ENGY.4340 Nuclear Reactor Theory
Exam #2 Fall 2016

Problem 1. 1-Group Criticality Calculations for Bare Systems (20 points)

Consider a bare homogeneous 1-group system with the following material properties (the fuel is Pu239 and the coolant is liquid sodium):

$$\eta = 2.61 \qquad \Sigma_a^F = 0.008\ \text{cm}^{-1} \qquad \Sigma_a^{Na} = 0.0005\ \text{cm}^{-1} \qquad \Sigma_{tr} = 0.100\ \text{cm}^{-1}$$

a. Using the parameters given above, calculate the critical dimension, a_o, for a 1-D bare slab reactor.

b. If the reactor dimension computed above is changed to a_o = 50 cm, what would be the corresponding change in the multiplication factor for the system (i.e. what is the Δk associated with increasing the reactor dimension as done here)? Also explain why you might want to have the reactor physically larger than the critical size computed in Part a.

   Note: If you didn't get a reasonable value for Part a, use a dimension of a_o = 40 cm for the reference just-critical system.

c. If the average differential worth of a bank of control rods in this system is 0.018 $\Delta k/k$ per cm, what is the approximate insertion length needed to offset the excess reactivity associated with the reactor configuration from Part b?

Problem 2. Modified 1-Group Theory Calculations (20 points)

a. Using modified 1-group theory, estimate k_∞ for a homogenous mixture of U235 and water with a fuel concentration of 0.0145 g/cm^3 and a water density of 1.0 g/cm^3. Use the following data in your calculations:

$$\bar{\sigma}_{aF} = 590\ \text{b} \qquad \bar{\sigma}_{aM} = 0.588\ \text{b} \qquad L_{TM}^2 = 8.1\ \text{cm}^2 \qquad \tau_T = 27\ \text{cm}^2$$

$$p\varepsilon \approx 1 \qquad \eta_T = 2.065$$

   Note: If you are unsuccessful with Part a, use f = 0.60 and k_∞ = 1.10 to continue with the rest of this problem.

b. If the material in Part a is put into a finite bare cylindrical geometry configuration with R_o = 60 cm where $H_o = 1.82R_o$, estimate k_{eff} for this configuration. Assume that the extrapolation distance, d, is small for these calculations.

c. If the bare core in Part b is fully surrounded by an infinite water reflector, estimate the excess reactivity (k_{eff} -1) of the un-poisoned reflected system.

Useful Relationships:

buckling for a bare finite RZ reactor: $B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$

reflector savings for a water moderated and reflected system: $\delta \approx 7.2 + 0.10\left(M_T^2 - 40.0\right)$

Problem 3. The Four Factor Formula (20 points)

A set of 2-group macroscopic cross section data appropriate for a specific research reactor fuel assembly is given below:

$D_1 = 1.56$ cm $\qquad D_2 = 0.283$ cm

$\Sigma_{a1} = 1.64\times10^{-3}\ \text{cm}^{-1}$ $\qquad \Sigma_{a2} = 5.72\times10^{-2}\ \text{cm}^{-1}$ $\qquad \left(\Sigma_{a2}^{F} = 4.32\times10^{-2}\ \text{cm}^{-1}\right)$

$\nu\Sigma_{f1} = 1.97\times10^{-3}\ \text{cm}^{-1}$ $\qquad \nu\Sigma_{f2} = 8.92\times10^{-2}\ \text{cm}^{-1}$ $\qquad \Sigma_{1\to2} = 2.74\times10^{-2}\ \text{cm}^{-1}$

Precisely define each term in the four-factor formula in equation form and, using the above data, numerically evaluate each term -- eventually computing k_∞ for the assembly.

Problem 4. Understanding the Neutron Life Cycle via the Six Factor Formula (15 points)

The neutron balance in a nearly critical system that has a **fast neutron leakage rate of 2.1×10^9 neutrons/sec in the current generation** is characterized by the following parameters associated with the 6-factor formula:

$\eta_T = 2.050$ $\qquad f = 0.802$ $\qquad p = 0.907$ $\qquad \varepsilon = 1.046$ $\qquad P_F = 0.700$ $\qquad P_T = 0.916$

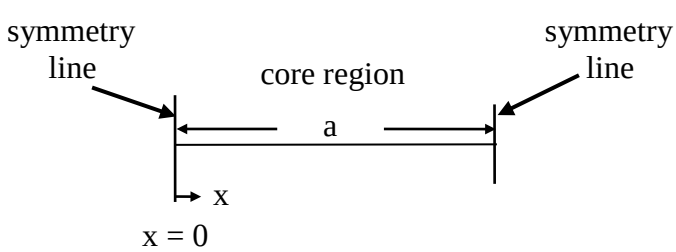
Based on the information given, determine the following quantities (show your logic/work):

1. The total fast neutron production rate from all fission in neutrons/sec in the current generation.
2. The total number of neutrons/sec absorbed at thermal energies in the current generation.
3. The fast neutron production rate in neutrons/sec in the next generation from thermal fission in the current generation.

Note: Read each description very carefully to be sure you are computing the requested quantity!!!

Problem 5. Critical Slab Core with Symmetry BCs on Both Sides (25 points)

Consider the slab reactor shown in the sketch. This geometry is "special" in that symmetry boundary conditions (BCs) are applicable on both the left and right sides (at x = 0 and at x = a). Note that reflected (symmetry) BCs on both sides of the homogeneous slab make this system essentially infinite -- thus, this is one way to represent an infinite homogeneous system.



The goal for this problem is to **formally develop the full solution to the 1-group diffusion equation for this situation**. To guide the solution, you should perform the following steps in the order given:

a. Write the appropriate 1-group diffusion equation for this 1-D slab homogeneous critical reactor problem.

b. Determine the general solution to the diffusion equation from Part a.

c. **Formally apply** the symmetry BCs at both x = 0 and at x = a to determine the eigencondition (criticality condition) for this problem -- **be formal here**!!! Explain/justify your work here.

   Note that this is the most sensitive part to this problem -- so be careful and be rigorous here. Although the result is somewhat different, the procedure is identical to what we did in class and in the HWs -- just be formal and things will work perfectly!!!

d. Write explicit expressions for the first three (3) modes to this eigenvalue problem -- identify both the eigenvalue and the eigenfunction for each mode. Also carefully sketch the three profiles making sure that the curves satisfy the appropriate BCs.

e. As we have indicated several times, only the fundamental mode solution (i.e. the smallest eigenvalue and corresponding eigenfunction) remains at steady state. What is the fundamental mode buckling for this system? What does this imply about the leakage from this system? What does this mean physically?

f. If the power density (i.e. power per unit volume, P/V) in this system is PD (with units of W/cm^3), write an expression for the steady state flux normalization factor for this 1-group homogeneous system.

g. Finally, what is the expression for k_{eff} for this system? Is this consistent with your expectations for an infinite system?