Nuclear Reactor Theory

Lesson 4: Cross Sections for Preliminary Calculations

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Lesson 4 Objectives

Discuss the interpretation of the diffusion length and extend this basic concept to 2-group problems.

Explain the important terminology, assumptions, and basic procedure needed to develop the expression for the thermally averaged absorption (or fission) cross section:

$$\bar{\Sigma}_a(T) = \Sigma_{a2}(T) = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_o)\left(\frac{T_o}{T}\right)^{1/2}$$

Explain and apply the expressions that account for density and temperature corrections to the cross sections tabulated in several Lamarsh tables.

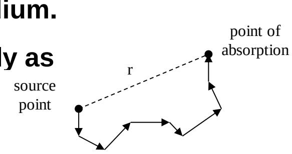
Obtain 1-group and 2-group cross sections for preliminary calculations from the Lamarsh tables in the Lecture Notes and/or via the cross_sections_gui code and apply these in various situations.

Interpretation of the Diffusion Length

We have seen that the diffusion area, L^2, or diffusion length, L, is an important material property for characterizing the behavior of neutrons diffusing within a moderating medium.

For 1-group problems, L^2 was defined simply as

$$L^2 = \frac{D}{\Sigma_a}$$



However, it is easy to show (see detailed Lecture Notes) that L^2 can be interpreted as 1/6th the square of the crow flight distance, r, from the source point to the point of absorption.

Or, more concisely, the diffusion length, L, is a measure of the distance traveled from birth to death of the neutron.

Thus, materials with large values of L have more diffusion and/or less absorption than materials with smaller L, and neutrons tend to diffuse farther with less attenuation in materials with large L.

Diffusion Length in 2-Group Problems

The above physical interpretation can also be extended to 2-group diffusing media problems.

In particular, for a homogeneous non-multiplying medium (with no upscatter), we have

$$-D_1\nabla^2\phi_1 + (\Sigma_{a1} + \Sigma_{1\to 2})\phi_1 = Q_1$$

$$-D_2\nabla^2\phi_2 + \Sigma_{a2}\phi_2 - \Sigma_{1\to 2}\phi_1 = Q_2$$

Since the coefficients are constant, we can divide each equation by the diffusion coefficient, giving

$$\nabla^2\phi_1 - \frac{1}{L_1^2}\phi_1 = -\frac{Q_1}{D_1}$$

$$\nabla^2\phi_2 - \frac{1}{L_2^2}\phi_2 + \frac{\Sigma_{1\to 2}}{D_2}\phi_1 = -\frac{Q_2}{D_2}$$

Diffusion Length in 2-Group Problems

where the fast and thermal diffusion areas are defined explicitly as

$$L_1^2 = \frac{D_1}{\Sigma_{a1} + \Sigma_{1\to2}} \quad \text{and} \quad L_2^2 = \frac{D_2}{\Sigma_{a2}}$$

in general $L_g^2 = \frac{D_g}{\Sigma_{Rg}}$

In a moderating medium, the primary interaction at high energy is neutron scattering. Thus, $\Sigma_{1\to2}$ is usually much greater than Σ_{a1} (i.e. resonance absorption is not as important in low A nuclides as it is in high A fuel and/or structural materials).

With this assumption, the expression for the fast diffusion area reduces to $D_1/\Sigma_{1\to2}$ which is typically called the thermal neutron age, τ_T, or

$$L_1^2 = \frac{D_1}{\Sigma_{a1} + \Sigma_{1\to2}} \quad \Rightarrow \quad \tau_T = \frac{D_1}{\Sigma_{1\to2}}$$

Diffusion Length in 2-Group Problems

The thermal neutron age (or fast diffusion area) is very similar to the thermal diffusion area, except that it applies to the fast group instead of the thermal group.

Thus,

fast group

$L_1^2 \approx \tau_T =$ measure of the distance traveled from the point where a neutron is born as a fast neutron to the point where it dies as a fast neutron (gets absorbed or slows down to thermal)

$L_2^2 = L_T^2 =$ measure of the distance traveled from the point where a thermal neutron is born to the point where it is finally absorbed

thermal group

Cross Sections for Preliminary Calculations

Besides the diffusion lengths, we also need a variety of additional 1-group and 2-group cross sections for performing preliminary calculations.

Lamarsh has a series of data tables that can serve as the basis for much of the needed cross section data, and these are included as an Appendix in the formal Lecture Notes, as follows:

1-group data for a sodium-cooled fast reactor:

- Table 6.1 in Lamarsh

2-group data in Lamarsh for thermal reactors:

- Table 5.3 for fast data for moderators
- Table 5.2 for thermal data for moderators
- Table 6.3 for η_T data versus temperature
- Table 3.2 for the non-1/v factors for some materials

Thermally Averaged Cross Sections

What about the 2200 m/s data?

There are also several possible sources for 2200 m/s cross sections, including:

- The Tables of Nuclear Data at the Japan Nuclear Data Center (JNDC) at http://wwwndc.jaea.go.jp/NuC/index.html
- The cross section tables vs. energy within the JANIS program (can be obtained from the www.nea.fr/janis/ website)
- Appendix II in Lamarsh (if available)

Of course, we know that, for use in the analysis of the thermal energy region, we really need cross sections averaged over the energy region of interest.

We cannot use the 2200 m/s cross section directly in most computations -- we need a thermally averaged cross section !!!

Thermally Averaged Cross Sections

For example, for the absorption cross section (for each isotope), proper averaging over the thermal region gives

$$\bar{\Sigma}_a = \frac{\int_T \Sigma_a(E)\phi(E)dE}{\int_T \phi(E)dE} = \frac{1}{\phi_T}\int_T \Sigma_a(E)\phi(E)dE$$

where the T implies an integration over thermal energies and ϕ_T is the thermal flux.

From previous discussions (from Fundamentals of NSE) concerning 1/v and non-1/v cross sections, we know that

$$\int_T \Sigma_a(E)\phi(E)dE = g_a(T)\Sigma_a(E_o)\phi_o$$

where ϕ_o is the 2200 m/s flux.

Combining these expressions gives

$$\bar{\Sigma}_a = g_a(T)\Sigma_a(E_o)\frac{\phi_o}{\phi_T}$$

Thermally Averaged Cross Sections

Now recall that ϕ_o is given by

$$\phi_o = nv_o = n\left(\frac{2E_o}{m}\right)^{1/2} = n\left(\frac{2kT_o}{m}\right)^{1/2}$$

where n is the total neutron density, and ϕ_T is given by

$$\phi_T = \int_T n(E)v(E)dE = \int_T nf(E)v(E)dE$$

Now, if we make the assumption that the thermal flux spectrum, f(E), is given by the thermal Maxwellian distribution function, then we can actually perform the integrations implied in the expression for ϕ_T .

Recall that $v(E) = \sqrt{2E/m}$ and that the Maxwellian is given by

$$f(E) = \frac{2\pi}{(\pi kT)^{3/2}}E^{1/2}e^{-E/kT}$$

Thermally Averaged Cross Sections

Using these expressions, ϕ_T becomes

$$\phi_T = \left[\frac{2\pi n}{(\pi kT)^{3/2}}\right]\left(\frac{2}{m}\right)^{1/2}\int_0^\infty E e^{-E/kT} dE$$

$$\phi_T = \left[\frac{2\pi n}{(\pi kT)^{3/2}}\right]\left(\frac{2}{m}\right)^{1/2}(kT)^2 = \frac{2}{\sqrt{\pi}} n \left(\frac{2}{m}\right)^{1/2}(kT)^{1/2}$$

where we have used the following integral result as part of the above manipulations,

$$\int_0^\infty E e^{-E/kT} dE = \frac{e^{-E/kT}}{(1/kT)^2}\left(-\frac{E}{kT}-1\right)\Bigg|_0^\infty = -(kT)^2[0-1] = (kT)^2$$

Note also that we have integrated the thermal flux expression over the range $0 \le E \le \infty$ since there is negligible contribution to the overall integral above thermal (about 1 eV).

Thermally Averaged Cross Sections

Now, with expressions for ϕ_o and ϕ_T

$$\phi_o = n\left(\frac{2kT_o}{m}\right)^{1/2} \qquad \phi_T = \frac{2}{\sqrt{\pi}} n \left(\frac{2}{m}\right)^{1/2}(kT)^{1/2}$$

we have

$$\frac{\phi_o}{\phi_T} = \frac{\sqrt{\pi}}{2}\left(\frac{T_o}{T}\right)^{1/2}$$

and the desired equation for the thermally averaged absorption cross section becomes

We can determine thermally averaged cross sections from 2200 m/s data

$$\bar{\Sigma}_a = g_a(T)\Sigma_a(E_o)\frac{\phi_o}{\phi_T}$$

$$\bar{\Sigma}_a(T) = \Sigma_{a2}(T) = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_o)\left(\frac{T_o}{T}\right)^{1/2}$$

Similarly, for the fission cross section, we have

$$\bar{\Sigma}_f(T) = \Sigma_{f2}(T) = \frac{\sqrt{\pi}}{2} g_f(T)\Sigma_f(E_o)\left(\frac{T_o}{T}\right)^{1/2}$$

What about the Fast Group Data for Fuel and Structure?

An obvious omission from the previous list of data resources concerns the fast group cross sections for fuel and structural nuclides in a thermal system.

The primary phenomena of interest here are fast fission and resonance absorption, and both effects are tough to quantify!

The best method is to use a sophisticated computer code to obtain the desired data. However, for quick hand calculations, this is not feasible -- thus, we need simple approximate method.

For preliminary analyses, one often assumes that the fast fission and resonance absorption effects are relatively small and these are treated via correction factors.

In particular, the fast fission factor, ε, is used to account for fast fission and the resonance escape probability, p, is used to handle resonance absorption (these will be defined explicitly in a later lesson…).

Density and Temperature Corrections

It is also important to note that most of the data tabulated in the Lamarsh data tables are given for nominal density and room temperature conditions.

Since the macroscopic cross section is proportional to the nuclide density, Σ is a direct function of the physical density, ρ.

The diffusion coefficient, on the other hand, is inversely proportional to ρ and it is only a weak function of temperature.

Based on these considerations, the appropriate relationships for accounting for density and temperature effects in preliminary computations are [from Lamarsh]:

$$\bar{\Sigma}_a(\rho,T) = \bar{\Sigma}_a(\rho_o,T_o)\left(\frac{\rho}{\rho_o}\right)\left(\frac{T_o}{T}\right)^{1/2}$$

These expressions do not include the temperature dependence of the non-1/v factor, so don't forget to include this effect when needed.

Density and Temperature Corrections

$$\bar{D}(\rho,T)=\bar{D}(\rho_o,T_o)\left(\frac{\rho_o}{\rho}\right)\left(\frac{T}{T_o}\right)^m \quad \text{with } m=\begin{cases}0.470 \text{ for } H_2O\\ 0.112 \text{ for } D_2O\\ \approx 0 \text{ otherwise (solids)}\end{cases}$$

$$L_T^2(\rho,T)=\frac{\bar{D}(\rho,T)}{\bar{\Sigma}_a(\rho,T)}=L_T^2(\rho_o,T_o)\left(\frac{\rho_o}{\rho}\right)^2\left(\frac{T}{T_o}\right)^{m+1/2}$$

and

$$\tau_T(\rho)=\frac{D_1(\rho)}{\Sigma_{1\to 2}(\rho)}=\tau_T(\rho_o)\left(\frac{\rho_o}{\rho}\right)^2$$

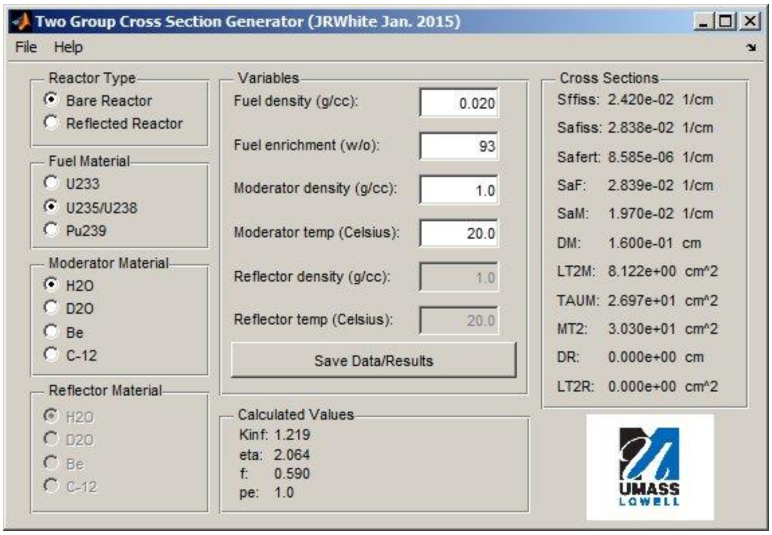
where the thermal neutron age, τ_T, has no explicit temperature dependence since both D_1 and $\Sigma_{1\to 2}$ are only weakly dependent on the temperature of the material.

The cross_sections_gui Interface

In addition to the raw data tables in Lamarsh and the base 2200 m/s data (from various sources), a Matlab graphical user interface, called cross_sections_gui, has also been developed to compute many of the averaged cross sections (and other useful auxiliary information) that are needed for performing a variety of preliminary analyses for thermal systems (see sample GUI screen on next slide).

This GUI simply automates the process of obtaining cross sections for preliminary calculations (using the above formulas and the referenced data tables from Lamarsh).

The cross_sections_gui Interface



Lesson 4 Summary

In this Lesson we have briefly discussed the following subjects:

The interpretation of the diffusion length and how to extend this basic concept to 2-group problems.

The important terminology, assumptions, and procedure needed to develop the expression for the thermally averaged absorption (or fission) cross section:

$$\bar{\Sigma}_a(T) = \Sigma_{a2}(T) = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_o)\left(\frac{T_o}{T}\right)^{1/2}$$

The expressions needed to account for density and temperature corrections to the cross sections tabulated in Lamarsh.

How to obtain 1-group and 2-group cross sections for preliminary calculations from the Lamarsh tables and/or via the cross_sections_gui code and apply these in various situations.