ENGY.4340 Nuclear Reactor Theory
Exam #1 Fall 2016

Problem 1. Energy Dependence of Flux and Neutron Cross Sections (30 points)

a. Carefully sketch the typical flux spectrum, $\phi(E)$ vs. E, expected in a thermal reactor. In doing this, be sure to identify the three key energy regions where distinct spectra are expected and their corresponding shapes.

b. With the definition of lethargy as $u(E) = \ln(E_{ref}/E)$, formally show that $\phi(u) = E\phi(E)$.

c. Carefully sketch the typical flux spectrum, $\phi(u)$ vs. E, in a thermal reactor.

d. Make separate sketches of the typical $\sigma_x(E)$ behavior for absorption and elastic scattering for many nuclides. Note here that the overall magnitude is not important (since this varies significantly by nuclide) but be careful with the expected shape of $\sigma_x(E)$ vs. E for both absorption and elastic scattering). Also, briefly describe the three primary energy regions where distinct $\sigma_x(E)$ behavior is expected.

e. Within a multigroup formulation, formally define the groupwise flux, ϕ_g, and the group averaged cross section, σ_{xg}, in terms of their continuous energy representations, $\phi(E)$ and $\sigma_x(E)$.

Problem 2. Neutron Leakage in a Bare Slab Reactor (20 points)

In a 1-D bare critical slab reactor the flux distribution is given by

$$\phi(x) = A\cos(Bx) \qquad \text{for} \quad 0 \le x \le a_o/2$$

where A is a normalization constant, B is the geometric buckling, a_o is the full thickness of the slab, and x is measured from the center of the reactor (center of the semi-infinite slab). This configuration is symmetric about the point x = 0 -- thus the above expression is also valid over the full x-domain from $-a_o/2$ to $a_o/2$. However, this problem only focuses on the right half of the slab from $0 \le x \le a_o/2$.

a. For the system described above, **formally develop** an expression for the leakage per unit area out of the right face of the slab at $x = a_o/2$ using the following formula:

$$\text{Leakage} = \int_A \vec{J} \bullet \hat{n}\, dA$$

Be sure to explain/show each step of the development.

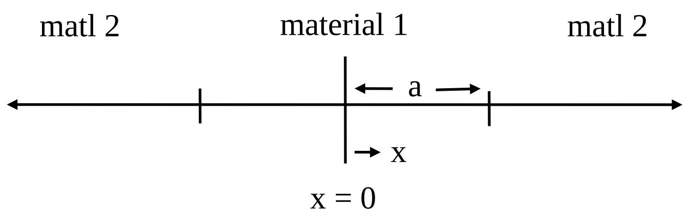
b. Now, **formally develop** an expression for the leakage per unit area out of the right face at $x = a_o/2$ using the following formula:

$$\text{Leakage} = \int_V \vec{\nabla} \bullet \vec{J}\, d\vec{r}$$

Again, be sure to explain/show all the key steps.

Problem 3. Neutron Diffusion in a 3-Region Symmetric 1-D Slab Geometry (30 points)

A three-region infinite slab is shown below. Material 1 has a uniform source of S n/cm^3-s and it extends a length of 2a. Material 2 is on either side of zone 1 and it extends in both directions to infinity. Material 2 has no external source. The system is symmetric with respect to the centerline at x = 0. Both materials are non-multiplying.



a. Using 1-group theory, set up the differential equations that describe the flux distribution in each of the two material zones. What is the general solution to the equation in each region?

b. Describe in detail how each of the unknown coefficients in the general solutions can be determined. In particular, identify the specific boundary conditions and how they would affect/simplify the general solutions. Be specific here -- however, you do not need to solve the resultant set of simultaneous equations.

c. Carefully make a sketch of the expected flux solution versus position for x > 0 for the situation described above. In your sketch, explicitly show the boundary between material 1 and material 2. Sketch the flux profile carefully and make sure it follows the form of the solution from above.

Problem 4. Material Data for Zirconium Hydride Fuel (20 points)

Uranium-Zirconium Hydride fuel, U-$ZrH_{1.6}$, is an alloy of uranium and zirconium hydride, $ZrH_{1.6}$, with an average H/Zr ratio of 1.6. The goal of this problem is to compute the atom densities and thermally averaged macroscopic cross sections for this fuel using the following base data (obtained from the JAEA Tables of Nuclear Data):

Nuclide	Molecular Weight (g/gmol)	$\sigma_c(E_o)$ (barns)	$\sigma_f(E_o)$ (barns)
H	1.008	0.332	0.0
Zr_{nat}	91.22	0.196	0.0
U235	235.0	98.7	585.1
U238	238.1	2.68	0.0

The thermal capture and fission cross section data given above are appropriate for E_o = 0.0253 eV and T_o = 20 C. Also note that all the nuclides are approximately 1/v absorbers.

a. If the content of the U-$ZrH_{1.6}$ mix is 45 w/o uranium, the uranium is enriched to 8 w/o U235, and the physical density of the fuel mix is 8.26 g/cm^3, determine the atom densities of the U235, U238, Zr, and H for this particular fuel composition.

b. Calculate the macroscopic absorption and fission cross sections for this fuel material at E_o and T_o.

c. Calculate the thermally averaged macroscopic absorption and fission cross sections at a temperature of 100 C.