Nuclear Reactor Theory

Lesson 8: The Critical Reactor III

2-Group Diffusion Theory for Critical Systems

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Lesson 8 Objectives

Setup and solve the 2-group diffusion equation for bare critical systems.

Develop formal expressions for the 2-group k_{eff} and fast-to-thermal flux ratio for bare critical systems.

Reduce the finite reactor expressions for k_{eff} and ϕ_1/ϕ_2 to be applicable for an infinite system (k_∞ and $\phi_1/\phi_{2\infty}$ are very useful material properties).

Define each term within the 4-factor formula in words and in symbols.

Show the equivalency of the 4-factor formula for k_∞ and the expression derived formally from the 2-group diffusion equation.

Develop an approximate expression for k_{eff} for bare homogeneous systems and, with proper definition of the fast and thermal non-leakage probabilities, develop the so-called 6-factor formula.

Lesson 8 Objectives (cont.)

Explain the neutron life cycle within a thermal system in terms of the elements of the 6-factor formula.

Convert the 6-factor formula into the modified 1-group theory expression for k_{eff}.

Explain the approximations associated with the assumption of a "dilute homogeneous system".

Define the term reflector savings and explain how this is used to determine the "effective core size" of a reflected thermal reactor.

Perform modified 1-g theory critical size and critical composition calculations for bare homogeneous systems and simple core-reflector systems via hand calculations and with the use of the diluteh_gui code.

2-Grp Theory for Critical Systems

For the two-group approximation to multigroup diffusion theory, one usually assumes no upscatter ($\Sigma_{2\to1} = 0$) and no fission source in group 2 ($\chi_1 = 1.0$ and $\chi_2 = 0.0$).

With these specifications, the group 1 and 2 diffusion equations for a critical homogeneous system become

$$-D_1\nabla^2\phi_1 + \Sigma_{R1}\phi_1 - \lambda\left(\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2\right) = 0$$

$$-D_2\nabla^2\phi_2 + \Sigma_{a2}\phi_2 - \Sigma_{1\to2}\phi_1 = 0$$

Solution of these equations for the general case is quite complicated (and beyond our current scope -- note the fully coupled-structure of the two ODEs).

2-Grp Theory for Critical Systems

However, for a 1-region bare homogeneous reactor, a number of simplifying assumptions lead to a system that is easy to solve and interpret.

This procedure leads to formal expressions for k_{eff} and the fast-to-thermal flux ratio in bare critical systems.

Also, with the assumption of a large region, we get expressions for k_∞ and $\phi_1/\phi_{2\infty}$ for the material of interest (these neutronic material properties are very important in the design and analysis of real thermal reactor systems).

Bare Homogeneous Systems

To develop the desired formulations, we restrict our analysis to the case of a bare homogeneous 1-region critical system.

In addition, we argue that the extrapolation distance in each energy group is the same (recall that $d_g \approx 2.13\ D_g$, so the extrapolation distance is really energy dependent).

This latter approximation can be justified by the fact that the diffusion coefficient is not a strong function of energy, and that d is often small compared to the reactor dimensions anyway. Thus, the minor variation of d with energy is usually negligible.

With this assumption, for the group 1 and group 2 equations to be valid at every point in the reactor, the spatial forms of $\phi_1(\vec{r})$ and $\phi_2(\vec{r})$ must be identical.

This is easy to see when the leakage term is small, since all the other terms simply have constant coefficients.

Bare Homogeneous Systems (cont.)

From 1-group theory, we know that the flux curvature, $\nabla^2\phi$, is proportional to the flux shape -- that is $\nabla^2\phi = -B^2\phi$, where B^2 is a constant.

Thus, the spatial profile of the flux is the same for each energy group, and the full solution to the group 1 and group 2 equations can be written as

$$\phi_1(\vec{r}) = c_1\phi(\vec{r}) \quad \text{and} \quad \phi_2(\vec{r}) = c_2\phi(\vec{r})$$

assumes space-energy separability

where $\phi(\vec{r})$ satisfies an equation of the form

$$\nabla^2\phi(\vec{r}) + B^2\phi(\vec{r}) = 0 \quad \text{or} \quad \nabla^2\phi(\vec{r}) = -B^2\phi(\vec{r})$$

In these expressions, $\phi(\vec{r})$ (without a group subscript) represents only the spatial distribution of the flux, and $\phi_g(\vec{r}) = c_g\phi(\vec{r})$ (with a group subscript) represents the full space-energy solution.

These relationships assume space-energy separability!!!

Bare Homogeneous Systems (cont.)

In addition, since $\phi(\vec{r})$ represents the spatial profile from a 1-group bare system, we already know this distribution function for all the common bare reactor geometries.

Thus, our challenge reduces to finding the discrete energy dependence of the flux (i.e. the c_1 and c_2 values).

Now, with the above separability assumption, substituting

$$\phi_g(\vec{r}) = c_g\phi(\vec{r})$$

into the matrix form of the 2-group balance equation gives

$$\begin{bmatrix} D_1B^2 + \Sigma_{R1} - \lambda\nu\Sigma_{f1} & -\lambda\nu\Sigma_{f2} \\ -\Sigma_{1\to2} & D_2B^2 + \Sigma_{a2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where the spatial flux profile, $\phi(\vec{r})$, has canceled from the expression since we have a homogeneous system of equations.

Bare Homogeneous Systems (cont.)

This set of homogeneous algebraic equations will have a non-trivial solution if and only if the determinant of the coefficient matrix vanishes (i.e. the matrix must be singular).

This is the criticality condition for the 2-group bare homogeneous problem.

Forcing the determinant of the 2×2 matrix to zero gives

$$\left(D_1B^2+\Sigma_{R1}-\lambda\nu\Sigma_{f1}\right)\left(D_2B^2+\Sigma_{a2}\right)-\lambda\nu\Sigma_{f2}\Sigma_{1\to2}=0$$

and, separating out the terms containing the eigenvalue λ, gives

$$\left(D_1B^2+\Sigma_{R1}\right)\left(D_2B^2+\Sigma_{a2}\right)-\lambda\left[\nu\Sigma_{f1}\left(D_2B^2+\Sigma_{a2}\right)+\nu\Sigma_{f2}\Sigma_{1\to2}\right]=0$$

Bare Homogeneous Systems (cont.)

Solving this last expression for the eigenvalue λ, gives

$$\lambda_n=\frac{\left(D_1B_n^2+\Sigma_{R1}\right)\left(D_2B_n^2+\Sigma_{a2}\right)}{\nu\Sigma_{f1}\left(D_2B_n^2+\Sigma_{a2}\right)+\nu\Sigma_{f2}\Sigma_{1\to2}}$$

where the n subscript denotes that there are an infinite number of B_n^2 's that satisfy the critical bare reactor problem.

Now, recalling that $\lambda=1/k_{eff}$, for the fundamental mode (i.e. n = 1), we have

$$k_{eff}=\frac{\nu\Sigma_{f1}\left(D_2B^2+\Sigma_{a2}\right)+\nu\Sigma_{f2}\Sigma_{1\to2}}{\left(D_1B^2+\Sigma_{R1}\right)\left(D_2B^2+\Sigma_{a2}\right)}$$

2-group k_{eff} for a bare homogeneous 'critical' system

Bare Homogeneous Systems (cont.)

Recall also that the geometric buckling, B^2, is inversely proportional to the square of the characteristic dimension for the system of interest.

Thus, as the system becomes large, B^2 approaches zero and, in the limit of an infinite system, we have

$$k_\infty = \frac{\nu\Sigma_{f1}\Sigma_{a2} + \nu\Sigma_{f2}\Sigma_{1\to 2}}{\Sigma_{R1}\Sigma_{a2}}$$

2-group k_∞ is a very important material property

Also recall that the two-group fluxes only differ by a constant factor, which is usually denoted by the fast-to-thermal flux ratio.

Bare Homogeneous Systems (cont.)

Thus, from the matrix representation given previously, we have

$$\frac{\phi_1}{\phi_2} = \frac{c_1}{c_2} = \frac{\lambda\nu\Sigma_{f2}}{D_1B^2 + \Sigma_{R1} - \lambda\nu\Sigma_{f1}} = \frac{D_2B^2 + \Sigma_{a2}}{\Sigma_{1\to 2}}$$

fast-to-thermal flux ratio

For an infinite system this can be written as a simple ratio of cross sections, or

$$\left.\frac{\phi_1}{\phi_2}\right|_\infty = \frac{\Sigma_{a2}}{\Sigma_{1\to 2}}$$

another very important material property

This ratio is a good indicator of the general flux spectrum in a thermal system.

Finally, to complete this problem, we need to normalize the spatial flux distribution to the reactor power, or

$$P = \kappa\sum_g \int \Sigma_{fg}(\vec{r})\phi_g(\vec{r})\,d\vec{r}$$

Bare Homogeneous Systems (cont.)

However, $\phi_g(\vec{r}) = c_g\phi(\vec{r})$ and, for a homogeneous system, the cross sections are spatially independent. Therefore, we can write the power as

$$P = \kappa\left(c_1\Sigma_{f1} + c_2\Sigma_{f2}\right)\int\phi(\vec{r})\,d\vec{r}$$

Letting $c_2 = A$, we have

$$\phi_1(\vec{r}) = \frac{c_1}{c_2}A\phi(\vec{r}) = A\frac{\phi_1}{\phi_2}\phi(\vec{r}) \qquad \text{and} \qquad \phi_2(\vec{r}) = A\phi(\vec{r})$$

Thus, the power expression becomes

$$P = \kappa\left[\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right]A\int\phi(\vec{r})\,d\vec{r}$$

flux normalization

and the flux normalization is given by

$$A = \frac{P}{\kappa\left[\Sigma_{f1}\dfrac{\phi_1}{\phi_2} + \Sigma_{f2}\right]\displaystyle\int\phi(\vec{r})\,d\vec{r}}$$

Bare Homogeneous Systems (cont.)

This development is now complete. Although relatively brief, there is a lot of meat here!

We have seen that the flux shape in a 2-group model of a bare reactor is the same as the 1-group profile for each geometry of interest.

However, the expressions for the 2-group k_{eff} and k_∞ are quite different from their 1-group counterparts, since they take into account the production and loss of neutrons in both the fast and thermal groups.

In addition, we introduced the fast-to-thermal flux ratio and saw that this quantity is needed to define the fast flux, and that it enters into consideration in the computation of the normalization.

Bare Homogeneous Systems (cont.)

If the cross section data are available for a particular system, these expressions are relatively easy to use to compute the following quantities:

- critical size (given the material composition)
- critical composition (given the reactor geometry)
- k_{eff} for a specific material-geometry combination
- maximum value of the flux for a given power level
- etc.

There is actually a lot of information that can be obtained here…

Bare Homogeneous Systems (cont.)

The above development of 2-group theory represents a formal treatment of this subject for bare homogeneous systems.

However, in many situations, the detailed cross section data needed to evaluate the above formal expressions are not readily available.

The difficulty here often lies with determining the fast cross sections for the fuel (primarily Σ_{f1} and Σ_{a1} which are associated with fast fission and resonance absorption effects, respectively).

Although these quantities can be computed accurately with sophisticated cross section processing codes, it would be nice to have an alternative, relatively simple approach for performing preliminary analyses.

Bare Homogeneous Systems (cont.)

In addition, there is NOT a lot of physical insight that can be gained from the formal expressions.

To address these concerns, we will introduce the four-factor and six-factor formulas and the basic ideas behind modified 1-group theory.

These concepts will give us some simple 2-group computational capability and provide additional insight and understanding of the neutron life cycle in thermal systems.

> Fission neutrons are born at high energy, they slow down via elastic and inelastic neutron scattering, and then, as thermal neutrons, they cause additional fissions to continue the cycle, but we also must consider both fast and thermal parasitic absorption, fast fission, and leakage from the system of interest…

Four Factor Formula

Let's focus on an infinite system for the moment.

In such systems, of course, there is no leakage, so the only ultimate loss term is absorption.

In the fast group, neutrons can get absorbed (primarily in the fuel and structure resonances) or scatter to thermal.

At thermal, all the neutrons that have scattered from group 1 get absorbed, where some of the absorptions involve fission in the fuel.

The fissions that occur (at both fast and thermal energies) produce neutrons at high energy, which start the neutron life cycle all over again.

To describe this process in a quantitative manner, let's define a number of terms (see next few slides):

Four Factor Formula (cont.)

$$\text{thermal utilization} = f = \frac{\text{thermal absorption rate in fuel}}{\text{total thermal absorption rate}}$$

thermal utilization

$$f = \frac{\int \Sigma_{a2}^{F}\phi_2\, d\vec{r}}{\int \Sigma_{a2}\phi_2\, d\vec{r}} = \frac{\bar{\Sigma}_{aF}\langle\phi_2\rangle}{\left(\bar{\Sigma}_{aF}+\bar{\Sigma}_{aM}\right)\langle\phi_2\rangle} = \frac{\bar{\Sigma}_{aF}}{\bar{\Sigma}_{aF}+\bar{\Sigma}_{aM}} = \frac{\bar{\Sigma}_{aF}}{\bar{\Sigma}_{a}}$$

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$$\text{reproduction factor} = \eta_T = \frac{\text{total neutrons emitted from thermal fission}}{\text{thermal neutrons absorbed in fuel}}$$

thermal reproduction factor

$$\eta_T = \frac{\langle \nu\Sigma_f\phi\rangle_T}{\langle \Sigma_a^F\phi\rangle_T} = \frac{\nu\Sigma_{f2}}{\Sigma_{a2}^{F}}\frac{\langle\phi_2\rangle}{\langle\phi_2\rangle} = \frac{\nu\bar{\Sigma}_f}{\bar{\Sigma}_{aF}}$$

Four Factor Formula (cont.)

$$\text{fast fission factor} = \varepsilon = \frac{\text{total neutrons emitted from fission (fast and thermal)}}{\text{neutrons emitted from thermal fission}}$$

fast fission factor

$$\varepsilon = \frac{\langle \nu\Sigma_{f1}\phi_1\rangle + \langle \nu\Sigma_{f2}\phi_2\rangle}{\langle \nu\Sigma_{f2}\phi_2\rangle} = \frac{\nu\Sigma_{f1}\langle\phi_1/\phi_2\rangle + \nu\Sigma_{f2}}{\nu\Sigma_{f2}}$$

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$$\text{resonance escape probability} = p = \text{probability that a fission neutron is not absorbed while slowing down}$$

resonance escape probability

$$p = \frac{\langle \Sigma_{1\to 2}\phi_1\rangle}{\langle \Sigma_{a1}\phi_1\rangle + \langle \Sigma_{1\to 2}\phi_1\rangle} = \frac{\Sigma_{1\to 2}}{\Sigma_{a1}+\Sigma_{1\to 2}}$$

Four Factor Formula (cont.)

Now let's define the thermal absorption rate as

$$\text{thermal absorption rate} = \int \Sigma_{a2}\phi_2 \, d\vec{r} = \Sigma_{a2}\langle\phi_2\rangle = \bar{\Sigma}_a\langle\phi_2\rangle = \left(\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}\right)\langle\phi_2\rangle$$

where $\bar{\Sigma}_{aM}$ represents the absorption cross section in the moderator, coolant, structure, etc. (includes everything but fuel).

With these definitions, we can write explicit expressions for the following quantities that help describe the neutron life cycle:

- # of thermal neutrons absorbed in reactor = $\bar{\Sigma}_a\langle\phi_2\rangle = \bar{\Sigma}_a\langle\phi_T\rangle$
- # of thermal neutrons absorbed in fuel = $f\bar{\Sigma}_a\langle\phi_T\rangle$
- # of neutrons emitted from thermal fission = $\eta_T f\bar{\Sigma}_a\langle\phi_T\rangle$
- # of neutrons emitted from all fission = $\varepsilon\eta_T f\bar{\Sigma}_a\langle\phi_T\rangle$
- # of thermal neutrons in next generation = $p\varepsilon\eta_T f\bar{\Sigma}_a\langle\phi_T\rangle$

Four Factor Formula (cont.)

Thus, we see that these terms can indeed be used to help define the overall neutron balance in a 2-group infinite system.

With the above parameters, we can also develop a simple expression for k_∞.

In particular, since that the downscatter rate is equal to the thermal absorption rate, the infinite multiplication factor can be written as

$$k_\infty = \frac{\text{production rate}}{\text{loss rate}} = \frac{\varepsilon\eta_T f\bar{\Sigma}_a\langle\phi_T\rangle}{\Sigma_{a1}\langle\phi_1\rangle + \bar{\Sigma}_a\langle\phi_T\rangle} = \frac{\varepsilon\eta_T f\bar{\Sigma}_a\langle\phi_T\rangle}{\Sigma_{a1}\langle\phi_1\rangle + \Sigma_{1\to2}\langle\phi_1\rangle}$$

From the definition of the resonance escape probability, the denominator of this expression can be written as

$$\langle\Sigma_{a1}\phi_1\rangle + \langle\Sigma_{1\to2}\phi_1\rangle = \frac{\langle\Sigma_{1\to2}\phi_1\rangle}{p}$$

Four Factor Formula (cont.)

Thus, the expression for k_∞ is simply

$$k_\infty = \frac{\varepsilon \eta_T f p \bar{\Sigma}_a \langle \phi_T \rangle}{\Sigma_{1\to 2} \langle \phi_1 \rangle} = \frac{\varepsilon \eta_T f p \bar{\Sigma}_a \langle \phi_T \rangle}{\bar{\Sigma}_a \langle \phi_T \rangle} = \eta_T f p \varepsilon$$

Thus, the so-called four factor formula is just a simple expression for the multiplication factor in an infinite system

$$k_\infty = \eta_T f p \varepsilon$$

k_∞ is an important material property

This is an important property that characterizes the reactivity potential of a given material composition.

It is also very instructive to show that the two expressions for k_∞ are identical [derived from formal 2-group theory and the 4-factor formula].

Four Factor Formula (cont.)

To show this, we expand the expression for $k_\infty = \eta_T f p \varepsilon$ in terms of the four factors written in full detail, or

$$k_\infty = \left[\frac{\langle \nu\Sigma_{f2}\phi_2 \rangle}{\langle \Sigma^F_{a2}\phi_2 \rangle}\right]\left[\frac{\langle \Sigma^F_{a2}\phi_2 \rangle}{\langle \Sigma_{a2}\phi_2 \rangle}\right]\left[\frac{\langle \Sigma_{1\to 2}\phi_1 \rangle}{\langle \Sigma_{R1}\phi_1 \rangle}\right]\left[\frac{\langle \nu\Sigma_{f1}\phi_1 \rangle + \langle \nu\Sigma_{f2}\phi_2 \rangle}{\langle \nu\Sigma_{f2}\phi_2 \rangle}\right]$$

and, cancelling the common factors contained in both the numerator and denominator, gives

$$k_\infty = \left[\frac{\langle \nu\Sigma_{f1}\phi_1 \rangle + \langle \nu\Sigma_{f2}\phi_2 \rangle}{\langle \Sigma_{a2}\phi_2 \rangle}\right]\left[\frac{\langle \Sigma_{1\to 2}\phi_1 \rangle}{\langle \Sigma_{R1}\phi_1 \rangle}\right]$$

But, in an infinite system, the downscatter rate from the fast group is equal to the thermal absorption rate. Thus, the above expression reduces to

$$k_\infty = \frac{\langle \nu\Sigma_{f1}\phi_1 \rangle + \langle \nu\Sigma_{f2}\phi_2 \rangle}{\langle \Sigma_{R1}\phi_1 \rangle} = \frac{\nu\Sigma_{f1}(\phi_1/\phi_2) + \nu\Sigma_{f2}}{\Sigma_{R1}(\phi_1/\phi_2)} = \frac{\nu\Sigma_{f1}(\Sigma_{a2}/\Sigma_{1\to 2}) + \nu\Sigma_{f2}}{\Sigma_{R1}(\Sigma_{a2}/\Sigma_{1\to 2})}$$

Four Factor Formula (cont.)

or $$k_\infty = \frac{\nu\Sigma_{f1}\Sigma_{a2} + \nu\Sigma_{f2}\Sigma_{1\to 2}}{\Sigma_{R1}\Sigma_{a2}}$$

This is identical to the formal equation derived from 2-group theory

where we have used the expression for the fast-to-thermal flux ratio in an infinite system in the last manipulation step.

Therefore, with the proper definition of the individual terms, the four factor formula is identical to the expression derived from formal 2-group theory for infinite homogeneous systems.

Either expression for the 2-group k_∞ can be used as appropriate:

$$k_\infty = \frac{\nu\Sigma_{f1}\Sigma_{a2} + \nu\Sigma_{f2}\Sigma_{1\to 2}}{\Sigma_{R1}\Sigma_{a2}}$$

This form is used when detailed cross section data are available

$$k_\infty = \eta_T f p \varepsilon$$

This form is used when estimates of the four factors are available or when trying to explain the neutron life cycle in an infinite system

6-Factor Formula & Modified 1-Grp Theory

Using the terms from the four factor formula to rewrite the 2-grp diffusion model slightly, we can also develop an alternate expression for k_{eff} for a finite system.

To do this we will follow the procedure from Lamarsh.

In particular, we start with the formal 2-group neutron balance equation with the following changes/assumptions:

1. Rewrite the fission source term as

$$S^{fission} = \lambda\left(\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2\right) = \lambda\varepsilon\eta_T f\,\Sigma_{a2}\phi_2 = \lambda\frac{k_\infty}{p}\Sigma_{a2}\phi_2$$

2. Make the assumption that the fast absorption cross section is small compared to the downscatter cross section, or

$$\Sigma_{R1} = \Sigma_{a1} + \Sigma_{1\to 2} \approx \Sigma_{1\to 2}$$

6-Factor Formula & Modified 1-Grp Theory

3. Modify the downscatter source to account for the above assumption -- since the fast flux will be somewhat high due to the elimination of the fast absorption term, we reduce the downscatter rate by the resonance escape probability, or

$$S^{\text{downscatter}} = \Sigma_{1\to 2}\phi_1 \Rightarrow p\Sigma_{1\to 2}\phi_1$$

With these modifications, the fast and thermal balance equations become

$$-D_1\nabla^2\phi_1 + \Sigma_{1\to 2}\phi_1 - \lambda\frac{k_\infty}{p}\Sigma_{a2}\phi_2 = 0$$

$$-D_2\nabla^2\phi_2 + \Sigma_{a2}\phi_2 - p\Sigma_{1\to 2}\phi_1 = 0$$

Now, with these equations, we can follow the same procedure as before to derive an expression for k_{eff} for the system of interest.

6-Factor Formula & Modified 1-Grp Theory

Again we let $\phi_g(\vec{r}) = c_g\phi(\vec{r})$ where the spatial solution is given by an equation of the form $\nabla^2\phi + B^2\phi = 0$ (i.e. the 1-group critical reactor equation).

Then, upon substitution and putting the resultant algebraic equations into matrix form, we have

$$\begin{bmatrix} D_1B^2 + \Sigma_{1\to 2} & -\lambda\frac{k_\infty}{p}\Sigma_{a2} \\ -p\Sigma_{1\to 2} & D_2B^2 + \Sigma_{a2} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Again the determinant of the coefficient matrix must be zero for a non-trivial solution, or

$$\left(D_1B^2 + \Sigma_{1\to 2}\right)\left(D_2B^2 + \Sigma_{a2}\right) - \lambda k_\infty\Sigma_{1\to 2}\Sigma_{a2} = 0$$

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6-Factor Formula & Modified 1-Grp Theory

and solving for λ

$$\lambda = \frac{\left(D_1 B^2 + \Sigma_{1\to 2}\right)\left(D_2 B^2 + \Sigma_{a2}\right)}{k_\infty \Sigma_{1\to 2}\Sigma_{a2}}$$

or

$$k_{eff} = \frac{1}{\lambda} = \frac{k_\infty \Sigma_{1\to 2}\Sigma_{a2}}{\left(D_1 B^2 + \Sigma_{1\to 2}\right)\left(D_2 B^2 + \Sigma_{a2}\right)}$$

2-group theory with approximations from Lamarsh

This expression should be compared to the expression given previously -- derived from formal 2-group theory with minimal approximations,

$$k_{eff} = \frac{\nu\Sigma_{f1}\left(D_2 B^2 + \Sigma_{a2}\right) + \nu\Sigma_{f2}\Sigma_{1\to 2}}{\left(D_1 B^2 + \Sigma_{R1}\right)\left(D_2 B^2 + \Sigma_{a2}\right)}$$

formal 2-group diffusion theory

6-Factor Formula & Modified 1-Grp Theory

The current development (from Lamarsh) has some additional assumptions and, therefore, it is only an approximation to the formal 2-group result.

However, for preliminary estimates of k_{eff}, this approximate result is often much easier to apply because a formal set of 2-group macroscopic cross sections are not required.

To see this more clearly, recall the definitions of the thermal neutron age, τ_T, and thermal diffusion area, L_T^2, where

$$\tau_T = \frac{D_1}{\Sigma_{1\to 2}} \quad \text{and} \quad L_T^2 = \frac{D_2}{\Sigma_{a2}}$$

Using these definitions, the approximate k_{eff} expression becomes

obtained by manipulating the $\Sigma_{1\to 2}\Sigma_{a2}$ factor

$$k_{eff} = k_\infty \left(\frac{1}{1+\tau_T B^2}\right)\left(\frac{1}{1+L_T^2 B^2}\right)$$

2-group k_{eff} for a bare homogeneous system

6-Factor Formula & Modified 1-Grp Theory

Note that the two factors in parentheses are formal expressions for the fast and thermal non-leakage probabilities in the system, where

$$P_F = \frac{\text{downscatter rate}}{\text{total loss rate}} = \frac{\langle \Sigma_{1\to 2}\phi_1 \rangle}{\langle D_1 B^2 \phi_1 \rangle + \langle \Sigma_{1\to 2}\phi_1 \rangle} = \frac{1}{1+\tau_T B^2}$$

and

$$P_T = \frac{\text{absorption rate}}{\text{total loss rate}} = \frac{\langle \Sigma_{a2}\phi_2 \rangle}{\langle D_2 B^2 \phi_2 \rangle + \langle \Sigma_{a2}\phi_2 \rangle} = \frac{1}{1+L_T^2 B^2}$$

Thus, with these definitions, the expression for the multiplication factor in a bare homogeneous reactor can be written as

$$k_{eff} = k_\infty P_T P_F = \eta_T f p \varepsilon P_F P_T$$

six factor formula

This is called the six factor formula -- and it is often easier to estimate the terms within this expression.

6-Factor Formula & Modified 1-Grp Theory

The 6-factor formula is quite important when performing preliminary computations of critical size or critical composition, and it is also quite useful in describing the life cycle of neutrons in a thermal reactor.

To illustrate its usefulness for describing the neutron life cycle, consider the following example that comes directly from the DOE handbook (see next slide)…

Be sure you are very comfortable with this example!!!

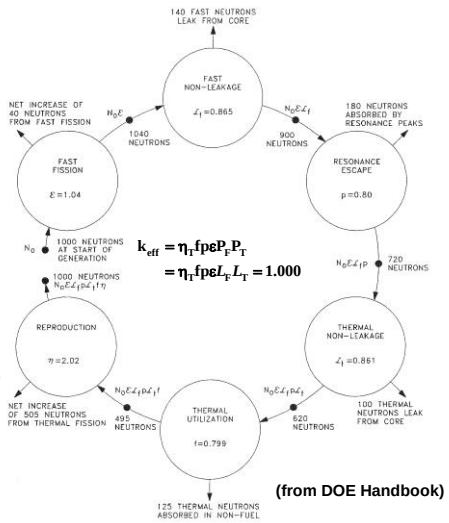
6-Factor Formula & Modified 1-Grp Theory

Let's start with N_o fast neutrons from thermal fission, then

- $N_o\varepsilon$ total fast neutrons
- $N_o\varepsilon P_F p$ neutrons make it to thermal
- $N_o\varepsilon P_F p P_T f$ n's get absorbed in fuel
- $N_o\varepsilon P_F p P_T f \eta_T$ fission n's produced in next generation from thermal fission

The multiplication factor is the ratio of neutrons in one generation to the neutrons in the previous generation at some common point in the life cycle, or

$$k_{eff} = \frac{N_o \varepsilon P_F p P_T f \eta_T}{N_o} = \eta_T f p \varepsilon P_F P_T$$



(from DOE Handbook)

6-Factor Formula & Modified 1-Grp Theory

Now, as a final task in our development of 2-group theory, we can also put the 6-factor formula into a form similar to 1-group theory -- that is, we want to develop the so-called modified 1-group theory formula for k_{eff}.

First we recall the 1-group expression

$$k_{eff} = \frac{k_\infty}{1 + L^2 B^2} = k_\infty P_{NL}$$

actual 1-group theory

where P_{NL} is the non-leakage probability and $k_\infty = \eta f$.

Now, for 2-group theory, let's expand the denominator of the P_F and P_T terms, or

$$k_{eff} = \frac{k_\infty}{\left(1 + L_T^2 B^2\right)\left(1 + \tau_T B^2\right)} = \frac{k_\infty}{1 + \left(L_T^2 + \tau_T\right) B^2 + L_T^2 \tau_T B^4}$$

6-Factor Formula & Modified 1-Grp Theory

For a large system, B^2 is quite small and B^4 is very small.

In this case,

$$L_T^2 \tau_T B^4 << \left(L_T^2 + \tau_T\right) B^2$$

and we can write the following "modified 1-group theory" critical condition as

$$k_{eff} = \frac{k_\infty}{1 + \left(L_T^2 + \tau_T\right) B^2} = \frac{k_\infty}{1 + M_T^2 B^2}$$

modified 1-group theory

where $M_T^2 = L_T^2 + \tau_T$ is called the migration area .

Note that the name "modified1-group theory" is somewhat misleading since this is really a specific approximation to a 2-group problem -- it just happens to "look like" the 1-group formula…

Critical Size and Composition

The previous development is used in preliminary analyses for the critical size and critical composition of bare core geometries.

In addition, the same basic techniques can also be used for reflected cores, if some information about the reflector savings, δ, is known (see below).

The hand computations rely on a number of assumptions that allow one to readily estimate the parameters within the modified 1-group theory expression for k_{eff}.

In particular, the assumption of a dilute homogeneous system is key to resolving a lot of the necessary data for the calculations -- where the word "dilute" implies that the fuel composition is only a small component of the overall homogeneous mixture.

Critical Size and Composition (cont.)

For example, for a two-component dilute homogeneous system (consisting of only fuel and moderator -- where the "moderator" here is everything but fuel), the macroscopic cross sections for the mixture can be approximated as follows:

The transport cross section is given as

$$\Sigma_{tr} = \Sigma_{tr}^{F} + \Sigma_{tr}^{M} = N^{F}\sigma_{tr}^{F} + N^{M}\sigma_{tr}^{M}$$

But, although the microscopic data are on the same order of magnitude (i.e. σ_{tr}^{F} is of the same order as σ_{tr}^{M}), the dilute nature of the mixture tells us that $N^{M} >> N^{F}$.

Thus, the expression for the transport cross section reduces to

$$\Sigma_{tr} \approx N^{M}\sigma_{tr}^{M} \approx \Sigma_{tr}^{M}$$

Critical Size and Composition (cont.)

and the diffusion coefficient for the mixture becomes

$$D = \frac{1}{3\Sigma_{tr}} \approx \frac{1}{3\Sigma_{tr}^{M}} = D_{M}$$

For dilute homogeneous systems, $D \approx D_M$

Similarly, for the downscatter cross section, we make the same set of assumptions, giving

$$\Sigma_{1\to 2} = \Sigma_{1\to 2}^{F} + \Sigma_{1\to 2}^{M} \approx \Sigma_{1\to 2}^{M}$$

Thus, we can write the thermal neutron age as

$$\tau_{T} = \frac{D_1}{\Sigma_{1\to 2}} \approx \frac{D_1^{M}}{\Sigma_{1\to 2}^{M}} = \tau_{TM}$$

For dilute homogeneous systems, $\tau_T \approx \tau_{TM}$

Also, for a homogeneous system, the basic definition of the thermal utilization gives

$$f = \frac{\bar{\Sigma}_a^{F}}{\bar{\Sigma}_a} = \frac{\bar{\Sigma}_a^{F}}{\bar{\Sigma}_a^{F} + \bar{\Sigma}_a^{M}} = \frac{\bar{\Sigma}_a^{F}/\bar{\Sigma}_a^{M}}{\bar{\Sigma}_a^{F}/\bar{\Sigma}_a^{M} + 1} = \frac{z}{z+1}$$

$$z = \frac{\bar{\Sigma}_a^{F}}{\bar{\Sigma}_a^{M}}$$

Critical Size and Composition (cont.)

where we have defined z as the ratio of the fuel and moderator thermal absorption cross sections (this is done simply for convenience in subsequent manipulations), or

$$z = \bar{\Sigma}_a^F / \bar{\Sigma}_a^M$$

With this definition of z and f, we can write an expression for the thermal diffusion area as

$$L_T^2 = \frac{D_2}{\Sigma_{a2}} \approx \frac{\bar{D}_M}{\bar{\Sigma}_a} = \frac{\bar{D}_M}{\bar{\Sigma}_a^F + \bar{\Sigma}_a^M} = \frac{\bar{D}_M / \bar{\Sigma}_a^M}{\bar{\Sigma}_a^F / \bar{\Sigma}_a^M + 1} = \frac{L_{TM}^2}{z+1}$$

and, noting that $1 - f = 1 - \frac{z}{z+1} = \frac{1}{z+1}$ we have

$$L_T^2 \approx (1-f) L_{TM}^2$$

Note that appropriate density and temperature corrections must be applied within all these formulas, as needed.

For dilute homogeneous systems, $L_T^2 = (1-f) L_{TM}^2$

Critical Size and Composition (cont.)

One final assumption, concerns the resonance escape probability and the fast fission factor.

In dilute homogeneous systems, the resonance escape probability, p, is slightly less than unity and the fast fission factor, ε, is slightly greater than unity.

However, it is not easy to get a good quantitative estimate for either of these quantities (not without a fair amount of effort).

Thus, for preliminary calculations, it is often assumed that the product of these two factors is approximately unity, or

$$p\varepsilon \approx 1.0$$

For dilute homogeneous systems, $p\varepsilon \approx 1.0$

Now, with some background on the various approximations involved and the above expressions for several of the needed intermediate quantities, we are ready to outline the actual computations required in typical analyses...

Critical Size and Composition (cont.)

In particular, the problem solution scheme is somewhat different for the two usual cases of interest here, as follows:

1. Given the Material Composition, Compute the 'Critical' Size

This is the easier of the two cases. Here, with the fuel and moderator compositions known, one can obtain cross section data for the fuel and moderator and compute the values of $k_\infty \approx \eta f(1.0)$ and $M_T^2 = L_T^2 + \tau_T$.

We then solve the modified 1-group equation for the buckling,

$$k_{eff} = \frac{k_\infty}{1 + M_T^2 B^2} \quad \longrightarrow \quad B^2 = \frac{k_\infty / k_{eff} - 1}{M_T^2}$$

and compute the desired critical dimension.

Let's do an example!!!

Critical Size and Composition (cont.)

2. Given the Core Size, Compute the 'Critical' Fuel Composition

For this case, we can immediately compute the buckling from the known core dimensions.

However, since we don't know the fuel composition, N^F, or $z = \bar{\Sigma}_a^F / \bar{\Sigma}_a^M$, we simply write the expression for k_{eff} in terms of one of these quantities, and then solve for this unknown quantity.

Doing this for a given k_{eff} with z as the desired quantity gives

$$k_{eff} = \frac{\eta_T z/(z+1)}{1 + B^2\left(\frac{L_{TM}^2}{z+1} + \tau_{TM}\right)}$$

and, solving this for z (see the Lecture Notes for details), gives

$$z = \frac{1 + B^2\left(L_{TM}^2 + \tau_{TM}\right)}{\eta_T / k_{eff} - 1 - B^2 \tau_{TM}}$$

Critical Size and Composition (cont.)

This expression for the fuel composition is used for simple hand computations when the geometry is known (i.e. B^2 is known) and one needs to compute the critical composition. Once z has been determined via the above equation, then

$$\bar{\Sigma}_a^F = z\bar{\Sigma}_a^M \qquad \text{and} \qquad N_F = z N_M \frac{\bar{\sigma}_a^M}{\bar{\sigma}_a^F}$$

which is the usual desired result from a "critical composition calculation".

Let's do an example!!!

The Parametric Approach

The methodology described above is based on Lamarsh…

An easier, and possibly more useful approach, would be to simply compute k_{eff} for a given geometry and material combination -- and, with this ability, one can easily perform a parametric study for any parameter of interest.

Critical Size and Composition (cont.)

For the Case 1 scenario, one holds the composition fixed and varies the core size, each time computing a different k_{eff}.

Then, a simple plot of k_{eff} vs. core size will easily show the critical core size (i.e. when $k_{eff} = 1.0$) as well as show how rapidly the multiplication factor changes with the core dimensions.

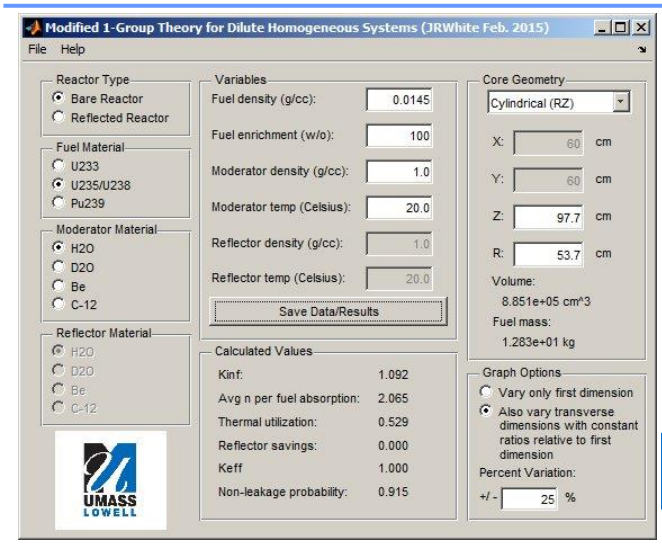
Similarly, for the Case 2 situation, one holds the core size fixed and varies the fuel composition -- giving information on how k_{eff} varies with the fuel loading as well as the critical composition.

This parametric approach is actually more intuitive!!!

One simply puts the computed values of B^2, $k_\infty \approx \eta f(1.0)$, and M_T^2 into the modified 1-group formula and computes k_{eff} directly for the given geometry and materials combination -- this is a much more straightforward approach to the problem.

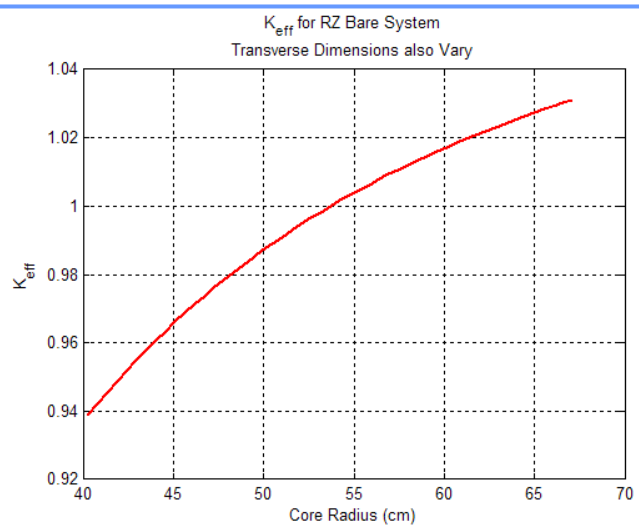
This parametric approach is the basis of the diluteh_gui code…

The diluteh_gui Interface



How about a demo???

Parametric Study: k_{eff} vs. Core Size



Reflected Core Calculations

As a final note, we should emphasize that the above analyses are only directly applicable for the solution of bare homogeneous critical core problems, since the modified 1-group formula for k_{eff} was derived explicitly for this situation.

However, it is possible to view a core-reflector system as a bare core with an "effective core size" that is increased to account for the effect of the reflector on the system.

In particular, the reflector savings, δ, is defined as the difference in the critical dimension of the bare and reflected systems.

Clearly, since the reflector reduces the net core leakage, the critical size of a reflected core will be smaller than the size of a bare critical core.

Or, from a different perspective, if a bare core with a given k_{eff} is surrounded by an infinite reflector, then the system multiplication factor will increase.

Reflected Core Calculations (cont.)

With this simple rationalization, one can estimate k_{eff} for the reflected system by effectively increasing the physical core size by the reflector savings and then use modified 1-group theory for a hypothetical bare core with the increased "effective core size".

In particular, Lamarsh gives approximate correlations for estimating the reflector savings in thermal systems, where

$$\delta \approx \frac{\bar{D}_c}{\bar{D}_r} L_{Tr} \qquad \text{(for all but water systems)}$$

and

$$\delta \approx 7.2 + 0.10\left(M_T^2 - 40.0\right) \qquad \text{(for water moderated and reflected systems)}$$

Now, with a known δ, the effective size of a reflected reactor becomes $a_o + 2\delta$, or $R + \delta$, etc., and this increased dimension would then be used to compute the effective buckling for the reflected system.

Reflected Core Calculations (cont.)

For example, the effective buckling for a reflected parallelepiped reactor becomes

$$B^2 = \left(\frac{\pi}{a+2\delta}\right)^2 + \left(\frac{\pi}{b+2\delta}\right)^2 + \left(\frac{\pi}{c+2\delta}\right)^2$$

where a, b, and c are the real core dimensions, and a + 2δ, b + 2δ, and c + 2δ are the effective dimensions used to account for the reduced leakage that will occur in the reflected system.

With this simple change, the above modified 1-group theory for bare cores can also be easily applied to reflected systems.

This simple "effective core size" approach for reflected systems has been implemented into the diluteh_gui code so that approximate critical size and composition calculations can be made for both bare and reflected systems.

You should definitely explore the capabilities of diluteh_gui…

Lesson 8 Summary

In this Lesson we have briefly discussed the following subjects:

The setup and solution of the 2-group diffusion equation for bare critical systems.

The formal expressions for the 2-group k_{eff} and fast-to-thermal flux ratio for bare critical systems.

How to reduce the finite reactor expressions for k_{eff} and ϕ_1/ϕ_2 to be applicable for an infinite system (k_∞ and $\phi_1/\phi_{2\infty}$ are very useful material properties).

The definition of each term within the 4-factor formula in words and in symbols.

The equivalency of the 4-factor formula for k_∞ and the expression derived formally from the 2-group diffusion equation.

How to develop an approximate 6-factor formula for k_{eff} for bare homogeneous systems.

Lesson 8 Summary (cont.)

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How to explain the neutron life cycle within a thermal system in terms of the elements of the 6-factor formula.

The conversion of the 6-factor formula into the modified 1-group theory expression for k_{eff}.

The approximations associated with the assumption of a "dilute homogeneous system".

The term reflector savings and how this is used to determine the "effective core size" of a reflected thermal reactor.

How to perform modified 1-group theory critical size and critical composition calculations for bare homogeneous systems and simple core-reflector systems via hand calculations and with the use of the diluteh_gui code.