ENGY.4340 Nuclear Reactor Theory
Final Exam Fall 2016

Problem 1. The Multigroup Balance Equation (15 points)

Consider a typical 3-group energy structure for a thermal reactor system, where group 1 represents the highest energy group and group 3 is the thermal group (the intermediate energy region, denoted as group 2, is often referred to as the epithermal region).

|-------- **3** ------------|----------- **2** -----------|---------- **1** ------------|

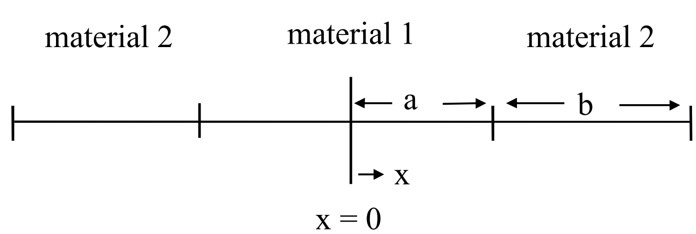
0 eV   1 eV   1 keV   20 MeV

Answer the following questions within the context of this group structure. Use standard multigroup notation for the group fluxes and the group-averaged macroscopic cross sections, as needed. However, **do not use any summation notation** in your final responses (write each quantity explicitly). Be precise with your notation and state any assumptions:

1. What is a reasonable distribution of numerical values for the multigroup fission spectrum for the three energy groups?
2. What is the inscatter rate to group 3?
3. What is the removal rate from group 2?
4. What is the neutron production rate in group 1?
5. For an infinite homogeneous system, write an expression for k_∞ for a 3-group energy model.

Problem 2. Neutron Diffusion in a 3-Region Symmetric 1-D Slab Geometry (20 points)

Consider the 1-D three-region finite slab geometry shown below. Both materials are moderating media (no fission) and material #2 within the outer region has a uniformly distributed source of S n/cm^3-s (i.e. S is a constant in the region defined by $a \le x \le a+b$). There is no external source in material #1 within the inner region. The system is symmetric about x = 0 and there is a vacuum boundary at x = a+b.



a. Using 1-group theory, set up the differential equations that describe the flux distribution in each of the two material zones. What is the general solution to the equation in each region?

b. Describe in detail how each of the unknown coefficients in the general solutions can be determined. In particular, identify the specific boundary conditions and how they would affect/simplify the general solutions. Be specific here; that is, you should set up each of the pertinent equations. However, you do **not** need to solve the resultant set of simultaneous equations.

c. Put the equations developed in Part b into standard matrix form and discuss briefly how you would go about solving this system to evaluate and plot the resultant flux solutions for a particular problem (in a Matlab program, for example). No code is required here -- just give an outline of the steps involved.

d. If dimension a is roughly L_1 and b is roughly $10L_2$, carefully sketch the expected 1-group flux profile for $x > 0$ in this system (here L_1 and L_2 refer to the diffusion lengths in material 1 and material 2, respectively).

   Now, if the size of the inner region (i.e. dimension a) is increased to be about $5L_1$, how would this change the expected flux profile? Sketch this carefully on a separate plot.

   For both plots, clearly identify the origin of the system, the location of the interface between materials 1 and 2, and the outer boundary at $x = a+b$. The flux profiles for $x > 0$ should be sketched carefully so that one can see the expected flux behavior at these boundary locations as well as within the interior of the geometry for both regions.

Problem 3. Cross Sections for Preliminary Calculations (15 points)

a. The 2200 m/s absorption and fission cross sections for U235 are

$$\sigma_a = 687\,\text{b} \qquad \text{and} \qquad \sigma_f = 587\,\text{b}$$

   Using the data in the Appendix, as needed, estimate the **thermally averaged microscopic cross sections** for U235 **absorption** and **fission** at 300 °C.

b. The density of water in an operating PWR is about 0.73 g/cm^3 (for T = 300 ºC and P = 2250 psia). Using the information in the Appendix, as needed, estimate the **thermal neutron age** and the **thermal diffusion area** of water at typical PWR operating conditions.

Problem 4. Six Factor Formula (15 points)

An instantaneous neutron balance in a nearly critical system has a **thermal absorption rate in the fuel of 4.646×10^{10} neutrons/sec.** The system is characterized by the following factors associated with the 6-factor formula:

$$\eta_T = 2.050 \qquad f = 0.800 \qquad p = 0.850 \qquad \varepsilon = 1.050 \qquad P_F = 0.772 \qquad P_T = 0.885$$

Based on the information given, determine the following parameters (show your logic/work):

1. The total fission source in the next generation in neutrons/sec.
2. The number of neutrons/sec absorbed in the resonance peaks in the next generation.
3. The thermal leakage rate from the system in neutrons/sec in the next generation.

Problem 5. Modified 1-Group Theory Calculations (15 points)

Consider a reflected reactor geometry with a homogeneous mix of U235 and water within the core and a large water reflector completely surrounding the core. The core consists of a 6x7 array of standard rectangular fuel assemblies which has a moderator density of about 1.0 g/cm^3 and a fuel density of 0.016 g/cm^3. The water reflector also has a density of 1.0 g/cm^3. The dimensions of an individual fuel assembly are: 10 cm x 10 cm in the xy plane and 80 cm in height. The system will be used as a small University research reactor whose primary purpose is the generation of neutrons for experimental purposes.

The system operates near room temperature and the appropriately averaged cross section data for the homogenized assemblies and water reflector are as follows (note that the water moderator properties are appropriate for both the core and the reflector regions):

$$\bar{\sigma}_{aF} = 590 \text{ b} \qquad \eta_T = 2.065$$

$$\bar{\sigma}_{aM} = 0.588 \text{ b} \qquad L^2_{TM} = 8.1 \text{ cm}^2 \qquad \tau_T = 27 \text{ cm}^2$$

a. Estimate k_∞ for the fuel assembly material.

   Note: If you are unsuccessful with Part a, use f = 0.6 and k_∞ = 1.239 to continue with the rest of this problem.

b. Using the result from above, estimate k_{eff} for a bare 6x7 array of these assemblies (you can neglect the extrapolation distance here).

c. Now, estimate the k_{eff} of the 6x7 array if it is fully surrounded by an infinite water reflector. Explain the logic used in your calculations…

Useful Relationships:

buckling for a bare parallelepiped reactor: $B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$

reflector savings for a water moderated and reflected system: $\delta \approx 7.2 + 0.10\left(M_T^2 - 40.0\right)$

Problem 6. Point Kinetics and Reactivity Feedback (20 points)

a. Define and discuss the term "temperature coefficient of reactivity" and **explain** why this should be negative. Be as explicit as possible and justify your remarks.

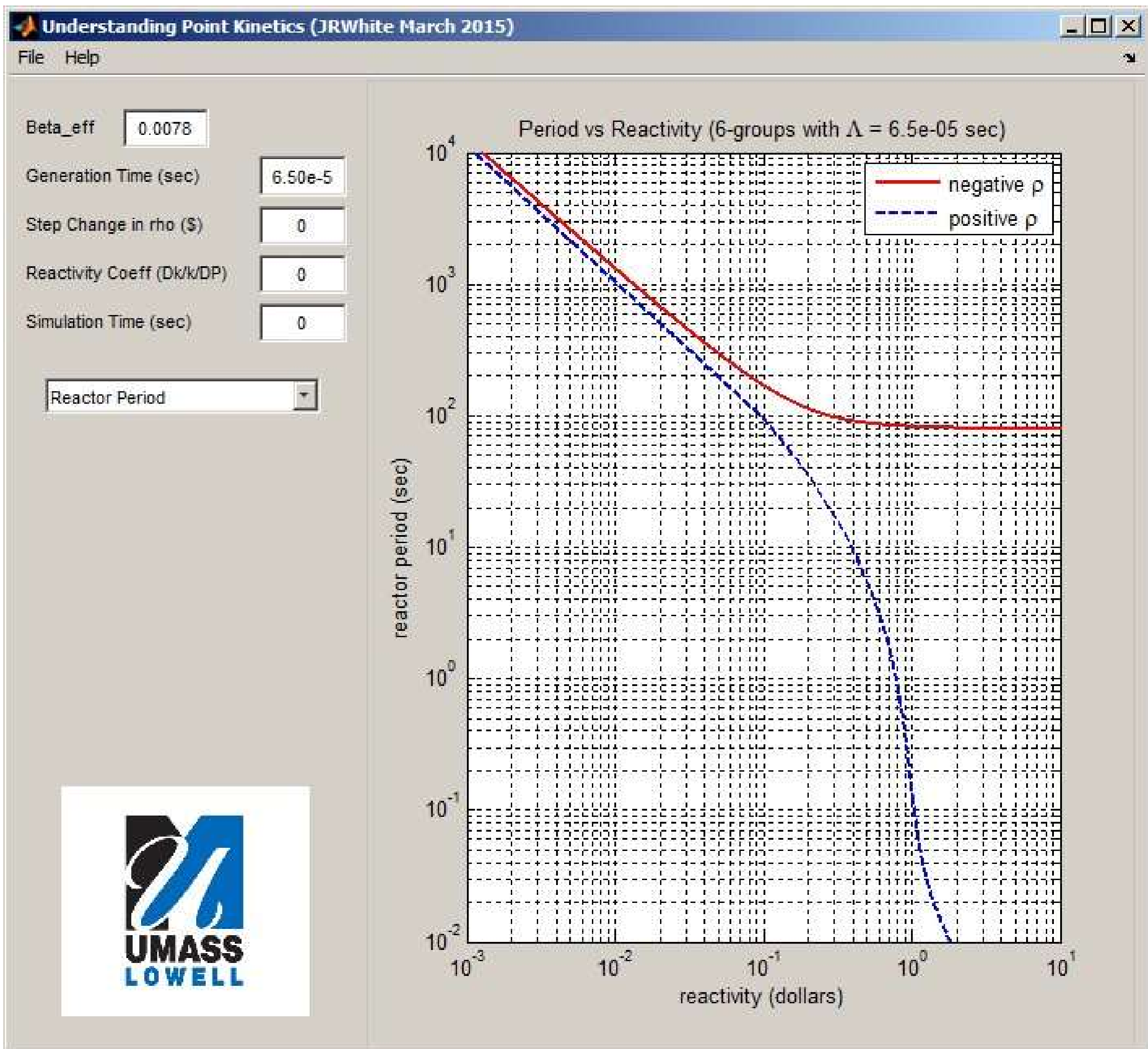
b. Discuss the general behavior of the power level, P(t), after a positive insertion of reactivity for the case of temperature feedback versus no feedback. Assume a negative temperature coefficient for the feedback case. Include sketches for the two cases in your discussion.

c. From an operations perspective, it is often convenient to lump the fuel and moderator temperature coefficients into a single reactivity coefficient for power changes -- which is defined as $\alpha_p = d\rho/dP$.

In the summer of 2007 a test was made within the UMLRR to determine this coefficient for natural convection operation -- a value of about -1.62×10^{-3} %Δk/k per kW was obtained. Using this value, if the reactor power was initially at 5 kW and an external reactivity change of +6 cents (with $\beta = 0.0078$) was made, estimate the new reactor power level after the transient is complete and a new critical steady state condition is achieved.

d. Blade #4 in the UMLRR is worth about 4.1 dollars of reactivity. From steady state operation at 1 MW the reactor is scrammed using just Blade #4. Under this scenario, neglecting feedback effects, estimate how long it takes for the fission power level to reach 50 W?

Data: In your calculations for Part d, use $\beta_{eff} = 0.0078$ for the UMLRR. Also recall that the prompt jump/drop is given as $P_1/P_0 = \beta/(\beta - \rho)$. Also, a plot of the most positive root of the reactivity equation vs. reactivity change (i.e. τ vs. ρ) is given below (from the Matlab-based **kinetics_gui** that we used in class). You may use this information, as appropriate, to answer Part d of this question.



Appendix -- Cross Section Data Tables (directly from Lamarsh)

Appendix Data: The data tabulated below are from the Lamarsh text, "***Introduction to Nuclear Engineering***". These data include some non-1/v factors (Table 3.2) for common materials, thermal and fast cross section data for some typical moderators (in Tables 5.2 and 5.3, respectively), some information for 1-g fast reactor analyses (Table 6.1), and some typical values for η_T (Table 6.3). Note that the group-averaged moderator data are for nominal density, ρ_o, and nominal temperature conditions (T_o = 20 ºC), and the usual density and temperature correction relationships for several important cross sections are as follows:

$$\bar{\Sigma}_a(\rho,T) = \bar{\Sigma}_a(\rho_o,T_o)\left(\frac{\rho}{\rho_o}\right)\left(\frac{T_o}{T}\right)^{1/2} \tag{1}$$

$$\bar{D}(\rho,T) = \bar{D}(\rho_o,T_o)\left(\frac{\rho_o}{\rho}\right)\left(\frac{T}{T_o}\right)^{m} \quad \text{with} \quad m = \begin{cases} 0.470 \text{ for } H_2O \\ 0.112 \text{ for } D_2O \\ \approx 0 \text{ otherwise (solid moderators)} \end{cases} \tag{2}$$

$$L_T^2(\rho,T) = \frac{\bar{D}(\rho,T)}{\bar{\Sigma}_a(\rho,T)} = L_T^2(\rho_o,T_o)\left(\frac{\rho_o}{\rho}\right)^2\left(\frac{T}{T_o}\right)^{m+1/2} \tag{3}$$

and

$$\tau_T(\rho) = \frac{D_1(\rho)}{\Sigma_{1\to 2}(\rho)} = \tau_T(\rho_o)\left(\frac{\rho_o}{\rho}\right)^2 \tag{4}$$

You may use the data and equations given here, as needed, during the exam…

TABLE 3.2 NON-1/V FACTORS*

	Cd	In	^{135}Xe	^{149}Sm	^{233}U		^{235}U		^{238}U	^{239}Pu	
T,°C	g_a	g_a	$g_a^{\dagger}$	g_a	g_a	g_f	g_a	g_f	g_a	g_a	g_f
20	1.3203	1.0192	1.1581	1.6170	0.9983	1.0003	0.9780	0.9759	1.0017	1.0723	1.0487
100	1.5990	1.0350	1.2103	1.8874	0.9972	1.0011	0.9610	0.9581	1.0031	1.1611	1.1150
200	1.9631	1.0558	1.2360	2.0903	0.9973	1.0025	0.9457	0.9411	1.0049	1.3388	1.2528
400	2.5589	1.1011	1.1864	2.1854	1.0010	1.0068	0.9294	0.9208	1.0085	1.8905	1.6904
600	2.9031	1.1522	1.0914	2.0852	1.0072	1.0128	0.9229	0.9108	1.0122	2.5321	2.2037
800	3.0455	1.2123	0.9887	1.9246	1.0146	1.0201	0.9182	0.9036	1.0159	3.1006	2.6595
1000	3.0599	1.2915	0.8858	1.7568	1.0226	1.0284	0.9118	0.8956	1.0198	3.5353	3.0079

*Based on C. H. Westcott, "Effective Cross-Section Values for Well-Moderated Thermal Reactor Spectra," Atomic Energy Commission report AECL-1101, January 1962.
†Based on E. C. Smith *et al.*, *Phys. Rev.* **115**, 1693 (1959).

TABLE 5.2 THERMAL NEUTRON DIFFUSION PARAMETERS OF COMMON MODERATORS AT 20°C*

Moderator	Density, g/cm³	$\overline{D}$, cm	$\overline{\Sigma}_a$, cm^{-1}	L_T^2, cm²	L_T, cm
H_2O	1.00	0.16	0.0197	8.1	2.85
D_2O†	1.10	0.87	$9.3 \times 10^{-\cdot}$	9.4×10^3	97
Be	1.85	0.50	1.04×10^{-3}	480	21
Graphite	1.60	0.84	2.4×10^{-4}	3500	59

*Based on *Reactor Physics Constants,* 2nd ed., Argonne National Laboratory report ANL-5800, 1963, Section 3.3.
†D_2O containing 0.25 weight/percent H_2O. These values are very sensitive to the amount of H_2O impurity (see Problem 5.28).

TABLE 5.3 FAST-GROUP CONSTANTS FOR VARIOUS MODERATORS

Moderator	D_1, cm	Σ_1, cm^{-1}	τ_T, cm²
H_2O	1.13	0.0419	~27
D_2O	1.29	0.00985	131
Be	0.562	0.00551	102
Graphite	1.016	0.00276	368

TABLE 6.1 NOMINAL ONE-GROUP CONSTANTS FOR A FAST REACTOR*

Element or Isotope	σ_γ	σ_f	σ_a	σ_{tr}	ν	η
Na	0.0008	0	0.0008	3.3		
Al	0.002	0	0.002	3.1		
Fe	0.006	0	0.006	2.7	—	—
^{235}U	0.25	1.4	1.65	6.8	2.6	2.2
^{238}U	0.16	0.095	0.255	6.9	2.6	0.97
^{239}P	0.26	1.85	2.11	6.8	2.98	2.61

*From *Reactor Physics Constants*, 2nd ed., Argonne National Laboratory report ANL-5800, 1963.

TABLE 6.3 VALUES OF η_T, THE AVERAGE NUMBER OF FISSION NEUTRONS EMITTED PER NEUTRON ABSORBED IN A THERMAL FLUX, AT THE TEMPERATURE T

T, °C	^{233}U	^{235}U	^{239}Pu
20	2.284	2.065	2.035
100	2.288	2.063	1.998
200	2.291	2.060	1.947
400	2.292	2.050	1.860
600	2.292	2.042	1.811
800	2.292	2.037	1.785
1000	2.292	2.033	1.770