What is the neutron flux where the two beams intersect?
- What is the neutron current where the two beams intersect?

$$\phi = nv$$

$$\text{and } \vec{J} = n\vec{v}$$

$$\phi_2 = n_2 v \quad \& \quad J_2 = n_2 v \cos \theta \hat{i} + n_2 v \sin \theta \hat{j}$$

$$\phi_1 = n_1 v \quad \& \quad J_1 = n_1 v \hat{i}$$

here $\theta = 60^\circ$ $\cos \theta = 0.5$ and $\sin \theta = 0.8660$

→ We also need v , where $E = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2E}{m}}$
for neutrons with E in eV

$$\begin{aligned} \frac{2E}{m} &= \frac{2}{1.6749 \times 10^{-27} \text{ kg}} \times \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} \times E \text{ eV} \\ &\quad \times \frac{1 \text{ kg m}^2/\text{s}^2}{J} \times \frac{(100 \text{ cm})^2}{(1 \text{ m})^2} \\ &= 1.913 \times 10^{12} \times E \frac{\text{cm}^2}{\text{s}^2} \end{aligned}$$

and $v(E) = \sqrt{\frac{2}{m}} \sqrt{E} = [1.383 \times 10^6 \sqrt{E} \frac{\text{cm}}{\text{s}}]$
with E in eV

∴ for $E = 2 \text{ eV}$ $v = 1.956 \times 10^6 \text{ cm/s}$

(a) $I = \phi = nv = \left(5 \times 10^7 \frac{n}{\text{cm}^3}\right) \left(1.956 \times 10^6 \frac{\text{cm}}{\text{s}}\right)$

beam intensity in both beams = $9.779 \times 10^{13} \frac{n}{\text{cm}^2 \cdot \text{s}}$

ans

b) Since flux is a scalar quantity

$$\begin{aligned}\Phi_{\text{intersection}} &= \Phi_A = \Phi_1 + \Phi_2 \\ &= 2 \left(9.779 \times 10^{13} \frac{n}{cm^2 \cdot s} \right) \\ \boxed{\Phi_A = 1.956 \times 10^{14} \frac{n}{cm^2 \cdot s}}\end{aligned}$$

ans

c) Since current is a vector quantity

$$\begin{aligned}\vec{J}_A &= \vec{J}_1 + \vec{J}_2 \\ &= \phi_1 \hat{i} + 0 \hat{j} \\ &\quad + \phi_2 \cos \theta \hat{i} + \phi_2 \sin \theta \hat{j} \\ &= 9.779 \times 10^{13} (1 + \cos \theta) \hat{i} \\ &\quad + 9.779 \times 10^{13} \sin \theta \hat{j}\end{aligned}$$

$$\begin{aligned}\cos 60^\circ &= 0.5 \\ \sin 60^\circ &= 0.866\end{aligned}$$

$$\therefore \boxed{\vec{J}_A = 1.467 \times 10^{14} \hat{i} + 8.469 \times 10^{13} \hat{j}} - n/cm^2 \cdot s$$

ans

note $|J_A| = \text{magnitude of current at pt. A}$

$$\begin{aligned}&= \sqrt{J_x^2 + J_y^2} \\ &= 1.694 \times 10^{14} n/cm^2 \cdot s\end{aligned}\quad \left. \begin{array}{l} \text{This is only} \\ \text{the magnitude} \end{array} \right\}$$

Consider two beams of monoenergetic neutrons traveling along the same guide tube. One beam has an energy of 10 keV and a density of $1 \times 10^6 \text{ n/cm}^3$ and the other beam contains neutrons at 1 eV with a density of $2 \times 10^7 \text{ n/cm}^3$.

Compute the ^{Total energy integrated} neutron flux and current in the guide tube for the following two cases:

1. beams travel in same direction
2. beams are in opposite direction

state any assumptions
and explain your results

Since the beams are monoenergetic, the flux and current associated with each beam are given by

$$\begin{aligned} \text{beam 1: } \phi_1 &= n_1 v_1 & v_1 &= 1.383 \times 10^6 \sqrt{E} & \left. \begin{array}{l} E \text{ in eV} \\ v \text{ in cm/s} \end{array} \right. \\ &= (1 \times 10^6)(138.3 \times 10^6) & &= 1.383 \times 10^6 \sqrt{10^4} \\ &= 1.383 \times 10^{14} \text{ n/cm}^2\text{-s} & &= 138.3 \times 10^6 \text{ cm/s} \end{aligned}$$

$$\begin{aligned} \bar{J}_1 &= n_1 \vec{v}_1 = 1.383 \times 10^{14} \text{ n/cm}^2\text{-s} \hat{i} & \leftarrow \begin{array}{l} \text{assume} \\ \text{going to the} \\ \text{right in the} \\ \text{guide tube} \end{array} \\ &= \hat{i} \phi_1 \end{aligned}$$

$$\begin{aligned} \text{beam 2: } \phi_2 &= n_2 v_2 & v_2 &= 1.383 \times 10^6 \sqrt{1} \\ &= (2 \times 10^7)(1.383 \times 10^6) & &= 1.383 \times 10^6 \text{ cm/s} \\ &= 2.766 \times 10^{13} \text{ n/cm}^2\text{-s} \end{aligned}$$

$$\begin{aligned} \bar{J}_2 &= n_2 \vec{v}_2 = 2.766 \times 10^{13} \text{ n/cm}^2\text{-s} \hat{i} & \leftarrow \begin{array}{l} \text{assume} \\ \text{going to right} \end{array} \\ &= \hat{i} \phi_2 \end{aligned}$$

Now, the neutron flux is a scalar variable and the neutron current is a vector quantity. Thus the total (energy integrated) neutron flux and net neutron current are given by

Case 1 (same direction)

$$\begin{aligned} \phi_{\text{tot}} &= \phi_1 + \phi_2 = 1.383 \times 10^{14} + 2.766 \times 10^{13} \\ &= 1.660 \times 10^{14} \text{ n/cm}^2\text{-s} \end{aligned} \quad \left. \begin{array}{l} \text{total # of neutrons/cm}^2\text{-s} \\ \text{crossing an area perpendicular} \\ \text{to the beams} \end{array} \right.$$

$$\begin{aligned} \bar{J}_{\text{net}} &= \bar{J}_1 + \bar{J}_2 \\ &= 1.660 \times 10^{14} \text{ n/cm}^2\text{-s} \hat{i} \end{aligned} \quad \left. \begin{array}{l} \text{same as above with the} \\ \text{added statement that} \\ \text{all neutrons are traveling} \\ \text{in the +x direction} \end{array} \right.$$

Case 2 (opposite direction)

$$\phi_T = \phi_1 + \phi_2 = 1.640 \times 10^{14} \text{ n/cm}^2\cdot\text{s}$$

} same as Case 1
since flux is
a scalar quantity

$$\begin{aligned}\overline{J}_{\text{net}} &= \overline{J}_1 - \overline{J}_2 \\ &= 1.106 \times 10^{14} \text{ n/cm}^2\cdot\text{s}\end{aligned}$$

assumes beam 1 is to the right and beam 2 is moving to the left

↑ This represents the net # of neutrons moving in the +x direction.

Note that $1.383 \times 10^{14} \text{ n/cm}^2\cdot\text{s}$ at 10 keV energies are moving to the right and $2.766 \times 10^{13} \text{ n/cm}^2\cdot\text{s}$ at 1 eV are moving to the left. If we had asked for the net currents at each energy, then this would have been the answer (one result for each energy).

However, since the energy integrated net current was requested, then we must "sum" up the currents over energy with proper allowance for their vector nature (i.e. direction).

Here, since the beams are in opposite directions, they simply subtract ...

The fuel for an experimental thermal reactor contains uranium carbide (UC) with the uranium enriched to 4.8 w/o. The density of UC is 13.6 g/cm³.

- With this information and suitable microscopic cross sections (see Note below), calculate the macroscopic thermal (at E = 0.0253 eV) absorption and fission cross sections for the uranium carbide fuel pin.
- Assuming 1/v behavior, also compute both macroscopic cross sections at E = 1 eV.

From the JNDc website (at E = 0.0253 eV)

$$\text{U235} \quad \sigma_f = 585.15 \quad \underline{\sigma_c = 98.75}$$

$$\text{U238} \quad \sigma_f \approx 0 \quad \underline{\sigma_c = 2.6835}$$

$$\sigma_a = \sigma_f + \sigma_c = 683.9 \text{ b}$$

$$\sigma_a = \sigma_c = 2.6835$$

$$e_{\text{nat}} \quad \sigma_a = \sigma_c = 3.86 \text{ mb}$$

R small

Now let's calculate the atom densities

$$\frac{1}{M_u} = \sum_i \frac{w_i}{m_i} = \frac{0.048}{235.04} + \frac{0.952}{238.05}$$

$$M_u = 237.90$$

$$M_{\text{UC}} = 237.90 + 12.01 = 249.91$$

$$\begin{aligned} N_{\text{U235}} &= \frac{13.6 \text{ g UC}}{\text{cm}^3} \times \frac{237.90 \text{ g U}}{249.91 \text{ g UC}} \times \frac{0.048 \text{ g U235}}{\text{g U}} \times \\ &\quad \underbrace{\frac{0.6022 \times 10^{-24} \text{ atoms U235}}{235.04 \text{ g U235}}} \times 10^{-24} \frac{\text{cm}^2}{\text{b}} \\ &= (13.6) \left(\frac{237.90}{249.91} \right) (0.048) \left(\frac{0.6022}{235.04} \right) \\ &= \boxed{1.592 \times 10^{-3} \text{ atoms U235 / b - cm}} \end{aligned}$$

$$\begin{aligned} N_{\text{U238}} &= (13.6) \left(\frac{237.90}{249.91} \right) (0.952) \left(\frac{0.6022}{238.05} \right) \\ &= \boxed{3.118 \times 10^{-2} \text{ atoms U238 / b - cm}} \end{aligned}$$

$$N_c = (13.6) \left(\frac{12.01}{249.91} \right) \left(\frac{0.6022}{12.01} \right) = \boxed{3.277 \times 10^{-2} \frac{\text{at} \text{ } c}{\text{b}-\text{cm}}}$$

(a) mass at $E = 0.0253 \text{ eV}$

$$\Sigma_f = \sum_i N_i \sigma_{fi} = N \sigma_f \Big|_{U235}$$

$$= (1.592 \times 10^3)(585.1) = \boxed{0.931 \text{ cm}^{-1}}$$

ans

$$\Sigma_a = \sum_i N_i \sigma_{ai}$$

$$= N \sigma_a \Big|_{U235} + N \sigma_a \Big|_{U238} + N \sigma_a \Big|_c$$

$$= (1.592 \times 10^3)(683.8) + (3.118 \times 10^{-2})(2.683)$$

$$+ (3.277 \times 10^{-2})(3.86 \times 10^{-3})$$

$$= 1.0886 + 0.0837 + 0.0001$$

$$= \boxed{1.172 \text{ cm}^{-1}}$$

ans R small

(b) for $\frac{1}{\sqrt{E}}$ absorbers

$$\sigma = \frac{c_1}{\sqrt{E}} = \frac{c_2}{\sqrt{E}} \quad \text{where } c_2 = \sigma(E_0) \sqrt{E_0}$$

$$\therefore \Sigma(E) = \Sigma(E_0) \left(\frac{E_0}{E} \right)^{\frac{1}{2}}$$

∴ at 1 eV

$$\Sigma_c = 0.931 \left(\frac{0.0253}{1} \right)^{\frac{1}{2}} = \boxed{0.148 \text{ cm}^{-1}}$$

$$\Sigma_a = 1.172 \left(\frac{0.0253}{1} \right)^{\frac{1}{2}} = \boxed{0.186 \text{ cm}^{-1}}$$

ans

- Let ϕ_0 be the mono-directional mono-energetic uniform neutron flux incident on a thick slab target of thickness L and surface area A. If the target material has $\sigma_a \gg \sigma_s$ so that $\sigma_t \approx \sigma_a$, derive an expression for the rate at which neutrons interact (i.e. get absorbed) in the full target.
- In a specific case, the incident beam intensity is 4×10^{10} neutrons/cm²-sec, the target cross sectional area is 1.2 cm² and the target is 5.6 cm thick. The target atom density is 2.5×10^{22} atoms/cm³ and the total cross section at the energy of the beam is 24.8 b. For this specific situation, how many neutron interactions per second occur in the target?

Hint: Integration over the spatial variable is usually necessary when computing total reaction rates when the collision density varies with position (i.e. which is the case with thick planar targets).

AMPAD®

②

We know that the uncolloided flux is given by

$$I(x) = I_0 e^{-\Sigma_t x}$$

(Note)

$$\text{or } \phi(x) = \phi_0 e^{-\Sigma_t x}$$

And the reaction rate is

$$R = \int_0^L \Sigma_t \phi A dx$$

thus,

$$\begin{aligned} R &= \Sigma_t A \phi_0 \int_0^L e^{-\Sigma_t x} dx \\ &= \Sigma_t A \phi_0 \left(-\frac{1}{\Sigma_t} e^{-\Sigma_t x} \right) \Big|_0^L = A \phi_0 \left(-e^{-\Sigma_t L} + 1 \right) \end{aligned}$$

$$R = A \phi_0 (1 - e^{-\Sigma_t L})$$

ans

A = cross sectional area

L = thickness

ϕ_0 = incident flux

Σ_t = total macro xs for target

(b) For the data given

$$\Sigma_t = N \sigma_t = \left(2.5 \times 10^{22} \frac{\text{at}}{\text{b} \cdot \text{cm}} \right) (24.8 \text{ b}) = 0.62 \text{ cm}^{-1}$$

$$\begin{aligned} \therefore R &= A \phi_0 (1 - e^{-\Sigma_t L}) \\ &= (1.2)(4 \times 10^{10})(1 - e^{-3.472}) \end{aligned}$$

$$R = 4.651 \times 10^{10} \text{ abs/sec}$$

ans

$$\begin{aligned} \frac{L}{2} &= \Sigma_t L = \frac{1}{2} \text{ mean free path} \\ &= (0.62)(5.6) \\ &= 3.472 \end{aligned}$$

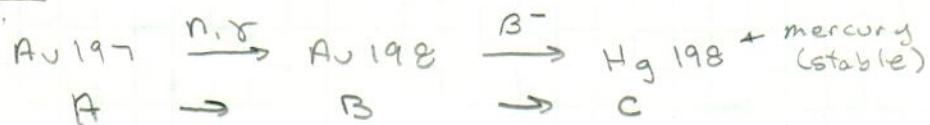
A small 0.12 g gold sample (pure Au197) is placed in an experimental location in a research reactor where the average neutron flux is $5 \times 10^{11} \text{ n/cm}^2\text{-s}$. The properly averaged capture cross section for Au197 is approximately 85 b. The sample is irradiated for 4 hr after which it is removed from the neutron field. During the irradiation, radioactive Au198 is produced at a constant rate via neutron capture in Au197 (that is, the flux is constant and the amount of Au197 does not change significantly during the irradiation interval). The half-life of Au198 is about 2.7 days and its neutron absorption cross section is small.

Based on the above description, estimate the activity (in curies) of the gold sample at the following times:

- immediately upon removal from the reactor and
- 72 hrs after removal from the reactor.

Explain your solution logic and support your results with a set of formal calculations/analyses.

Reaction



Element A (Au197) is stable and, over short irradiation times, the amount of A is essentially constant. Thus, the production rate of B (Au198) is constant and is given by

$$R = N_A \sigma_{cap} \phi \quad \text{where } N_A = \frac{\text{Total # of atoms}}{\text{volume}} = (\text{atom})(\text{vol})$$

However, nuclide B is radioactive, thus its balance eqn is given by

$$\frac{dN_B}{dt} = R - \lambda_B N_B \quad (\text{assumes a small } \sigma_{abs} \text{ value})$$

and solution of the eqn with $N_{B_0} = 0$ and $R = \text{const}$ is

$$N_B(t) = \frac{R}{\lambda_B} (1 - e^{-\lambda_B t})$$

$$\text{or } \alpha_B(t) = \lambda_B N_B(t) = R(1 - e^{-\lambda_B t})$$

\uparrow
activity of Au198 during irradiation phase

Now after removal from the reactor, material B simply decays with activity

$$\alpha_B(t) = \alpha_{B_0} e^{-\lambda_B t}$$

where t here is the time after removal

So to answer the questions we simply need to evaluate R and the above two expressions at the desired times.

activity

$$R = N_A \sigma_{CA} \phi$$

$$N_A = 0.12 \text{ g Au197} \times \frac{0.6022 \times 10^{24} \text{ atoms/mol}}{197 \text{ g/mol}} = 3.668 \times 10^{20} \text{ atoms}$$

$$R = (3.668 \times 10^{20} \text{ atoms}) (85 \times 10^{-24} \text{ cm}^2) (5 \times 10^{-11} \frac{\text{n}}{\text{cm}^2 \cdot \text{s}})$$

$$R = 1.559 \times 10^{-10} \text{ reactions/s}$$

$$\gamma_B = \frac{\ln 2}{T_{1/2, B}} = \frac{\ln 2}{2.7 \text{ day} \times \frac{24 \text{ hr}}{\text{day}}} = 1.0697 \times 10^{-2} \text{ hr}^{-1}$$

- a) for a 4-hr irradiation, the Au198 activity upon removal is given by

$$\begin{aligned} \alpha_B(4\text{hr}) &= R [1 - e^{-\gamma_B(4\text{hr})}] \\ &= (1.559 \times 10^{-10}) [1 - e^{-0.010697(4)}] \\ &= (1.559 \times 10^{-10})(0.0419) \\ &= 6.532 \times 10^{-10} \frac{\text{decays}}{\text{sec}} \times \frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ decays}} \\ &= 0.01765 \text{ Ci} \quad \underline{\text{ans}} \\ R &\approx 17.7 \text{ mCi} \end{aligned}$$

activity, t_i
upon removal

- b) 72 hrs after removal, the activity is given by

$$\begin{aligned} \alpha_B(72\text{hrs}) &= \alpha_0 e^{-\gamma_B(72\text{hr})} \\ &= (17.7 \text{ mCi}) [e^{-0.010697(72)}] \\ &= (17.7 \text{ mCi}) (0.4629) \\ &= 8.17 \text{ mCi} \quad \underline{\text{ans}} \end{aligned}$$

activity 72 hrs
after removal