# Nuclear Reactor Theory

## Lesson 12: The Time Dependent Reactor III

### Fission Product Poisoning and Fuel Depletion Effects

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# Lesson 12 Objectives

**Describe some of the key processes associated with fuel depletion and their affect on reactor operations.**

**Explain the difference between saturating and non-saturating fission products and sketch a set of typical time-dependent profiles.**

**Explain why Xe-135 is such an important fission product in thermal systems and identify some other important nuclides that required special consideration.**

**Given basic decay chain information, write the iodine-xenon balance equations and explain the typical behavior of the Xe reactivity for a number of different scenarios (startup, shutdown, power-level changes, etc.) with the use of the xenon_gui code.**

## Lesson 12 Objectives (cont.)

Perform similar analyses for other important FP chains (Pm149 → Sm149, Eu157 → Gd157, etc.)

Identify the key reactions and set up the basic matrix form of the fuel depletion equations assuming a quasi-static flux approximation.

Sketch a typical $k_{excess}$ vs. time curve for large power reactors and explain the importance of burnable absorbers, soluble boron (in PWRs), and the use of external control during the 18-24 month time span between refueling.

Explain why a uniform fuel loading leads to a non-optimum configuration, why loading of fresh fuel on the core periphery is no longer practiced, why BWRs typically have more control rods than PWRs, etc…



## Key Fuel Depletion Processes

During normal reactor operation several important nuclide transformation processes occur that affect the overall neutron balance, as follows:

Fuel Burnup -- both fission and parasitic capture within fissile material reduce the amount of fuel present in the system.

Fissile Production -- neutron capture in fertile isotopes eventually produce new fissile material, which tends to offset some of the fuel loss due to depletion.

FP Production -- each fission reaction produces two fission products, many of which are radioactive -- which leads to several subsequent parent–daughter sequences (eventually producing hundreds of different FPs).

## Key Fuel Depletion Processes

**Burnable Absorber Depletion** -- most reactors use burnable poisons to compensate for some of the initial excess reactivity associated with the fresh fuel and, as implied by the terminology, these isotopes deplete during the cycle, which tends to flatten out the decrease in reactivity versus time.

**Special FP Chains** -- some of the FPs produced, especially in thermal systems, have very large absorption cross sections and thus these FP chains can have a significant short-term impact on reactor operations.



## Saturating and Non-Saturating FPs

Aside from the several special FPs in thermal systems that have very large $\sigma_a$ (e.g. Xe-135, Sm-149, Gd-157, etc.), the average thermal absorption cross section is roughly 20 – 40 b.

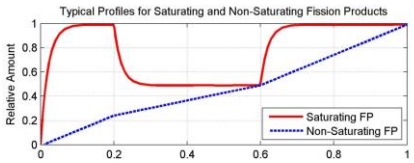
The several hundred FPs are often separated into two general groups -- saturating and non-saturating isotopes.

The FPs within the saturating group have decay constants and/or absorption cross sections such that they reach equilibrium after relatively short operational times.

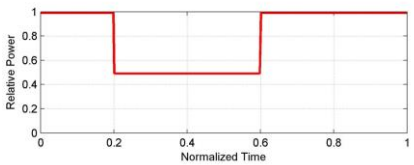
In contrast, the non-saturating FPs effectively have no loss term (i.e. capture and decay lead to another non-saturating FP) and the FP inventory simply grows linearly with the number of fissions that have occurred (i.e. increases linearly with burnup).

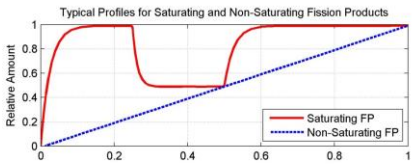
Although several key saturating FPs dominate the overall FP reactivity loss, the non-saturating FPs can become important at high burnup.

# Saturating and Non-Saturating FPs

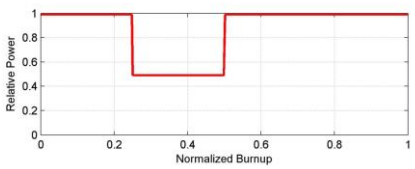



← Profiles vs. Time




Profiles vs. Burnup →






# Special Saturating FPs

**A few FP nuclides play an especially important role in thermal systems because of their extremely large thermal absorption cross sections (and relatively large equilibrium yields).**

**For example, for Xe-135, $\sigma_a(E_o) = 2.65\times10^6$ barns ($\gamma_{eq} \approx 0.0663$)**

**for Sm-149, $\sigma_a(E_o) = 41{,}000$ barns ($\gamma_{eq} \approx 0.0107$)**

**for Sm-151, $\sigma_a(Eo) = 15{,}200$ barns ($\gamma_{eq} \approx 0.0042$)**

**for Gd-157, $\sigma_a(E_o) = 2.4\times10^5$ barns ($\gamma_{eq} \approx 0.00007$)**

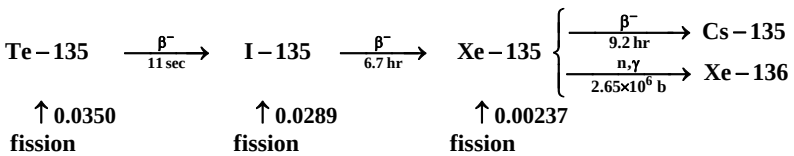
**for a typical fission product, $\sigma_a(E_o) \approx 20$ - 40 barns ($\gamma_{eq} \approx 1.92$)**

**The time constants associated with the dynamics of the important FP chains are on the order of hours to days (not a safety concern).**

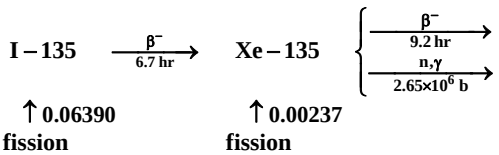
**Here, we highlight the dynamics of the I-Xe chain because of the large equilibrium yield and its extremely large thermal $\sigma_a$.**

# Dynamics of the I-Xe Chain

The detailed I-Xe chain can be visualized as follows:




But, because Te-135 decays so rapidly and, since the daughter products from Xe-135 decay and absorption are not of interest, we can simplify the above scheme, as follows:






# Dynamics of the I-Xe Chain (cont.)

Based on the simplified sketch, the basic isotope balance equations that define the I-Xe dynamics of interest in thermal systems can be written as (accumulation rate = production rate – loss rate):

I-135 balance equation:

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I$$

Xe-135 balance equation:

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \left(\lambda_X + \sigma_{aX}\phi\right) X$$

In these expressions, the thermal neutron flux is directly related to the operating power level.

Thus, a change in power can be treated as a driving force that moves the I-135 and Xe-135 densities from their equilibrium or steady-state values to some new state.

# Dynamics of the I-Xe Chain (cont.)

There are several subtle approximations built into the previous equations:

1. An averaging process has been applied to the spatial variable, leading to the spatial independence associated with the given equations.

2. For the energy dependence, if we interpret the $\Sigma_f\phi$ and $\sigma_{aX}\phi$ terms as integrals over energy, then we can accurately account for any energy variation within the system.

3. In practice, however, the Xe-135 cross section is extremely large at thermal energies relative to that at high energy -- thus, ignoring the fast component of the Xe-135 reaction rate is certainly justifiable.



# Dynamics of the I-Xe Chain (cont.)

4. Also, since most of the fissions in a thermal system occur at thermal energies, one often assumes that the fission term includes only thermal energies.

However, for 2-group theory, we can write

$$\Sigma_f\phi \Rightarrow \Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2 = \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi_2 \Rightarrow \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi$$

Thus, we will let $\Sigma_f$ represent the effective fission cross section as implied here.

# Equilibrium Xenon Reactivity

When the production and loss rates for a particular isotope become balanced (i.e. production rate = loss rate), the isotope concentration no longer changes with time, and it is said to be in equilibrium.

This condition occurs in a real system when the reactor has been operating at constant power for a relatively long period of time.

To determine the reactivity effect for the equilibrium condition, we set the derivatives, dI/dt and dX/dt, to zero, which gives

$$I_\infty = \frac{\gamma_I \Sigma_f \phi_\infty}{\lambda_I} \qquad \text{and} \qquad X_\infty = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty}$$



# Equilibrium Xenon Reactivity (cont.)

Working, in particular, with the expression for $X_\infty$, we can write the macroscopic Xe-135 absorption cross section at equilibrium as

$$\Sigma_{a\infty} = X_\infty \sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty}\sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\phi_X + \phi_\infty}$$

where $\phi_\infty$ is the average thermal flux at equilibrium conditions and $\phi_X$ is given by

$$\phi_X = \lambda_X / \sigma_{aX}$$

has the same units as neutron flux

Finally, we recall from the previous Lesson that the reactivity due to a homogenous poison in a thermal system can be given as

$$\rho(t) = \frac{\Sigma_{aP}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T} = \frac{X(t)\sigma_{aX}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T}$$

# Equilibrium Xenon Reactivity (cont.)

Thus, putting the expression for $X_\infty$ into this equation gives the reactivity effect of equilibrium xenon, or

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu p \varepsilon P_F P_T} \frac{\Sigma_f}{\Sigma_{f2}} \frac{\phi_\infty}{(\phi_X + \phi_\infty)}$$

equilibrium xenon reactivity

It is interesting to find an approximate numerical value for $\rho_\infty$.

To do this, we need to make a bunch of assumptions:

First assume a large U-235 fueled system ($P_F P_T \rightarrow 1.0$) with $p\varepsilon \approx 1.0$ and $\Sigma_f/\Sigma_{f2} \approx 1.0$.

With these rough approximations, the above expression reduces to

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu} \frac{\phi_\infty}{(\phi_X + \phi_\infty)}$$

for a large system with $p\varepsilon \approx 1.0$ and $\Sigma_f/\Sigma_{f2} \approx 1.0$



# Equilibrium Xenon Reactivity (cont.)

Now, the thermally averaged $\sigma_a$ for Xe-135 at temperature T is given by

$$\sigma_{aX}(T) = \frac{\sqrt{\pi}}{2} g_a(T) \sigma_{aX}(E_o) \left(\frac{T_o}{T}\right)^{1/2}$$

and, at $T = T_o = 293$ K (20 C), this can be evaluated to give

$$\sigma_{aX} = \frac{\sqrt{\pi}}{2}(1.158)\left(2.65\times10^6 \text{ b}\right)\left(10^{-24} \frac{\text{cm}^2}{\text{b}}\right) = 2.72\times10^{-18} \text{ cm}^2$$

Also, the decay constant for Xe-135 is given by

$$\lambda_X = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{9.2 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 2.09\times10^{-5} \text{ s}^{-1}$$

Using these latter two values in the expression for $\phi_X$ gives

$$\phi_X = \frac{2.09\times10^{-5} \text{ s}^{-1}}{2.72\times10^{-18} \text{ cm}^2} = 0.77\times10^{13} \text{ cm}^{-2}\text{s}^{-1}$$

has the same units as neutron flux

## Equilibrium Xenon Reactivity (cont.)

Now, we see that if the thermal flux at equilibrium is much greater than $\phi_X$, then the ratio $\phi_\infty/(\phi_X + \phi_\infty)$ approaches unity.

Thus, for the case of a large thermal flux, the equilibrium xenon reactivity becomes

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu}(1.0) = -\frac{0.06627}{2.43} = 2.73\ \%$$

This represents an approximate value for the equilibrium xenon reactivity in thermal systems with a large thermal flux (where $\phi_\infty >> \phi_X$).

This corresponds to about 4 dollars of negative reactivity -- which certainly represents an important consideration in the control and operation of thermal systems.



## Solution for the General Case

The above development for $\rho_\infty$ is only applicable for operation at constant power for a relatively long period of time.

All the other cases of interest involve solution of the full time-dependent I-Xe dynamics equations.

This means that, given some initial condition and the time-dependent thermal flux (or power) as input, we need to solve the I-135 and X-135 balance equations to give X(t), which can then be used to give $\rho(t)$.

For the general case, where a general P(t) or $\phi(t)$ is used, the easiest way to solve these equations is via numerical integration with a standard ODE solver (such as Matlab's ode45 routine).

However, when P(t) is constant over some interval, the balance equations represent a set of sequential, linear, constant coefficient ODEs, that can be solved analytically.

## Solution for the General Case (cont.)

The result for the constant thermal flux case, after suitable manipulation to get it into desired form, is given as follows:

$$I(t) = I_o e^{-\lambda_I t} + \frac{\gamma_I \Sigma_f \phi}{\lambda_I}\left(1 - e^{-\lambda_I t}\right)$$

$$X(t) = X_0 e^{-wt} + \frac{\left(\gamma_I + \gamma_X\right)\Sigma_f \phi}{w}\left(1 - e^{-wt}\right) + \frac{\gamma_I \Sigma_f \phi - I_0 \lambda_I}{w - \lambda_I}\left(e^{-wt} - e^{-\lambda_I t}\right)$$

where $w = \sigma_{aX}\phi + \lambda_X$ and $I_o$ and $X_o$ represent the initial I-135 and Xe-135 concentrations.

Note that that these equations are only valid over a time period of constant power/flux given by the value of $\phi$.

However, even with this restriction, these equations can be very useful, since for most situations of interest, the thermal flux (or power) can be given as a piecewise constant function.



## Some Case Studies

In particular, there are three situations:

1. Instantaneous startup to some constant power and flux level, with $I_o = X_o = 0$.

2. Instantaneous shutdown after a long period of full power operation, with $I_o = I_\infty$ and $X_o = X_\infty$ (using the full power thermal flux to determine the equilibrium concentrations).

3. Instantaneous change to some new constant power and flux after previous operation at a different constant power level (here the initial concentrations for each new time interval are obtained from the conditions at the end of the prior time interval).

Careful study of these three cases can give significant insight into the general behavior of the I-Xe chain, as well as a good understanding of how xenon buildup and decay can affect the reactivity in thermal systems.

# Application to the UMLRR

To quantify the Xe-135 reactivity effect for a given application, we require specific information about the particular system of interest.

In particular, we have used the six-factor formula to estimate the reactivity effect because it is relatively easy to approximate the terms within this expression for the multiplication factor, k, using appropriate 2-group cross sections and some geometry and material information for the system under study.

For the UMLRR, the needed parameters are summarized as follows (using 2-group cross sections generated in Aug. 2010):

$\nu = 2.43$ $\quad p = 0.879$ $\quad \varepsilon = 1.067$

$P_F = 0.665$ $\quad P_T = 0.969$

$\Sigma_{f1} = 1.21\times10^{-3}\ cm^{-1}$ $\quad \Sigma_{f2} = 5.04\times10^{-2}\ cm^{-1}$ $\quad \phi_1/\phi_2 = 2.75$



# Application to the UMLRR (cont.)

As noted previously, the power or flux represents the driving force for the I-Xe dynamics.

The thermal flux is given in terms of the relative power, rp, and this quantity can vary for the various cases studied, as follows:

$$P = \kappa\left(\Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2\right)V_{core} = \kappa\Sigma_f\phi V_{core}$$

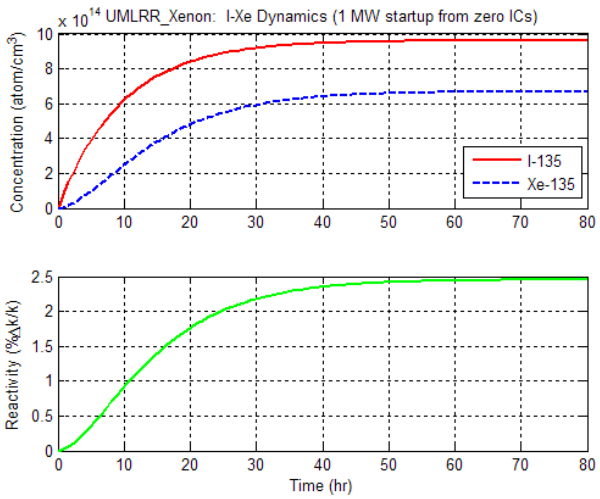
Now solving for $\phi$ gives

$$\phi = \frac{P}{\kappa\Sigma_f V_{core}} = \frac{rp\times P_{full}}{\kappa\Sigma_f V_{core}}$$

where $P_{full} = 1\ MW = 10^6\ W$ and the relative power, rp, is now the driving force for modeling the I-Xe dynamics within the UMLRR.

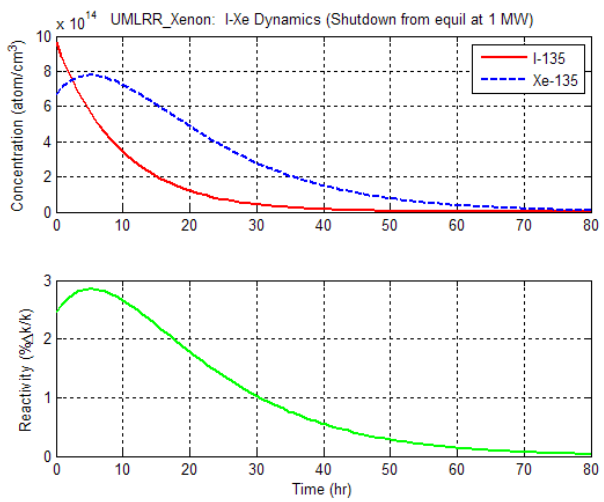
For the UMLRR, we used a 20 element core where the fuel assembly cross section is 7.7724 cm × 7.7724 cm and the active fuel height is 59.69 cm ($V_{core} = 7.212\times10^4\ cm^3$).

# Startup of the UMLRR from Zero ICs

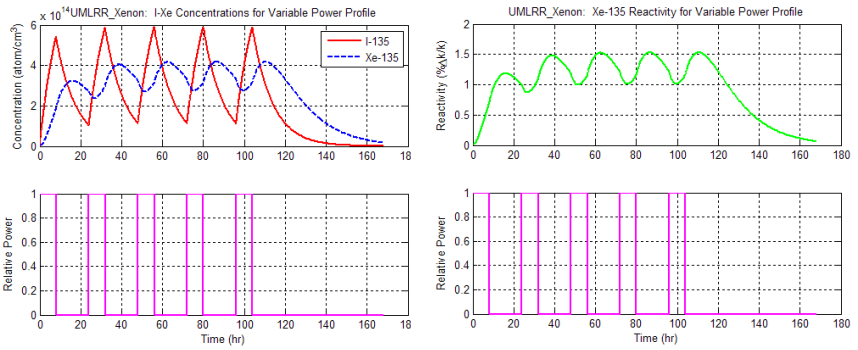





# Shutdown from Equilibrium ICs

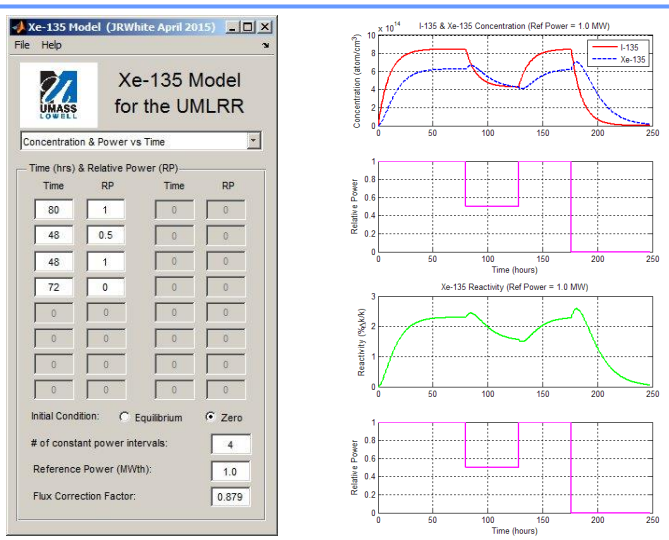


# UMLRR Operation (8-hr shift, 5-days/week)






# The xenon_gui Interface



# Some Other Important FP Chains

The simplified **Pm-Sm chain** is given by

$$\mathrm{Pm}-149 \xrightarrow[53\ \mathrm{hr}]{\beta^-} \mathrm{Sm}-149 \begin{cases} \text{stable} \\ 41{,}000\,\mathrm{b} \end{cases}$$

$\uparrow 0.01071$ fission

and the **appropriate balance equations** are:

$$\frac{dP}{dt} = \gamma_P \Sigma_f \phi - \lambda_P P \qquad \text{and} \qquad \frac{dS}{dt} = \lambda_P P - \sigma_{aS} \phi S$$



# Some Other Important FP Chains

Similarly, the simplified **Eu-Gd chain** is given by

$$\mathrm{Eu}-157 \xrightarrow[15.2\ \mathrm{hr}]{\beta^-} \mathrm{Gd}-157 \begin{cases} \text{stable} \\ 240{,}000\,\mathrm{b} \end{cases}$$

$\uparrow 0.00007$ fission

and the **appropriate balance equations** are:

$$\frac{dE}{dt} = \gamma_E \Sigma_f \phi - \lambda_E E \qquad \text{and} \qquad \frac{dG}{dt} = \lambda_E E - \sigma_{aG} \phi G$$

**Both Sm-149 and Gd-157 are stable isotopes -- thus, the dynamics here are somewhat different from the I-Xe chain.**

# Fuel Depletion

The last topic in our introductory treatment of Reactor Theory is fuel depletion.

In practice, since the time scale of interest for fuel depletion is long relative to the kinetics problem, the so-called Quasi-Static Burnup Approximation is almost always used.

This approach discretizes the time variable into multiple discrete steps:

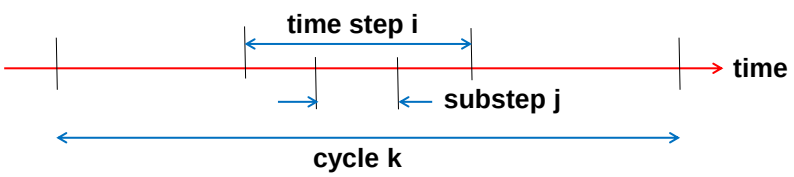
cycle k, time step i, and substep j

where various discrete operations are only allowed at the beginning of certain intervals and the nuclide vector is the only time dependent variable within the lowest substep level.

see next slide…



# Quasi-Static Burnup Approximation




| Operation | Time Boundary | | |
|---|---|---|---|
| | Cycle | Time Step | Substep |
| Refueling/Shuffling/Discharge | x | | |
| Control Movement/Nuclide Search | x | x | |
| Flux-Eigenvalue Calculation | x | x | |
| Cross Section Update | x | x | |
| Power Normalization | x | x | x |

# Quasi-Static Burnup Approximation

At each time step and cycle boundary:

neutron balance equation: $\left(L-\lambda F\right)\psi=0$

this gives $\psi_g(\vec{r}) \rightarrow \psi_{gz}=\frac{1}{v_z}\int_{v_z}\psi_g(\vec{r})\,d\vec{r}$ zone average flux

where $\sum_z v_z \sum_g \psi_{gz}=1$ shape normalization

At each substep, time step, and cycle boundary:

power normalization: $P=\alpha\sum_z v_z\left(\sum_\ell \kappa_\ell N_{\ell z}\sum_g \sigma_{f\ell gz}\psi_{gz}\right)$

where $\ell \rightarrow$ nuclide $z \rightarrow$ zone $g \rightarrow$ group flux magnitude

$\kappa \rightarrow$ energy/fission $\phi=\alpha\psi$ = flux shape & magnitude



# Quasi-Static Burnup Approximation

Within each substep j and for each zone z:

nuclide balance equation: $\frac{d}{dt}\underline{N}_z=\underline{\underline{M}}_z\underline{N}_z$

where $\underline{N}_z$ = nuclide vector for zone z

$\underline{\underline{M}}_z$ = transmutation matrix with loss terms along the main diagonal and the production routes in the off-diagonal elements

Can use stiff ODE Solver or Matrix Exponential Approach…

See Lesson on Radioactive Decay Calculations from Fundaments of NSE course…

# A Simple Example

**Consider the following abbreviated transmutation scheme:**

$$\underset{N_1}{Th232} \xrightarrow{n,\gamma} \underset{N_2}{Pa233} \xrightarrow[27.4\,d]{\beta^-} \underset{N_3}{U233} \xrightarrow{n,\gamma} \underset{N_4}{U234} \xrightarrow{n,\gamma} \underset{N_5}{U235}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

fission fission fission $N_6$ (only 1 FP)

$$\frac{d}{dt}\underline{N} = \underline{\underline{M}}\,\underline{N}$$

$$\frac{d}{dt}\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix} = \begin{bmatrix} -\langle\sigma_{a1}\phi\rangle & 0 & 0 & 0 & 0 & 0 \\ \langle\sigma_{c1}\phi\rangle & -(\langle\sigma_{a2}\phi\rangle+\lambda_2) & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & -\langle\sigma_{a3}\phi\rangle & 0 & 0 & 0 \\ 0 & 0 & \langle\sigma_{c3}\phi\rangle & -\langle\sigma_{a4}\phi\rangle & 0 & 0 \\ 0 & 0 & 0 & \langle\sigma_{c4}\phi\rangle & -\langle\sigma_{a5}\phi\rangle & 0 \\ \gamma_{16}\langle\sigma_{f1}\phi\rangle & 0 & \gamma_{36}\langle\sigma_{f3}\phi\rangle & 0 & \gamma_{56}\langle\sigma_{f5}\phi\rangle & 0 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}$$



# Two Different Algorithms

**Microscopic Depletion in Core Calculation**

1. **Calculate region-dependent multigroup (8-10 groups or more) microscopic cross section library.**

**Primarily used for fast reactor analysis…**

**Then for the full core calculations:**

2. **Microscopic data remains constant versus burnup.**
3. **Allow nuclide densities to vary according to nuclide transmutation equation.**
4. **Macroscopic cross sections vary with time**

$$\Sigma_{xz}(t) = \sum_{\ell} N_{\ell z}(t)\sigma_{x\ell z}$$

5. **Follow quasi-static burnup approximations as outlined above.**

# Two Different Algorithms

**Macroscopic Depletion in Core Calculation**

1. Calculate assembly-dependent few-group (2-4 groups) macroscopic cross section libraries versus several variables (burnup, void fraction, temperature, soluble boron, etc.).

**Note:** The calculation versus burnup on the cell and/or assembly scale is done similar to that described above. The macroscopic cross section data versus burnup, for example, are then tabulated for later use.

Then for the full core calculations:

Primarily used for LWR analysis…

2. Calculate burnup (BU = Power × Time).
3. Determine appropriate macroscopic cross section by interpolating from the data tables.
4. Do new flux/power distribution calculation.
5. Specify $\Delta t$ and go to Step 2 until end-of-cycle is reached.



# Lesson 12 Summary

In this Lesson we have briefly discussed the following subjects:

Some of the key processes associated with fuel depletion and their affect on reactor operations.

The difference between saturating and non-saturating fission products and their typical time-dependent profiles.

Why Xe-135 is such an important fission product in thermal systems and several other important nuclides that required special consideration.

How to write the iodine-xenon balance equations along with an explanation of the typical behavior of the Xe reactivity for a number of different scenarios (startup, shutdown, power-level changes, etc.) with the use of the xenon_gui code.

# Lesson 12 Summary (cont.)

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Learning with Purpose

Similar dynamic analyses for other important FP chains (Pm149 → Sm149, Eu157 → Gd157, etc.) .

The key reactions and structure of the basic matrix form of the fuel depletion equations assuming a quasi-static flux approximation.

The behavior of a typical $k_{excess}$ vs. time curve for large power reactors and the importance of burnable absorbers, soluble boron (in PWRs), and the use of external control during the 18-24 month time span between refueling.

Why a uniform fuel loading leads to a non-optimum core configuration, why loading of fresh fuel on the core periphery is no longer practiced, why BWRs typically have more control rods than PWRs, etc…