Nuclear Reactor Theory

Lesson 1: The Flux Spectrum in Thermal Systems

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Lesson 1 Objectives

Describe the general shape of the expected flux distribution versus energy in a thermal system.

Explain the differences between the various expressions for the scalar neutron flux: $\phi(E)$, ϕ_g, and $\phi(u)$.

Develop a relationship between $\phi(u)$ and $\phi(E)$.

Identify, by inspection of the spectrum plot, which form of the flux representation has been plotted.

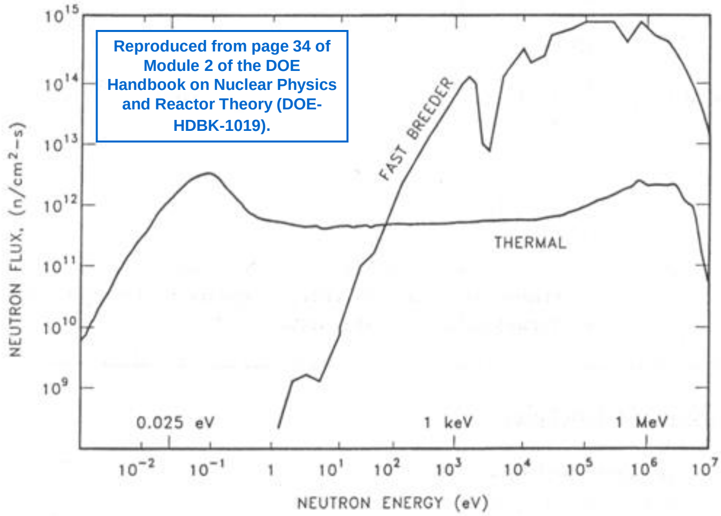
Motivation

Much of the motivation for this lesson is to explain the flux profiles on the next slide and, in particular, the description that accompanies the thermal flux profile.

The two curves given here contrast the typical spectra for a thermal system and for a fast reactor.

This example shows exactly what one might expect, with the fast reactor having essentially no thermal neutrons -- since it has no moderator to slow down the original fission neutrons.

Typical Flux Spectra



Motivation (cont.)

However, the partial quote that tries to explain the thermal flux profile does not appear to make any sense:

"In the thermal reactor, the flux in the intermediate energy region (1 eV to 0.1 MeV) has approximately a 1/E dependence. That is, if the energy (E) is halved, the flux doubles. This 1/E dependence is caused by the slowing down process …".

Clearly this behavior is not apparent in the given plot -- so what is going on here???

Three Broad Energy Regions

In Fundamentals of NSE, we described the general neutron life cycle in a thermal system as follows:

> In a thermal system, fission neutrons are born at high energy, they slow down via inelastic and elastic neutron scattering, and then, as thermal neutrons, they cause additional fissions to continue the cycle…

Here, we clearly identify three energy regions of focus:

1. The high energy region which is populated by the neutrons emitted from the fission process.

2. An intermediate energy region where neutron scattering and slowing down are the dominant processes (along with some resonance absorption).

3. The low energy thermal region, where the dilute neutron gas is nearly in thermal equilibrium with the surrounding materials.

Typical Spectra in Thermal Systems

The high energy and thermal energy regions were already discussed in some detail in Fundamentals of NSE, where

1. The high energy neutron energy dependence is described by the fission spectrum, $\chi(E)$.

2. The low energy profile is given by the thermal Maxwellian distribution, $n(E) = n_{tot}f(E)$ or $\phi(E) = n(E)v(E) = n_{tot}f(E)v(E)$.

In the slowing down region, things are more difficult to describe analytically, especially if resonance absorption dominates scattering.

However, in water moderated systems, although not negligible, resonance absorption does not have an overly significant effect on the flux spectrum, which can be described approximately by a $1/\Sigma_t E$ behavior at intermediate energies (a formal treatment of slowing down theory is beyond the scope of this course).

Spectra in Thermal Systems (cont.)

In particular, the basic profiles discussed above, assuming that $\Sigma_t(E)$ is roughly constant in the intermediate region, lead to the following approximate composite spectrum:

$$\phi(E) = \begin{cases} \phi_1(E) = c_1 e^{-1.036E} \sinh\sqrt{2.29E} & \text{for } E_2 \le E \le E_1 \\ \phi_2(E) = c_2 / E & \text{for } E_3 \le E \le E_2 \\ \phi_3(E) = c_3 E e^{-E/kT} & \text{for } 0 \le E \le E_3 \end{cases}$$

where we note that the thermal spectrum function, $\phi_3(E)$, is a product of n(E) and v(E) in the thermal region -- where the product of the $\sqrt{E}$ dependence in each of these separate functions leads to the E dependence given here [the exponential part is from the Maxwellian n(E) component].

In these expressions, E is given in MeV, the Boltzmann constant is 8.617×10^{-11} MeV/K, and the temperature is in absolute units, K.

An Example

To actually plot this functional dependence, we need to specify the normalization factors, c_1, c_2, and c_3, and the specific 3-region energy boundaries, E_1, E_2, and E_3.

Varying these quantities lead to considerably different spectra, so we need to specify a consistent set of constraints and conditions to achieve a reasonable thermal flux.

In doing this, we first determine the normalization coefficients by forcing continuity in the composite spectrum at E_2 and E_3 and by setting an overall (arbitrary) normalization of 10^{12} neutrons/cm^2-s.

This gives

Continuity at E_2: $\phi_1(E_2) = \phi_2(E_2)$ Continuity at E_3: $\phi_2(E_3) = \phi_3(E_3)$

Overall normalization: $\int_0^{E_1} \phi(E)dE = 1\times 10^{12}\,\frac{\text{neutrons}}{\text{cm}^2 - \text{s}}$

An Example (cont.)

These three conditions give three equations with three unknowns and, if written in matrix form, we have

$$\begin{bmatrix} e^{-1.036E_2}\sinh\sqrt{2.29E_2} & -\dfrac{1}{E_2} & 0 \\ 0 & \dfrac{1}{E_3} & -E_3 e^{-E_3/kT} \\ \int_{E_2}^{E_1} e^{-1.036E}\sinh\sqrt{2.29E}dE & \int_{E_3}^{E_2}\dfrac{dE}{E} & \int_0^{E_3} Ee^{-E/kT}dE \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10^{12} \end{bmatrix}$$

and, for a given set of energy boundaries and a given T, we can evaluate the matrix elements and solve the resultant equation for the desired normalization coefficients (Matlab's *quadl* routine, for example, could be used to perform the required integrals).

Note that this approach for computing $\phi(E)$ is certainly quite artificial, but it allows us to get a reasonable estimate of a typical spectrum in a thermal system…

An Example (cont.)

Here we set E_1 = 10 MeV (a convenient and reasonable choice).

For E_2 and E_3, these were set to give a fast-to-thermal flux ratio in the range of 2.5 to 3.5 (since this is consistent with the core region within the UMLRR).

In doing this we defined the fast-to-thermal flux ratio as

$$\frac{\phi_{fast}}{\phi_{thermal}} = \frac{\int_{E_2}^{E_1} \phi_1(E)\,dE + \int_{E_3}^{E_2} \phi_2(E)\,dE}{\int_0^{E_3} \phi_3(E)\,dE}$$

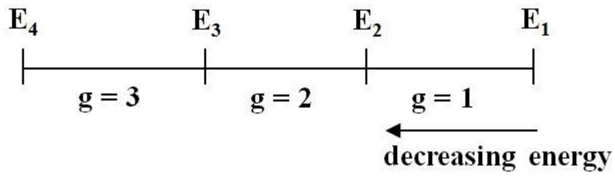
An Example (cont.)

Basic Procedure: make a reasonable guess for E_2 and E_3, solve for the normalization coefficients, and then evaluate the above equation to determine $\phi_{fast}/\phi_{thermal}$.

This was done with various choices of E_2 and E_3 until a suitable value of $\phi_{fast}/\phi_{thermal}$ was obtained, with the following results:

E_2 = 0.05 MeV and $E_3 = 2\times10^{-7}$ MeV gives

$\phi_{fast}/\phi_{thermal}$ = 3.35 for T = 20 C



An Example (cont.)

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With a reasonable estimate for the composite thermal flux spectrum $\phi(E)$, we can now turn our attention to the goal of plotting $\phi(E)$ in a variety of ways.

However, it should be noted that the resultant $\phi(E)$ is only a rough approximation to reality, since many assumptions were made in this development.

Nevertheless, the resultant profile is quite reasonable and, more importantly for this exercise, it is certainly reasonable enough to illustrate how the various plotting techniques affect our visualization of the flux spectrum.

The most common visualization techniques include plotting $\phi(E)$ vs. E, ϕ_g vs. E, and $\phi(u)$ vs. E, where $\phi(E)$ is the flux per unit energy, ϕ_g is the group flux (energy integrated flux over ΔE_g), and $\phi(u)$ is the flux per unit lethargy.

An Example (cont.)

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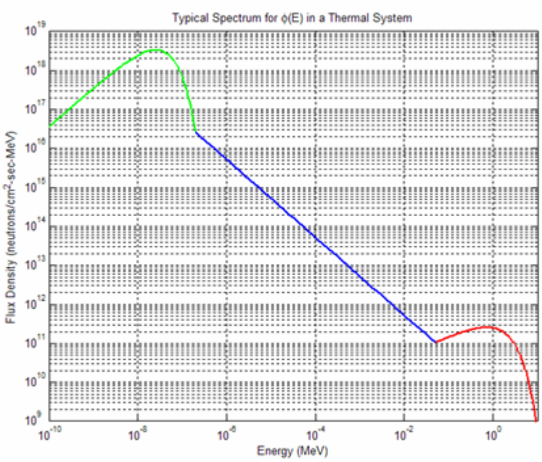


For plotting $\phi(E)$ vs. E, we evaluate the flux spectrum (as described above) and plot it versus energy using logarithmic axes for both $\phi(E)$ and E.

This distribution clearly shows the three distinct spectral functions that make up the composite $\phi(E)$ spectrum, with the fission spectrum at high energies, a clearly observable 1/E behavior at intermediate energies, and a Maxwellian profile at low energy.

An Example (cont.)

To plot ϕ_g vs. E, we first must create a fine-group energy grid, compute the discrete ϕ_g values, and then plot these values versus energy using a histogram-type plot that shows a constant group flux in each energy bin, with discontinuous changes at the group boundaries.

For the energy grid, let's use roughly equal lethargy intervals.

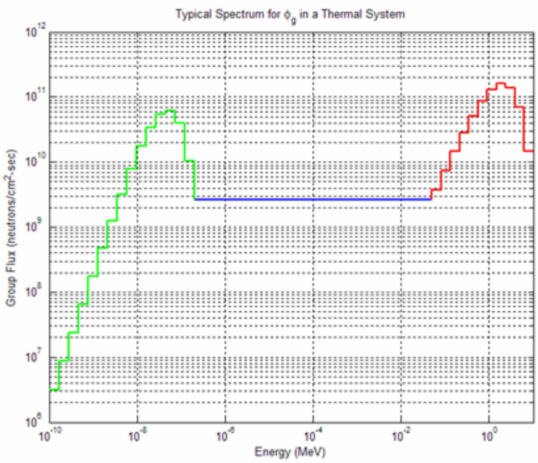
Thus, for plotting over a range of 10^{-10} MeV to 10 MeV, the total change in lethargy is given by

$$\Delta u_{tot} = \ln\left(\frac{10\,\text{MeV}}{10^{-10}\,\text{MeV}}\right) = \ln 10^{11} = 25.33$$

and, for a $\Delta u_g \approx 0.5$, this would lead to about 50 or 51 total groups.

Note: The reader should see the formal Lecture Notes for further details here.

An Example (cont.)



The resultant plot of ϕ_g vs. E is shown here, where logarithmic axes for both ϕ_g and E are still used because of the large variation in both these quantities.

Clearly this distribution for ϕ_g vs. E looks quite different from the $\phi(E)$ vs. E plot shown previously.

We still see the three distinct spectral functions that make up the composite flux spectrum, but now the group flux values represent integrals over each energy bin.

An Example (cont.)

As a **final visualization method**, let's look at a **continuous plot of the flux per unit lethargy versus energy, $\phi(u)$ vs. E**.

Here we note that **both $\phi(E)$ and $\phi(u)$ are density functions**, and **the total number of neutrons/cm^2-s in a given energy interval, dE, and corresponding lethargy interval, du, must be the same**, or

$$\phi(u)\,du = -\phi(E)\,dE = \#\ \text{neutrons/cm}^2\text{-s in interval}$$

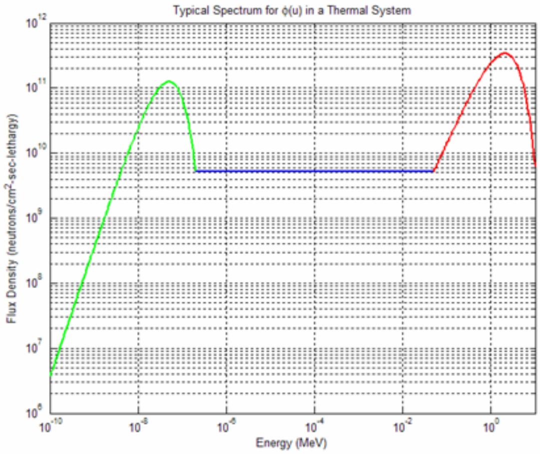
where the **negative sign** accounts for the fact that **a decrease in energy leads to an increase in lethargy**.

Now with this relationship and the **definition of u(E)** as $u(E) = \ln\left(E_{ref}/E\right)$, the **flux per unit lethargy** is simply given as

$$\phi(u) = -\phi(E)\frac{dE}{du} = -\phi(E)\left(-E\right) = E\phi(E) \qquad \text{or} \qquad \boxed{\phi(u) = E\phi(E)}$$

Finally, we note that, **in the intermediate energy** where **$\phi(E) \approx c_2/E$**, **the flux per unit lethargy becomes approximately constant**, or **$\phi(u) \approx c_2$**.

An Example (cont.)



For completeness, a plot of **$\phi(u)$ vs. E** is shown here.

Clearly, **this profile is similar to that for ϕ_g vs. E**, **except for the discontinuous versus discrete nature of the two profiles**.

This behavior is typical and these two formats, **ϕ_g vs. E** or **$\phi(u)$ vs. E**, are the **usual choices for visualization of the flux spectrum** (**depends on whether discrete or continuous distribution information is available and the multigroup structure**).

An Example (cont.)

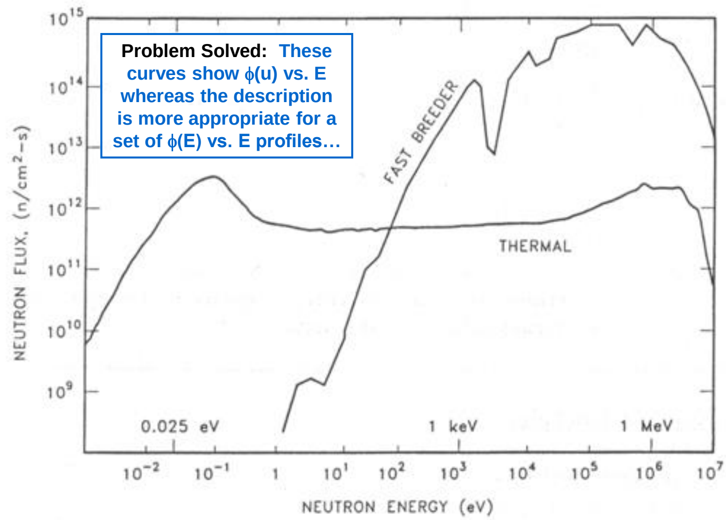
We have now accomplished our goal -- that is, highlighting the differences between $\phi(E)$, ϕ_g, and $\phi(u)$, for thermal systems.

The most common visualizations include plots of $\phi(u)$ or ϕ_g vs. E (i.e. continuous or discrete representations) -- and, more often than not, the labeling simply indicates that the neutron flux is being plotted (even though one should always be very explicit to avoid any possible ambiguities).

However, with this example, you should be able to distinguish between a plot of $\phi(E)$ or $\phi(u)$ (even if the plot is not properly labeled).

For example, it should be clear that the typical thermal spectrum plot from the DOE Handbook (see next slide) is clearly the flux per unit lethargy and, with this knowledge and the expected typical behavior of $\phi(E)$ vs. E, the quote that accompanies the plot also makes perfect sense…

Typical Flux Spectra



Lesson 1 Summary

In this Lesson we have briefly discussed the following subjects:

The general shape of the expected flux distribution versus energy in a thermal system.

The key differences between the various expressions for the scalar neutron flux: $\phi(E)$, ϕ_g, and $\phi(u)$.

The relationship between $\phi(u)$ and $\phi(E)$.

How to identify, by inspection of the spectrum plot, which form of the flux representation has been plotted.