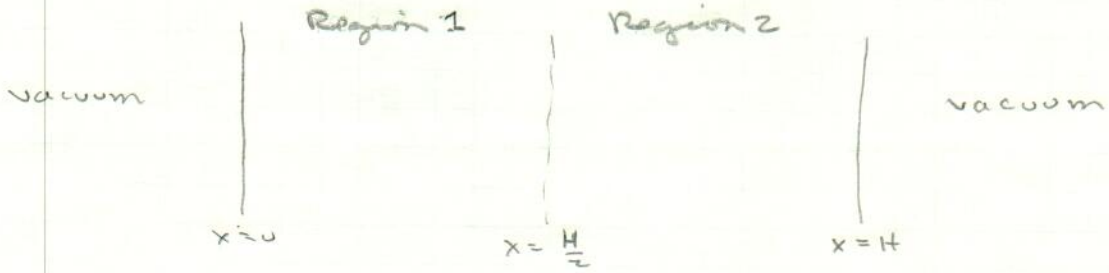
Consider the semi-infinite 2-region slab shown in the sketch. Both sides of the slab have vacuum BCs (i.e. at $x=0$ and $x=H$). The boundary between the two material regions is located at $x = H/2$, and standard interface BCs apply here. Both regions contain moderating mat'l.

There is an exponentially varying distributed source within the slab which is given by

$$S(x) = Ae^{-\alpha x} \qquad 0 \le x \le H$$

where A and α are known constants.



(a) Assuming 1-group theory, formally develop analytical expressions for the flux distributions, $\phi_1(x)$ and $\phi_2(x)$, throughout the 2-region slab. Denote the material properties and fluxes in the two regions with a '1' or '2' to denote the region of interest.

For this part of the problem, do not attempt to solve for the unknown coeffs — instead, simply setup the matrix equations needed to solve for these quantities.

Finally, since the value of H may be quite small, do not ignore the extrapolation distances for this problem.

Region #1

$$\nabla^2\phi_1 - \frac{1}{L_1^2}\phi_1 = -\frac{Q}{D_1}$$

1-D slab geometry

$$\frac{d^2}{dx^2}\phi_1 - \frac{1}{L_1^2}\phi_1 = -\frac{Q}{D_1}$$

homo soln (both finite regions)

$$\phi_{1H} \sim e^{\lambda x}$$

$$\lambda^2 - \frac{1}{L_1^2} = 0 \qquad \lambda_{1,2} = \pm\frac{1}{L_1}$$

Region #2

$$\nabla^2\phi_2 - \frac{1}{L_2^2}\phi_2 = -\frac{Q}{D_2}$$

$$\frac{d^2}{dx^2}\phi_2 - \frac{1}{L_2^2}\phi_2 = -\frac{Q}{D_2}$$

$$\phi_{2H} \sim e^{\lambda x}$$

$$\lambda^2 - \frac{1}{L_2^2} = 0 \qquad \lambda_{1,2} = \pm\frac{1}{L_2}$$

Region #1

$$\boxed{\phi_{1H}(x) = C_1 \cosh \frac{x}{L_1} + C_2 \sinh \frac{x}{L_1}}$$

Region #2

$$\boxed{\phi_{2H}(x) = C_3 \cosh \frac{x}{L_2} + C_4 \sinh \frac{x}{L_2}}$$

<u>particular soln</u> (both regions have an exponential source)

$$\boxed{\phi_{1p} = A_1 e^{-\alpha x}}$$

$$\phi_{1p}' = -\alpha A_1 e^{-\alpha x}$$

$$\phi_{1p}'' = \alpha^2 A_1 e^{-\alpha x}$$

upon subst.

$$A_1 \alpha^2 e^{-\alpha x} - \frac{1}{L_1^2} A_1 e^{-\alpha x} = -\frac{A}{D_1} e^{-\alpha x}$$

$$A_1 \left(\alpha^2 - \frac{1}{L_1^2} \right) = -\frac{A}{D_1}$$

or $$A_1 = \frac{-A/D_1}{\alpha^2 - \frac{1}{L_1^2}}$$

or $$\boxed{A_1 = \frac{-A L_1^2 / D_1}{\alpha^2 L_1^2 - 1}}$$ <u>known</u>

Note: This is only valid for $\alpha^2 \neq \frac{1}{L_1^2}$

$$\boxed{\phi_{2p} = A_2 e^{-\alpha x}}$$

(<u>identical analysis</u>)

$$\downarrow$$

$$\boxed{A_2 = \frac{-A L_2^2 / D_2}{\alpha^2 L_2^2 - 1}}$$ <u>known</u>

Note: Only valid for $\alpha^2 \neq \frac{1}{L_2^2}$

<u>general soln</u>

$$\phi_1(x) = \phi_{1H}(x) + \phi_{1p}(x)$$

$$\boxed{\phi_1(x) = C_1 \cosh \frac{x}{L_1} + C_2 \sinh \frac{x}{L_1} + A_1 e^{-\alpha x} \quad \text{for } 0 \le x \le \frac{H}{2}}$$

$$\phi_2(x) = \phi_{2H}(x) + \phi_{2p}(x)$$

$$\boxed{\phi_2(x) = C_3 \cosh \frac{x}{L_2} + C_4 \sinh \frac{x}{L_2} + A_2 e^{-\alpha x} \quad \text{for } \frac{H}{2} \le x \le H}$$

unique soln

Now we need 4 BCs to help determine the four unknowns (C_1, C_2, C_3, and C_4) in the general solution

BC #1 vacuum at $x = 0$

$$\phi_1(-d_1) = 0 \qquad \text{where } d_1 = 2.13\, D_1$$

$$\boxed{C_1 \cosh \frac{-d_1}{L_1} + C_2 \sinh \frac{-d_1}{L_1} = -A_1 e^{\alpha d_1}} \qquad \text{(eqn 1)}$$

BC #2 continuity of flux at $x = \frac{H}{2}$

$$\phi_1\left(\frac{H}{2}\right) = \phi_2\left(\frac{H}{2}\right)$$

or $$\phi_1\left(\frac{H}{2}\right) - \phi_2\left(\frac{H}{2}\right) = 0$$

$$\boxed{C_1 \cosh \frac{H}{2L_1} + C_2 \sinh \frac{H}{2L_1} - C_3 \cosh \frac{H}{2L_2} - C_4 \sinh \frac{H}{2L_2} = A_2 e^{-\alpha \frac{H}{2}} - A_1 e^{-\alpha \frac{H}{2}}} \qquad \text{(eqn 2)}$$

BC #3 continuity of current at $x = \frac{H}{2}$

$$J_1\left(\frac{H}{2}\right) - J_2\left(\frac{H}{2}\right) = 0$$

$$J_1(x) = -D_1 \frac{d}{dx} \phi_1(x)$$

$$= -D_1 \left[\frac{C_1}{L_1} \sinh \frac{x}{L_1} + \frac{C_2}{L_1} \cosh \frac{x}{L_1} - A_1 \alpha e^{-\alpha x} \right]$$

$$J_2(x) = -D_2 \frac{d}{dx} \phi_2(x)$$

$$= -D_2 \left[\frac{C_3}{L_2} \sinh \frac{x}{L_2} + \frac{C_4}{L_2} \cosh \frac{x}{L_2} - A_2 \alpha e^{-\alpha x} \right]$$

$\therefore$

$$\boxed{-\frac{D_1}{L_1} C_1 \sinh \frac{H}{2L_1} - \frac{D_1}{L_1} C_2 \cosh \frac{H}{2L_1} + \frac{D_2}{L_2} C_3 \sinh \frac{H}{2L_2} + \frac{D_2}{L_2} C_4 \cosh \frac{H}{2L_2} = D_2 A_2 \alpha e^{-\alpha \frac{H}{2}} - D_1 A_1 \alpha e^{-\alpha H/2}} \qquad \text{(eqn 3)}$$

BC #4

Consider at $x = H$

$$\phi_2(H+d_2) = 0 \quad \text{where } d_2 = 2.13\, D_2$$

$$C_3 \cosh \frac{H+d_2}{L_2} + C_4 \sinh \frac{H+d_2}{L_2} = -A_2 e^{-\alpha(H+d_2)}$$

eqn 5

Putting eqns 1-4 into matrix form gives

$$\begin{bmatrix} \cosh \frac{-d_1}{L_1} & \sinh \frac{-d_1}{L_1} & 0 & 0 \\ \cosh \frac{H}{2L_1} & \sinh \frac{H}{2L_1} & -\cosh \frac{H}{2L_2} & -\sinh \frac{H}{2L_2} \\ -\frac{D_1}{L_1} \sinh \frac{H}{2L_1} & -\frac{D_1}{L_1} \cosh \frac{H}{2L_1} & \frac{D_2}{L_2} \sinh \frac{H}{2L_2} & \frac{D_2}{L_2} \cosh \frac{H}{2L_2} \\ 0 & 0 & \cosh \frac{H+d_2}{L_2} & \sinh \frac{H+d_2}{L_2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} -A_1 e^{\alpha d_1} \\ (A_2 - A_1)\, e^{-\alpha \frac{H}{2}} \\ (D_2 A_2 - D_1 A_1)\, \alpha\, e^{-\alpha H/2} \\ -A_2 e^{-\alpha(H+d_2)} \end{bmatrix}$$

soln of this matrix eqn $\underline{\underline{M}}\ \underline{c} = \underline{b}$ will give the needed coeffs to formally define $\phi_1(x)$ and $\phi_2(x)$ over their appropriate domains.

(b) Within the context of an overall neutron balance for this system, develop formal expressions for the following quantities.

1. leakage rate per unit area across the yz plane at $x = 0$,
2. leakage rate per unit area across the yz plane at $x = H$.
3. the absorption rate per unit area in the yz plane within the full volume, $0 \le x \le H$.
4. the total source production rate within the full volume $0 \le x \le H$.

$\boxed{\text{leakage at } x = 0}$

$$L_{\substack{\text{left}\\\text{face}}} = \int_{\substack{\text{left}\\\text{face}}} \vec{J}_1 \cdot \hat{n}\, dA = -J_1(0)$$

where the neg sign comes from the $\hat{n} = -\hat{i}$ term

$$\therefore L_{\substack{\text{left}\\\text{face}}} = D_1 \left[\frac{C_1}{L_1} \underbrace{\sinh \frac{0}{L_1}}_{\to 0} + \frac{C_2}{L_1} \underbrace{\cosh \frac{0}{L_1}}_{\to 1} - A_1 \alpha \underbrace{e^{-\alpha(0)}}_{\to 1} \right]$$

$$\boxed{\text{leakage out of left face} = D_1 \left[\frac{C_2}{L_1} - A_1 \alpha \right]}$$

$\boxed{\text{Leakage at } x = H}$

$$L_{\substack{\text{right}\\\text{face}}} = \int_{\substack{\text{right}\\\text{face}}} \vec{J}_2 \cdot \hat{n}\, dA = J_2(H)$$

here $\hat{n} = +\hat{i}$

$$\therefore \boxed{L_{\substack{\text{right}\\\text{face}}} = -D_2 \left[\frac{C_3}{L_2} \sinh \frac{H}{L_2} + \frac{C_4}{L_2} \cosh \frac{H}{L_2} - A_2 \alpha e^{-\alpha H} \right]}$$

$\boxed{\text{Absorption rate within slab}}$

$$\text{abs rate} = \underbrace{\int_0^{H/2} \Sigma_{a1} \phi_1(x)\, dx}_{\text{I}} + \underbrace{\int_{H/2}^{H} \Sigma_{a2} \phi_2(x)\, dx}_{\text{II}}$$

$$\text{I} = \int_0^{H/2} \Sigma_{a_1} \phi_1(x)\, dx$$

$$= \Sigma_{a_1}\left[C_1 \int \cosh\frac{x}{L_1}\, dx + C_2 \int \sinh\frac{x}{L_1}\, dx + A_1 \int e^{-\alpha x}\, dx\right]$$

$$= \Sigma_{a_1}\left[C_1 L_1 \sinh\frac{x}{L_1}\Big|_0^{\frac{H}{2}} + C_2 L_1 \cosh\frac{x}{L_1}\Big|_0^{\frac{H}{2}} - \frac{A_1}{\alpha} e^{-\alpha x}\Big|_0^{H/2}\right]$$

$$\boxed{\text{I} = \Sigma_{a_1}\left[C_1 L_1 \sinh\frac{H}{2L_1} + C_2 L_1\left(\cosh\frac{H}{2L_1} - 1\right) - \frac{A_1}{\alpha}\left(e^{-\alpha\frac{H}{2}} - 1\right)\right]}$$

similarly

$$\text{II} = \int_{H/2}^{H} \Sigma_{a_2} \phi_2(x)\, dx$$

$$= \Sigma_{a_2}\left[C_3 \int \cosh\frac{x}{L_2}\, dx + C_4 \int \sinh\frac{x}{L_2}\, dx + A_2 \int e^{-\alpha x}\, dx\right]$$

$$= \Sigma_{a_2}\left[C_3 L_2 \sinh\frac{x}{L_2}\Big|_{\frac{H}{2}}^{H} + C_4 L_2 \cosh\frac{x}{L_2}\Big|_{\frac{H}{2}}^{H} - \frac{A_2}{\alpha} e^{-\alpha x}\Big|_{\frac{H}{2}}^{H}\right]$$

$$\boxed{\text{II} = \Sigma_{a_2}\left[C_3 L_2\left(\sinh\frac{H}{L_2} - \sinh\frac{H}{2L_2}\right) + C_4 L_2\left(\cosh\frac{H}{L_2} - \cosh\frac{H}{2L_2}\right) - \frac{A_2}{\alpha}\left(e^{-\alpha H} - e^{-\alpha H/2}\right)\right]}$$

and finally

$\boxed{\text{Source production rate}}$

$$\text{Source rate} = \int_0^H S(x)\, dx$$

$$= \int_0^H A e^{-\alpha x}\, dx = -\frac{A}{\alpha} e^{-\alpha x}\Big|_0^H$$

$$\text{source rate} = -\frac{A}{\alpha}\left(e^{-\alpha H} - 1\right)$$

or

$$\boxed{\text{source rate} = \frac{A}{\alpha}\left(1 - e^{-\alpha H}\right)}$$

(c) Using the parameters given below, evaluate the expressions derived in Part a and b for the flux distribution throughout the 2-region slab and for the individual components of the neutron balance eqn.

Formally plot the flux profile and tabulate the components of the neutron balance for the following four cases.

Case 1: region 1 contains graphite
region 2 contains water
source decay rate $\alpha = \alpha_1$

Case 2: Same as Case 1 but $\alpha = \alpha_2$

Case 3: region 1 contains water
region 2 contains graphite
source decay rate $\alpha = \alpha_1$

Case 4: same geometry as Case 3 but $\alpha = \alpha_2$

(d) Briefly discuss the overall results of this analysis – does everything behave as expected? Explain.

Data for the problem

Source magnitude: $A = 1\ n/cm^3\text{-}s$

size of slab: $H = 50$ cm

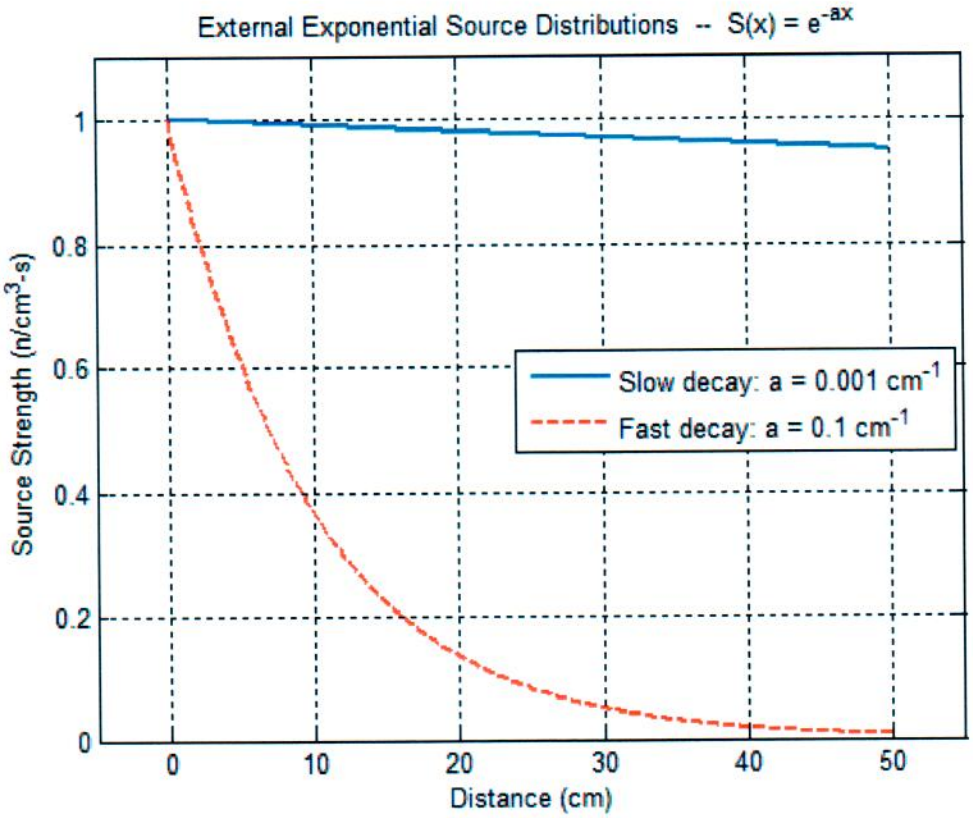
source decay rates: $\alpha_1 = 0.001\ cm^{-1}$
$\alpha_2 = 0.1\ cm^{-1}$

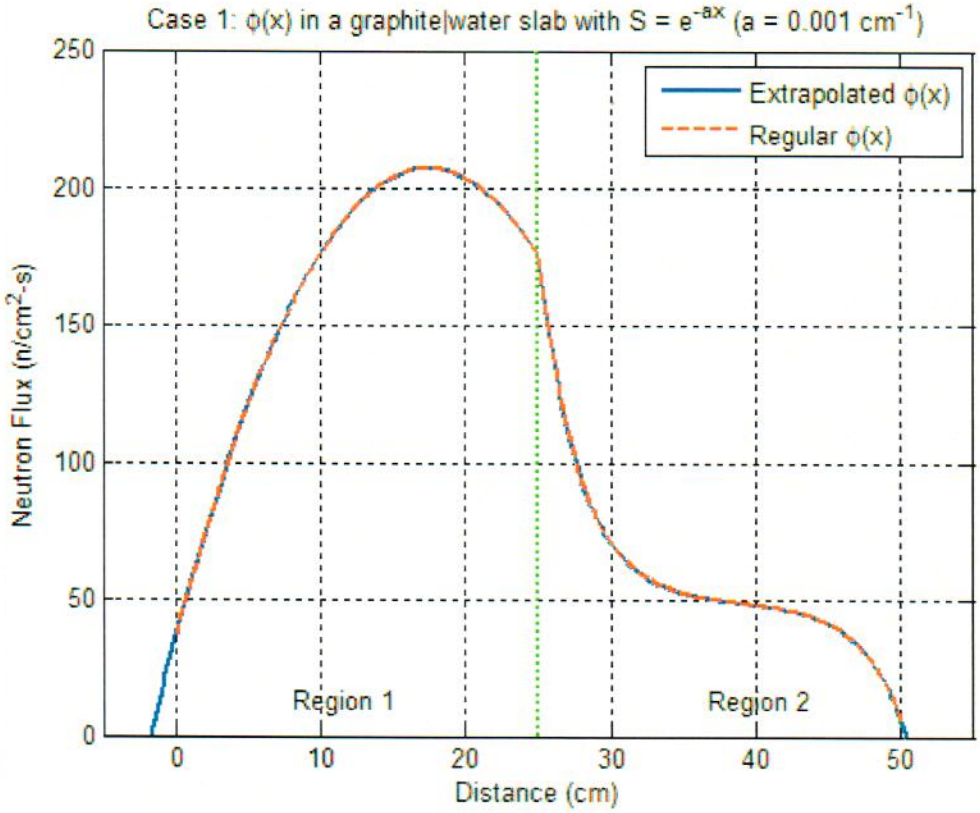
graphite properties: $D = 0.84$ cm $L^2 = 3500\ cm^2$

water properties: $D = 0.16$ cm $L^2 = 8.1\ cm^2$

---

for implementation see slab_2region_expsrc.m


External Exponential Source Distributions -- S(x) = e^-ax
Source Strength (n/cm^3-s)
Distance (cm)
Slow decay: a = 0.001 cm^-1
Fast decay: a = 0.1 cm^-1
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50

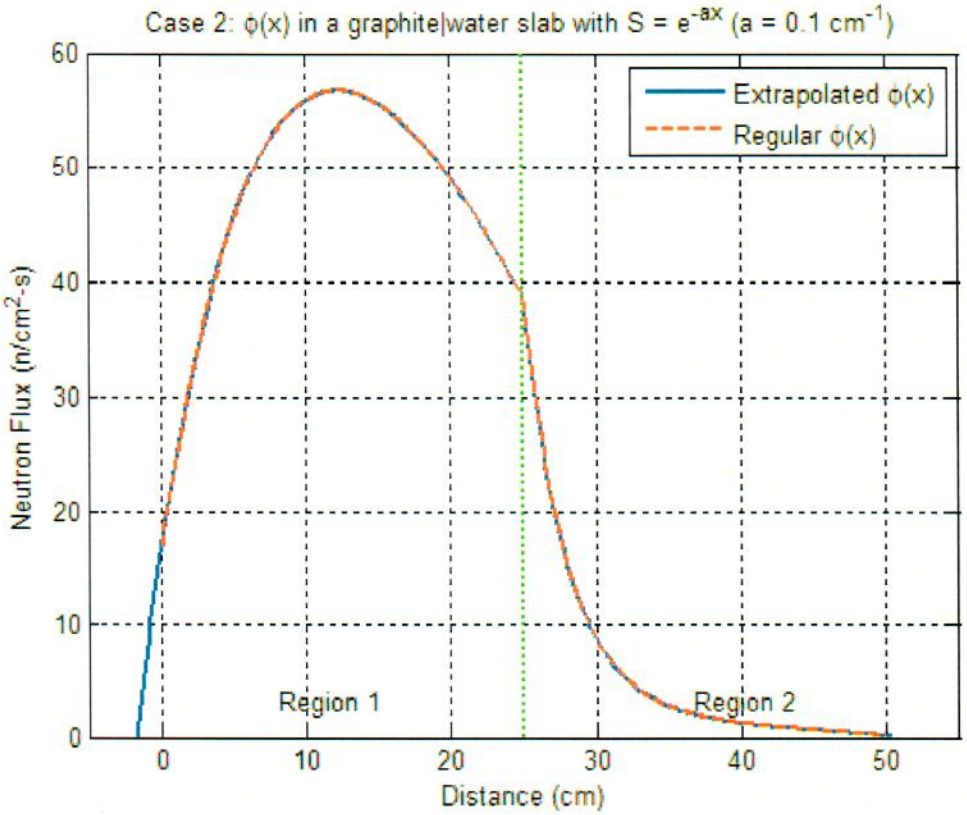



```
>> slab_2region_expsrc

   Neutron Balance Information for Finite 2-Region Slab with
    Exponentially-Varying Source (decay factor = 0.001 1/cm)

   Geometry Arrangement: graphite|water slab (50.0 cm)

   Components (neutrons/sec per unit area in yz plane):
     Left Leakage  (at x = 0):        1.661e+001
     Right Leakage  (at x = H):       2.411e+000
     Absorption Rate in Region 1:     9.895e-001
     Absorption Rate in Region 2:     2.876e+001
     Total Loss (leak + absorp):      4.877e+001
     Total Source within slab:        4.877e+001
     Balance Check (Prod - Loss):    -3.290e-005
>>
```

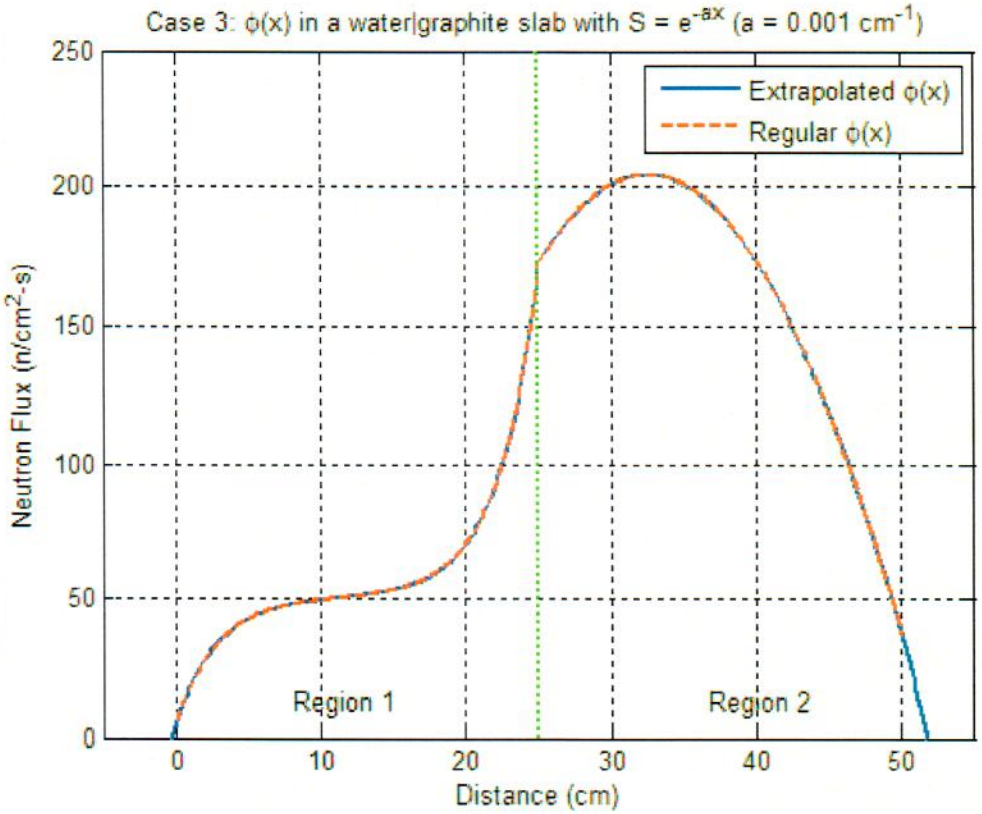



```
>> slab_2region_expsrc

   Neutron Balance Information for Finite 2-Region Slab with
    Exponentially-Varying Source (decay factor = 0.100 1/cm)

   Geometry Arrangement: graphite|water slab (50.0 cm)

   Components (neutrons/sec per unit area in yz plane):
     Left Leakage  (at x = 0):        6.918e+000
     Right Leakage  (at x = H):       2.434e-002
     Absorption Rate in Region 1:     2.850e-001
     Absorption Rate in Region 2:     2.705e+000
     Total Loss (leak + absorp):      9.933e+000
     Total Source within slab:        9.933e+000
     Balance Check (Prod - Loss):    -1.749e-005
>>
```

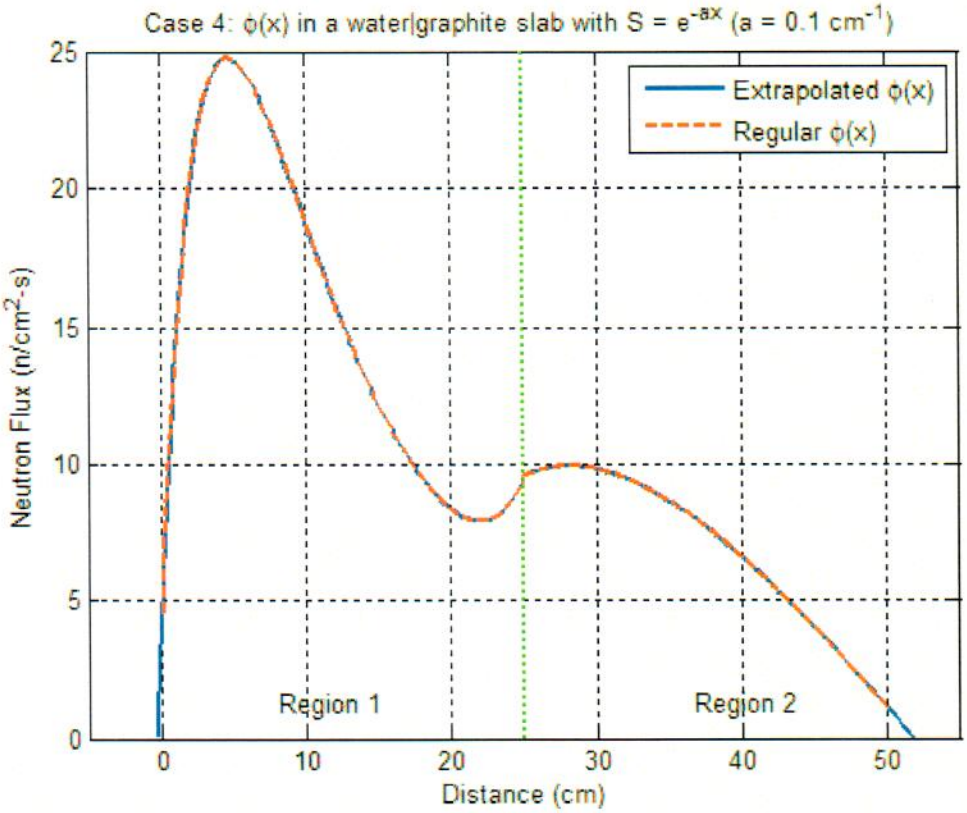



```
>> slab_2region_expsrc

   Neutron Balance Information for Finite 2-Region Slab with
    Exponentially-Varying Source (decay factor = 0.001 1/cm)

   Geometry Arrangement: water|graphite slab (50.0 cm)

   Components (neutrons/sec per unit area in yz plane):
     Left Leakage  (at x = 0):        2.520e+000
     Right Leakage  (at x = H):       1.617e+001
     Absorption Rate in Region 1:     2.911e+001
     Absorption Rate in Region 2:     9.705e-001
     Total Loss (leak + absorp):      4.877e+001
     Total Source within slab:        4.877e+001
     Balance Check (Prod - Loss):    -5.258e-013
>>
```




```
>> slab_2region_expsrc

   Neutron Balance Information for Finite 2-Region Slab with
    Exponentially-Varying Source (decay factor = 0.100 1/cm)

   Geometry Arrangement: water|graphite slab (50.0 cm)

   Components (neutrons/sec per unit area in yz plane):
     Left Leakage  (at x = 0):        1.961e+000
     Right Leakage  (at x = H):       5.019e-001
     Absorption Rate in Region 1:     7.428e+000
     Absorption Rate in Region 2:     4.139e-002
     Total Loss (leak + absorp):      9.933e+000
     Total Source within slab:        9.933e+000
     Balance Check (Prod - Loss):    -2.053e-012
>>
```

```
SLAB_2REGION_EXPSRC.M   Solution to a 2-Region 1-D Slab Geometry with
                              an Exponentially-Varying Source

This problem looks at the flux distribution and neutron balance associated with
a 2-region bare slab of width H.  The volume has an exponentially varying
distributed source, S(x) = A*exp(-a*x), where A and a are specified. Region 1
occupies half the slab (up to H/2) and Region 2 occupies the remainder (from H/2
to H).  Graphite and water are the two matierals of interest here.  In addition,
two different values of the source decay factor, a, are considered.

Various flux plots are compared.  In addition, the absorption rate in both
regions and leakage rates out the right and left sides the slab are tabulated
for each case.  A check on the overall neutron balance is also made.

File prepared by J. R. White, UMass-Lowell (last update: Oct. 2016)


  clear all,   close all,   nfig = 0;

define material properties from Lamarsh Table 5.2 (1 -> water, 2 -> graphite)
  Dm = [0.16 0.84];          % diffusion coeff (cm)
  L2m = [8.1 3500];          % diffusion area (cm^2)

define source strength and decay rate
  A = 1;                     % source magnitude (n/cm^3-s)
  aa = [0.001  0.1];         % decay factor (1/cm)

select case of interest
  ic = menu('Select Case of interest:', ...
            'Case 1: graphite | water     with SLOW source decay', ...
            'Case 2: graphite | water     with FAST source decay', ...
            'Case 3:  water   | graphite  with SLOW source decay', ...
            'Case 4:  water   | graphite  with FAST source decay', ...
            'Case 5: graphite | graphite  with SLOW source decay', ...
            'Case 6: graphite | graphite  with FAST source decay', ...
            'Case 7:    water | water     with SLOW source decay', ...
            'Case 8:    water | water     with FAST source decay');

  switch ic
  case 1
    D1 = Dm(2);  L21 = L2m(2);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(1);  L22 = L2m(1);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(1);   % slow decay
    titl = ['Case 1: \phi(x) in a graphite|water slab with S = e^{-ax} (a = ', ...
             num2str(a),' cm^{-1})'];
    titl2 = 'graphite|water slab';
  case 2
    D1 = Dm(2);  L21 = L2m(2);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(1);  L22 = L2m(1);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(2);   % fast decay
```

```
    titl = ['Case 2: \phi(x) in a graphite|water slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'graphite|water slab';
  case 3
    D1 = Dm(1);  L21 = L2m(1);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(2);  L22 = L2m(2);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(1);   % slow decay
    titl = ['Case 3: \phi(x) in a water|graphite slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'water|graphite slab';
  case 4
    D1 = Dm(1);  L21 = L2m(1);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(2);  L22 = L2m(2);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(2);   % fast decay
    titl = ['Case 4: \phi(x) in a water|graphite slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'water|graphite slab';
  case 5
    D1 = Dm(2);  L21 = L2m(2);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(2);  L22 = L2m(2);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(1);   % slow decay
    titl = ['Case 5: \phi(x) in a graphite|graphite slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'graphite|graphite slab';
  case 6
    D1 = Dm(2);  L21 = L2m(2);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(2);  L22 = L2m(2);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(2);   % fast decay
    titl = ['Case 6: \phi(x) in a graphite|graphite slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'graphite|graphite slab';
  case 7
    D1 = Dm(1);  L21 = L2m(1);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(1);  L22 = L2m(1);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(1);   % slow decay
    titl = ['Case 7: \phi(x) in a water|water slab with S = e^{-ax} (a = ', .W
            num2str(a),' cm^{-1})'];
    titl2 = 'water|water slab';
  case 8
    D1 = Dm(1);  L21 = L2m(1);  L1 = sqrt(L21);  d1 = 2.13*D1;  sa1 = D1/L21;
    D2 = Dm(1);  L22 = L2m(1);  L2 = sqrt(L22);  d2 = 2.13*D2;  sa2 = D2/L22;
    a = aa(2);   % fast decay
    titl = ['Case 8: \phi(x) in a water|water slab with S = e^{-ax} (a = ', ...
            num2str(a),' cm^{-1})'];
    titl2 = 'water|water slab';
  end
%
% define geometry (distance in cm)
    H = input('    Input full width, H, of the bare slab (cm): ');
    if H > 60, H = 60; disp('    Warning: H set to Hmax = 60 cm');  end
  H = 50;      % total witdh of slab (cm)  --  fixed for project
```

```
    He = H+d2;  % extrapolated H
    N1 = 101;   x1 = linspace(0,H/2,N1);    x1e = linspace(-d1,H/2,N1);
    N2 = 101;   x2 = linspace(H/2,H,N2);    x2e = linspace(H/2,He,N2);
%
%   plot both source distributions (just for reference)
    x = [x1 x2];
    S_slow = A*exp(-aa(1)*x);   S_fast = A*exp(-aa(2)*x);
    nfig = nfig+1;   figure(nfig)
    plot(x,S_slow,'b-',x,S_fast,'r--','LineWidth',2),grid
    title('External Exponential Source Distributions  --  S(x) = e^{-ax}')
    xlabel('Distance (cm)'),ylabel('Source Strength (n/cm^3-s)')
    rr = [-0.1*H 1.1*H 0 1.1*A];  axis(rr)
    legend(['Slow decay: a = ',num2str(aa(1)),' cm^{-1}'], ...
           ['Fast decay: a = ',num2str(aa(2)),' cm^{-1}'],'Location','East')
%
%   solve for coeffs in flux eqn, then evaluate/plot the flux profile (see development)
    A1 = -A*L21/D1/(a^2*L21-1);  A2 = -A*L22/D2/(a^2*L22-1);  % coeffs in part. soln
M = [  cosh(-d1/L1)          sinh(-d1/L1)                0                      0↙
;
       cosh(H/2/L1)          sinh(H/2/L1)          -cosh(H/2/L2)        -sinh(H/2/L2)↙
;
   -(D1/L1)*sinh(H/2/L1)  -(D1/L1)*cosh(H/2/L1)  (D2/L2)*sinh(H/2/L2)  (D2/L2)*cosh↙
(H/2/L2);
          0                       0                    cosh(He/L2)          sinh(He/L2)↙
;
b = [-A1*exp(a*d1)  (A2-A1)*exp(-a*H/2)   (D2*A2-D1*A1)*a*exp(-a*H/2)   -A2*exp(-a*He)]';
    c = M\b;
    c1 = c(1); c2 = c(2);  c3 = c(3);  c4 = c(4);
    phi1 = c1*cosh(x1/L1) + c2*sinh(x1/L1) + A1*exp(-a*x1);
    phi2 = c3*cosh(x2/L2) + c4*sinh(x2/L2) + A2*exp(-a*x2);
    phi1e = c1*cosh(x1e/L1) + c2*sinh(x1e/L1) + A1*exp(-a*x1e);
    phi2e = c3*cosh(x2e/L2) + c4*sinh(x2e/L2) + A2*exp(-a*x2e);
%
    nfig = nfig+1;   figure(nfig)
    plot(x1e,phi1e,'b-',x1,phi1,'r--',x2e,phi2e,'b-', ...
         x2,phi2,'r--','LineWidth',2),grid
    title(titl)
    xlabel('Distance (cm)'),ylabel('Neutron Flux (n/cm^2-s)')
    rr = axis;  rr(1:3) = [-0.1*H 1.1*H 0];  axis(rr)
    legend('Extrapolated \phi(x)','Regular \phi(x)')
    hold on
    plot([H/2 H/2],rr(3:4),'g:','LineWidth',2)
    text(H/6.2,0.05*rr(4),'Region 1');  text(2.2*H/3,0.05*rr(4),'Region 2');
    hold off
%
%   compute the components of the neutron balance (per unit area in yz plane)
    Qtot = (A/a)*(1-exp(-a*H));                                   % total source
    absorp1 = sa1*(c1*L1*sinh(H/2/L1) + c2*L1*(cosh(H/2/L1) - 1) ...
                              -  (A1/a)*(exp(-a*H/2) - 1));       % absorption #1
    absorp2 = sa2*(c3*L2*(sinh(H/L2) - sinh(H/2/L2)) ...
                 + c4*L2*(cosh(H/L2) - cosh(H/2/L2)) ...
```

```
                - (A2/a)*(exp(-a*H) - exp(-a*H/2)));                % absorption #2
    leakL = D1*(c2/L1 - A1*a);                                      % left leakage
    leakR = -D2*(c3/L2*sinh(H/L2) + c4/L2*cosh(H/L2) ...
                - A2*a*exp(-a*H));                                  % right leakage
    loss = absorp1 + absorp2 + leakL + leakR;                       % total loass
    chkbal = Qtot - loss;                                           % near zero???
%
%   edit results
    fprintf(1,'\n   Neutron Balance Information for Finite 2-Region Slab with  \n');
    fprintf(1,'    Exponentially-Varying Source (decay factor = %5.3f 1/cm)\n\n',a);
    fprintf(1,'   Geometry Arrangement: %s (%4.1f cm)\n\n',titl2,H);
    fprintf(1,'   Components (neutrons/sec per unit area in yz plane):\n');
    fprintf(1,'     Left Leakage  (at x = 0):     %12.3e \n',leakL);
    fprintf(1,'     Right Leakage  (at x = H):    %12.3e \n',leakR);
    fprintf(1,'     Absorption Rate in Region 1:  %12.3e \n',absorp1);
    fprintf(1,'     Absorption Rate in Region 2:  %12.3e \n',absorp2);
    fprintf(1,'     Total Loss (leak + absorp):   %12.3e \n',loss);
    fprintf(1,'     Total Source within slab:     %12.3e \n',Qtot);
    fprintf(1,'     Balance Check (Prod - Loss):  %12.3e \n',chkbal);
%
%   end of simulation
```