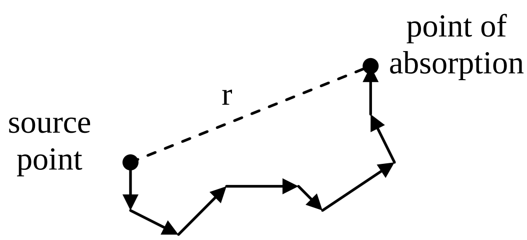
Interpretation of the Diffusion Length

The diffusion area, L^2, or diffusion length, L, is a material property that is important for characterizing the behavior of neutrons diffusing within a moderating medium. Recall that, for the 1-group problem, L^2 was defined in Ref. 1 as the ratio of the diffusion coefficient, D, and the macroscopic absorption cross section, Σ_a, or

$$L^2 = \frac{D}{\Sigma_a} \tag{1}$$

To gain a better interpretation of this quantity, let's consider the case of an isotropic point source in an infinite diffusing medium (see Ref. 2). In such a system, a neutron typically has several scattering interactions before finally being absorbed at some distance r from the source. A typical path might be as shown in the sketch given below:



r = crow flight distance from source to the point of absorption

Note that the absorption rate in a differential volume is given by

$$\begin{matrix}\#\text{ of neutrons absorbed per sec}\\ \text{in the volume } d\vec{r} \text{ around r}\end{matrix} = \Sigma_a \phi(r) d\vec{r} = \Sigma_a \phi(r) 4\pi r^2 dr \tag{2}$$

and that the 1-group flux due to an isotropic point source is

$$\phi(r) = \frac{Q}{4\pi D r} e^{-r/L} \tag{3}$$

Combining eqns. (2) and (3) gives

$$\begin{matrix}\#\text{ of neutrons absorbed per sec}\\ \text{in the volume } d\vec{r} \text{ around r}\end{matrix} = \frac{Q}{L^2} r e^{-r/L} dr \tag{4}$$

where we have also used the definition of the diffusion area from eqn. (1).

Now, if we divide this last expression by the number of source neutrons emitted per sec, Q, the result is the probability of a source neutron being absorbed in $d\vec{r} = 4\pi r^2 dr$ around r, or

$$\begin{matrix}\text{probability of a source}\\ \text{neutron being absorbed in } d\vec{r}\end{matrix} = p(r)dr = \frac{r e^{-r/L} dr}{L^2} \tag{5}$$

where, by definition, $\int_0^\infty p(r)dr = 1$.

Now, with this relationship, the average of the square crow flight distance traveled by a neutron before absorption is given as

$$< r^2 > = \frac{\int_0^\infty r^2 p(r)dr}{\int_0^\infty p(r)dr} = \frac{1}{L^2}\int_0^\infty r^3 e^{-r/L} dr = \frac{1}{L^2}\left[\frac{3!}{(1/L)^4}\right] = 6L^2 \qquad (6)$$

or $$L^2 = \frac{1}{6} < r^2 > \qquad (7)$$

Note that the integral given above was simply obtained from a standard table of integrals, where

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \qquad \text{for } a > 0 \text{ and } n = \text{positive integer}$$

Equation (7) gives a simple physical interpretation of the diffusion area, L^2, as 1/6th the square of the crow flight distance from the source point to the point of absorption. Or, more concisely, ***the diffusion length, L, is a measure of the distance traveled from birth to death of the neutron***. Materials with large values of L have more diffusion and/or less absorption than a material with smaller L, or with a slight rewording, one can say that ***neutrons tend to diffuse farther with less attenuation in materials with large L***.

The above physical interpretation of the diffusion length can also be extended to multigroup diffusing media problems. In particular, the 2-group diffusion equation for a homogeneous non-multiplying medium (with no upscatter) can be written as (see Ref. 1)

$$-D_1\nabla^2\phi_1 + (\Sigma_{a1} + \Sigma_{1\to 2})\phi_1 = Q_1 \qquad (8a)$$

$$-D_2\nabla^2\phi_2 + \Sigma_{a2}\phi_2 - \Sigma_{1\to 2}\phi_1 = Q_2 \qquad (8b)$$

Since the coefficients are constant, we can divide each equation by the diffusion coefficient, giving

$$\nabla^2\phi_1 - \frac{1}{L_1^2}\phi_1 = -\frac{Q_1}{D_1} \qquad (9a)$$

$$\nabla^2\phi_2 - \frac{1}{L_2^2}\phi_2 + \frac{\Sigma_{1\to 2}}{D_2}\phi_1 = -\frac{Q_2}{D_2} \qquad (9b)$$

where the fast and thermal diffusion areas, L_1^2 and L_2^2, respectively, are defined explicitly as

$$L_1^2 = \frac{D_1}{\Sigma_{a1} + \Sigma_{1\to 2}} = \frac{D_1}{\Sigma_{R1}} \qquad \text{and} \qquad L_2^2 = \frac{D_2}{\Sigma_{a2}} = \frac{D_2}{\Sigma_{R2}} \qquad (10)$$

However, in a moderating medium, the primary interaction at high energy is neutron scattering. Therefore, $\Sigma_{1\to 2}$ is usually much greater than Σ_{a1} (note that resonance absorption is not as important in low A nuclides as it is in high A fuel nuclides). If we make this assumption, then the removal cross section becomes $\Sigma_{R1} \approx \Sigma_{1\to 2}$, and the expression for the fast diffusion area reduces to $D_1/\Sigma_{1\to 2}$ which is typically called the ***thermal neutron age***, τ_T, (e.g. as done in Ref. 3),

$$\tau_T = \frac{D_1}{\Sigma_{1\to 2}} \tag{11}$$

Clearly the thermal neutron age (or fast diffusion area) is very similar to the thermal diffusion area, L_2^2 or L_T^2, except that it applies to the fast group instead of the thermal group. Thus, performing an analysis similar to that given above for the 1-group case, one can contrast $L_1^2 \approx \tau_T$ and $L_2^2 = L_T^2$ as follows:

$$L_1^2 \approx \tau_T = \begin{array}{c} 1/6^{\text{th}} \text{ the average of the square crow-flight distance from the} \\ \text{point where a neutron is born as a fast neutron to the point} \\ \text{where it dies (gets absorbed or slows down to thermal)} \end{array}$$

$$L_2^2 = L_T^2 = \begin{array}{c} 1/6^{\text{th}} \text{ the average of the square crow-flight distance} \\ \text{from the point where a thermal neutron is born to} \\ \text{the point where it is finally absorbed} \end{array}$$

where, in both cases, the system is assumed to be large so that leakage out of the physical volume of interest is negligible.

Thus, as above, ***the square root of the thermal neutron age,*** $\sqrt{\tau_T}$ ***, (or the fast diffusion length,*** L_1***) is a measure of the distance traveled from birth to death of a fast neutron*** and ***the thermal diffusion length,*** $L_2 = L_T$***, is a measure of the distance traveled from birth to death of a thermal neutron***. Also note that the fast and thermal diffusion lengths for the same material can be quite different – this just depends on the relative magnitudes of the diffusion coefficient and the removal cross sections for the material and group of interest. However, with information on the appropriate diffusion lengths for a given material, one can get a good picture of neutron diffusion in that particular material. Thus, ***the diffusion length is one of the most important neutronic property of a moderating material…***

Note: Several authors routinely use the subscript T to denote thermal energy -- so either L_2^2 or L_T^2 can be used interchangeably in a 2-group problem, depending on preference (for example, Ref. 3 consistently uses the T notation to refer to the thermal group).

References

1. J. R. White, "Solutions to the Steady State Diffusion Equation," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.
2. J. R. White, "Point Source in a Moderating Medium," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell. This set of Lecture Notes also provides documentation for the ***spheremm_gui*** Matlab program.
3. J. R. Lamarsh and A. J. Baratta, ***Introduction to Nuclear Engineering***, 3rd Edition, Prentice Hall (2001).