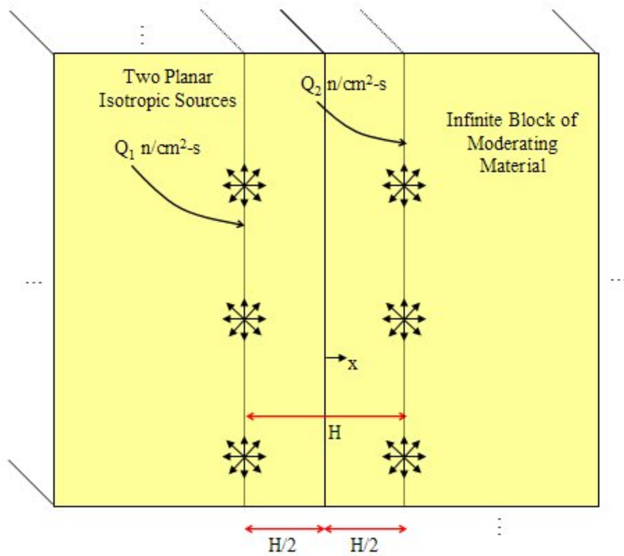
Two Planar Sources in an Infinite Moderating Medium

We have seen in a previous example (see Ref. 1) that the neutron flux and the magnitude of the current due to a single planar source in an infinite moderating medium are given by

$$\phi(x) = \frac{QL}{2D} e^{-x/L} \qquad \text{and} \qquad J(x) = \frac{Q}{2} e^{-x/L} \tag{1}$$

where x is the distance from the source to the point of interest (to the right of the source location), Q is the source strength (n/cm^2-s), and L and D are the diffusion length and diffusion coefficient, respectively (which characterize the moderating medium of interest).

One relatively simple extension to this classic situation might involve the case of two planar sources. In particular, one might pose the question: "Given two infinite planar sources each emitting Q neutrons/cm^2-s which are parallel at a distance H apart in an infinite moderator, what are the neutron flux and current as a function of distance from a plane midway between the two planar sources?".



This situation can be visualized as shown in the sketch to the right, and we can address the solution to this case by noting that the neutron flux is a ***scalar*** quantity and that the neutron current is a ***vector*** quantity. With this reminder, we can simply add the two fluxes and currents due to each source individually, being careful to take into account the direction as well as magnitude for the neutron current.

In doing this, we first rewrite eqn. (1) in a more general way to account for both positive and negative distances, giving

$$\phi(z) = \frac{QL}{2D} e^{-|z|/L} \qquad \text{and} \qquad J(z) = \frac{z}{|z|}\frac{Q}{2} e^{-|z|/L} \tag{2}$$

for the neutron flux and net neutron current, respectively. In eqn. (2), z is the distance from the source and the $z/|z|$ term in the current equation simply accounts for the sign of the x-directed current (i.e., if the point of interest z is to the left of the source location, then $z/|z| = -1$, and the current is in the negative x direction).

Now, for a coordinate system centered between the two planar sources (i.e. x = 0 is in the center of the two sources), the total flux is given by

$$\phi(x) = \phi_1(z_1) + \phi_2(z_2) \tag{3}$$

where z_1 is the distance from plane 1 and $\phi_1(z_1)$ is the flux due to planar source #1 (on the left), and z_2 is the distance from plane 2 and $\phi_2(z_2)$ is the flux due to planar source #2 (on the right), or

$$z_1(x) = x + \frac{H}{2} \qquad \text{and} \qquad z_2(x) = x - \frac{H}{2} \tag{4}$$

With these expressions, the total flux becomes

$$\phi(x) = \frac{QL}{2D}\left[e^{-|x+H/2|/L} + e^{-|x-H/2|/L}\right] \tag{5}$$

Similarly, the x-directed net neutron current is

$$J(x) = J_1(z_1) + J_2(z_2) \tag{6}$$

or
$$J(x) = \frac{Q}{2}\left[\frac{x+H/2}{|x+H/2|}e^{-|x+H/2|/L} + \frac{x-H/2}{|x-H/2|}e^{-|x-H/2|/L}\right] \tag{7}$$

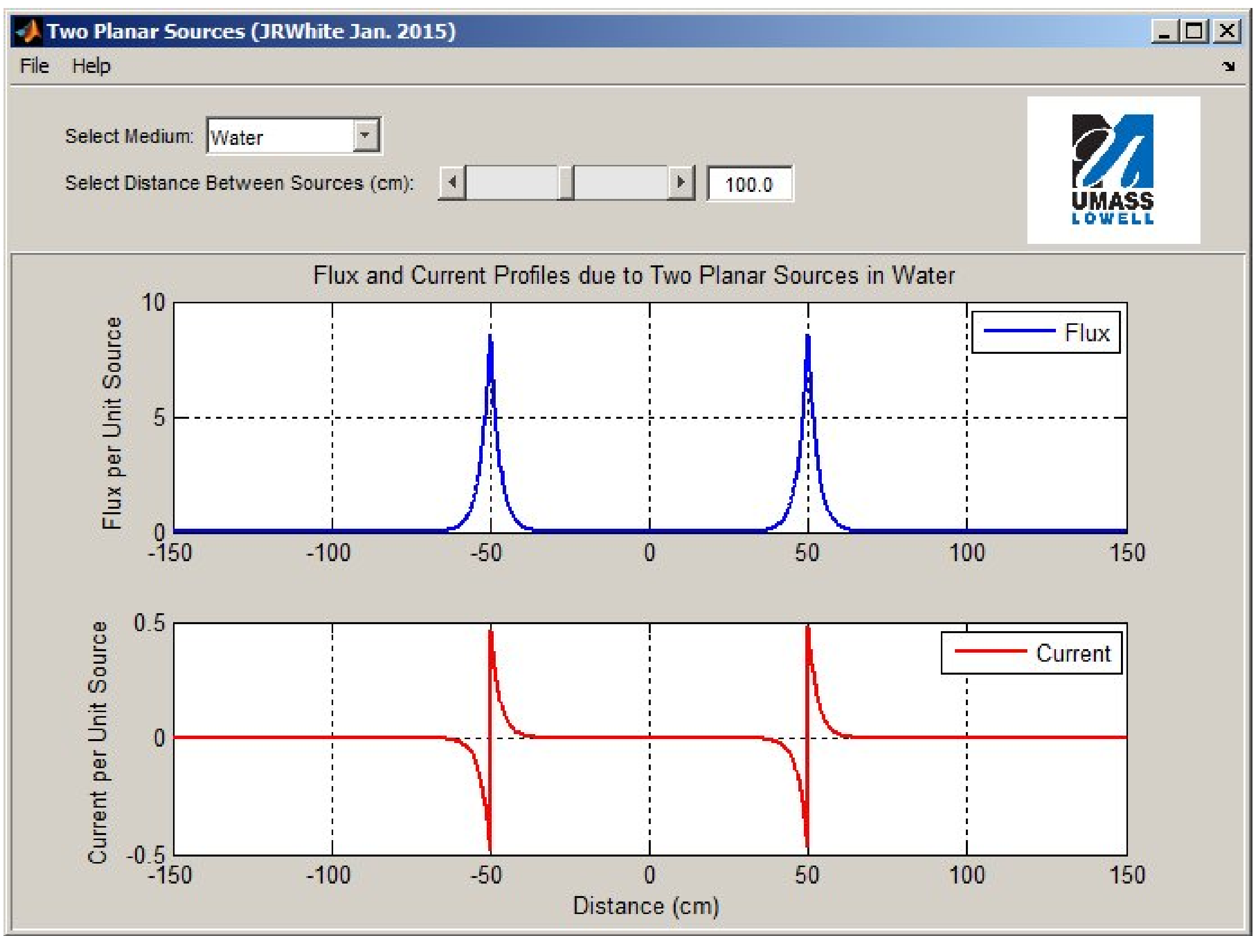


Fig. 1 User interface for the two_planar_sources GUI.

A Matlab graphical user interface called **two_planar_sources_gui** was designed to evaluate and plot eqns. (5) and (7) for a variety of moderator materials (water, graphite, or beryllium) for a user-specified value of H (the distance between the two sources). The GUI is pretty simple, as shown in Fig.1, but it is quite useful for understanding this particular situation. One should, in particular, observe the shape of the flux solution in the region between the sources for different moderator materials and different distances, H, and also carefully rationalize the sign of the neutron current in the various regions of this problem. The overall goal, of course, is to better 'visualize' the neutron diffusion processes that are at work here and to really understand the vector nature of the neutron current in this simple 1-D Cartesian geometry.

References

1. J. R. White, "Planar Source in a Moderating Medium," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell. This set of Lecture Notes also provides documentation for the ***slabmm_gui*** Matlab program.