

# Nuclear Reactor Theory

## Lesson 11: The Time Dependent Reactor II

### Reactivity Coefficients and Reactor Control Considerations

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## Lesson 11 Objectives

Define the term **reactivity coefficient** and explain **why this must be negative at hot conditions** within any reactor.

Show that the **total reactivity coefficient** is simply the **sum of the coefficients associated with individual effects**.

Explain the **basic concepts** associated with **justifying the sign** of the **fuel temperature** or **Doppler coefficient**.

Explain the **competing effects** that are often associated with **establishing the sign** of the **coolant temperature coefficient**.

Develop **expressions for the reactivity worth** of a **homogeneous poison** or **control material** within either a fast or thermal system.

Explain the **shape** of the **idealized integral and differential rod worth curves**.

Use the **blade\_worth\_gui program** and explain **why the measured differential blade worth curves for the UMLRR are bottom-peaked**.

## Reactivity Coefficients

In the last Lesson, a **generic power feedback coefficient** was applied to treat, in a **collective fashion**, a number of feedback effects.

In practice, however, **the individual coefficients for each separate effect are needed** since the **associated time constants can be significantly different**.

For example, **in a power excursion**, the **fuel temperature is the first to respond to an increased fission power**, then the **coolant temperature**, and finally the **temperature of the structural components**, and the **time delay associated with the various heat transport mechanisms can be important**.

Each reactivity coefficient is **defined in a similar fashion...**

## Reactivity Coefficients (cont.)

For a **temperature effect**, for example, we write the **temperature coefficient of reactivity** as

$$\alpha_T = \frac{\partial \rho}{\partial T}$$

where the temperature might be associated with the **fuel**, **coolant**, or **structural materials**.

Since  $\rho = (k - 1)/k$ , the **reactivity coefficient** can be written as

$$\alpha_T = \frac{\partial \rho}{\partial T} = \frac{\partial}{\partial T} \left( 1 - \frac{1}{k} \right) = \frac{1}{k^2} \frac{\partial k}{\partial T} \approx \frac{1}{k} \frac{\partial k}{\partial T}$$

where the last approximation ( $k^2 \approx k$ ) is valid for a near-critical or critical reference state.

## Reactivity Coefficients (cont.)



Now, from the **basic definition of a reactivity coefficient**,  $\alpha_T$ , we see that

$$\rho_f(t) = \Delta\rho(t) \approx \frac{\partial\rho}{\partial T} \Delta T(t) = \alpha_T \{T(t) - T_{\text{ref}}\}$$

Thus, **once the reactivity coefficients are known**, they can be used to approximate the **inherent (feedback) reactivity** within the system.

Note that the **units of the temperature coefficient** are  **$\Delta k/k$  per unit temperature** -- for example,  $\Delta k/k$  per  $^{\circ}\text{C}$ .

## Computation of Reactivity Coeffs.



In practice, the various **temperature coefficients are not easy to quantify**.

Often these are **computed using sophisticated computer codes** that attempt to model the reactor configuration in as much detail as possible.

Usually **two discrete temperatures are chosen** and the **appropriate cross sections and atom densities are determined for each case**.

The **neutron balance equation is then solved using these data sets to obtain two values of  $k_{\text{eff}}$** .

## Computation of Reactivity Coeffs.

For example, given the T -  $k_{\text{eff}}$  combinations,

$T_1$  = reference temperature  $\rightarrow k_1$  = reference  $k_{\text{eff}}$

$T_2$  = perturbed temperature  $\rightarrow k_2$  = perturbed  $k_{\text{eff}}$

The **average temperature coefficient over the given temperature range** is

$$\bar{\alpha}_T = \frac{\int_{T_1}^{T_2} \alpha_T(T) dT}{\int_{T_1}^{T_2} dT} = \frac{\int_{T_1}^{T_2} \frac{1}{k} \frac{\partial k}{\partial T} dT}{T_2 - T_1} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \frac{dk}{k} = \frac{\ln(k_2/k_1)}{T_2 - T_1}$$

This approach is **somewhat tedious** since a new complete set of collapsed cross sections and a new core  $k_{\text{eff}}$  calculation are needed for each temperature -- but this is really the **only way to compute the average temperature coefficient...**

## Qualitative Treatment

For rough **qualitative estimates** or to simply **help physically explain some observed behavior**, one can **use the 6-factor formula to break  $\alpha_T$  into its various subcomponents**.

Recall that the **6-factor formula** is given by  $k_{\text{eff}} = k_{\infty} P_T P_F$  and, taking the **natural logarithm of both sides**, gives

$$\ln k_{\text{eff}} = \ln k_{\infty} + \ln P_T + \ln P_F$$

Now, taking the **partial derivative with respect to temperature (holding all other variables constant)**, gives

$$\alpha_T = \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial T} = \frac{1}{k_{\infty}} \frac{\partial k_{\infty}}{\partial T} + \frac{1}{P_T} \frac{\partial P_T}{\partial T} + \frac{1}{P_F} \frac{\partial P_F}{\partial T}$$

or  $\alpha_T = \alpha_T|_{k_{\infty}} + \alpha_T|_{P_T} + \alpha_T|_{P_F}$  and  $\alpha_T|_{k_{\infty}} = \alpha_T|_{\eta_I} + \alpha_T|_f + \alpha_T|_p + \alpha_T|_g$

## Qualitative Treatment (cont.)

Thus, the **total temperature coefficient** can be obtained by **summing the individual effects associated** with how a change in temperature affects the **neutron reproduction factor**, the **fuel utilization**, the **resonance escape probability**, etc...

**One often discusses the importance of these individual terms** to the overall temperature coefficient of reactivity in a qualitative way -- **with a focus on establishing/justifying the sign (positive, negative, or essentially zero) of the separate components.**

As examples, we will discuss the **two most important cases** here:

1. The **fuel temperature coefficient** (which is often referred to as the **prompt temperature coefficient** or **Doppler coefficient** since the fuel temperature feedback usually has the **fastest response time**), and
2. The **moderator/coolant temperature coefficient.**

## Doppler Coefficient

For the **Doppler coefficient**, the **dominate contribution** here is due to the **change in the resonance escape probability.**

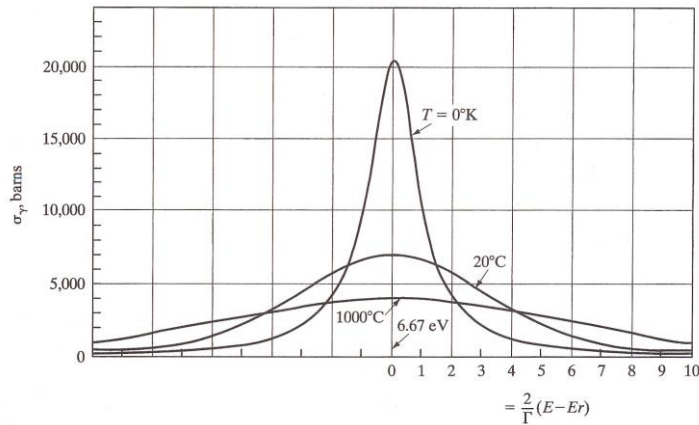
To see this, we note that **in thermal systems using low enriched uranium**, there is a **significant amount of U238 present** and a single particularly **large resonance at about 6.67 eV plays a dominant role** in the overall inherent safety of these systems.

In particular, as shown in the sketch (**next slide**) of the U238 capture cross section in the vicinity of the 6.67 eV resonance, **the peak cross section tends to decrease and the wings of the resonance tend to broaden as the temperature is increased.**

**This broadening is due to the increased relative motion of the U238 nuclei as the temperature and average kinetic energy increase** (this is often referred to as **Doppler broadening**).

## Doppler Coefficient (cont.)

Note, however, that although the **shape of the resonance changes**, the **integral under the  $\sigma_c(E)$  curve remains constant**.



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## Doppler Coefficient (cont.)

Thus, the **absorption rate** associated with this single resonance can be written as

$$F_a = \int N\sigma_c(E)\phi(E)dE = \phi_{\text{ave}} \int \Sigma_c(E)dE = \text{constant} \times \phi_{\text{ave}}$$

where

$$\phi_{\text{ave}} = \frac{\int \Sigma_c(E)\phi(E)dE}{\int \Sigma_c(E)dE}$$

Now, the **key observation here** is that, **as the resonance peak decreases**, we see less of a dip in the local flux,  $\phi(E)$ , within the resonance (i.e. **less resonance self shielding**), so the **effective average flux defined above increases** -- which, in turn, **increases the overall absorption rate in the resonance**.

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## Doppler Coefficient (cont.)

Thus, for an **increase in the fuel temperature**, we see

1. a **broadening of the resonance**,
2. which **increases the average flux and overall absorption rate**,
3. ultimately **resulting in a decreased resonance escape probability and reactivity**, or

$$\alpha_{T_f} = \frac{1}{k} \frac{\partial k}{\partial T_f} \approx \frac{1}{\rho} \frac{\partial \rho}{\partial T_f} < 0$$

## Moderator Temperature Coefficient

The **coolant/moderator temperature coefficient** is also very important and, along with the Doppler coefficient, it tends to **drive the ultimate behavior of the system over longer periods of time**.

In particular, in **water-cooled and water-moderated systems**, a **change in the moderator temperature**, which either increases or decreases the water density, **affects the multiplication factor in several ways**:

1. It changes the value of the **thermal utilization**,

$$f = \Sigma_{aF} / (\Sigma_{aF} + \Sigma_{aM})$$

by **changing the relative absorption rates of the fuel and non-fuel (moderator) materials**,

## Moderator Temperature Coefficient

2. It changes the **resonance escape probability**,

$$p = \Sigma_{1 \rightarrow 2} / (\Sigma_{a1} + \Sigma_{1 \rightarrow 2})$$

by **changing the relative distribution between the fast absorption rate and downscatter rate**, and

3. It changes the **non-leakage probability**,

$$P_F P_T = 1 / (1 + B^2 M_T^2)$$

since **neutrons diffuse more readily through less dense materials**.

Thus, we can write the **moderator temperature coefficient** in terms of these components as

$$\alpha_{T_m} = \alpha_{T|f} + \alpha_{T|p} + \alpha_{T|P_F} + \alpha_{T|P_T}$$

## Moderator Temperature Coefficient

For the **fuel utilization component** in water-moderated systems, an **increase in moderator temperature leads to a decrease in density** which **tends to reduce the number of absorptions in the moderator**.

Thus, **f usually tends to increase with an increase in moderator temperature**, and  $\alpha_{T|f}$  is positive (usually).

However, for the **resonance escape and non-leakage probabilities**, just **the opposite is true**.

For example, an **increase in moderator temperature decreases the moderator density**, which **decreases  $\Sigma_{1 \rightarrow 2}$** , with a subsequent **decrease in the resonance escape probability, p**.

Similarly, **this same decrease in density increases the neutron leakage and decreases the  $P_F$  and  $P_T$  non-leakage probabilities**.



## Moderator Temperature Coefficient

Thus, the remaining three components,  $\alpha_{T|p} + \alpha_{T|p_f} + \alpha_{T|p_T}$ , of the overall temperature coefficient,  $\alpha_{Tm}$ , are all negative.

For the ultimate sign of  $\alpha_{Tm}$ , one must balance the positive fuel utilization component with the other three negative terms.

Here, the question becomes: “Does the moderator absorb more that it moderates?” or “Does it moderate more than it absorbs?”.

For the usual case, the second option is true, since the main purpose of the moderator is to slow down neutrons while minimizing parasitic absorption -- thus, here  $\alpha_{Tm} < 0$ .

But, the amount of soluble boron can change this ...

## Affect of Soluble Boron on MTC

In a PWR with a high soluble boron loading (especially at the beginning of a new fuel cycle), the usual situation discussed in the previous slide may be reversed (i.e. the moderator and soluble poison material may absorb more than it moderates) and  $\alpha_{Tm}$  can become slightly positive.

This can occur if a decrease in water density leads to a substantial decrease in the relative absorption rate in the coolant (moderator plus poison), which increases  $f$  to an extent that it more than offsets the other negative components -- and ultimately increases  $k_{eff}$ .

Clearly this situation needs to be avoided under hot operational conditions -- since all the reactivity coefficients should be negative to guarantee inherent safety.

## Affect of Soluble Boron on MTC



Thus, there is usually an **upper limit on the soluble boron loading that is allowed to assure that  $\alpha_{Tm}$  remains negative under most conditions.**

The upper limit is often in the range of **1800 – 2200 ppm** (where ppm refers to **grams of boron per 1 million grams of water**).

**Note:** A **small positive temperature coefficient is sometimes allowed under very restricted conditions during reactor startup at the BOC (cold zero power conditions).**

## Reactor Control Considerations



**Reactivity control via burnable poisons, soluble boron, and/or discrete control rods or blades is necessary to allow full operator control of the fission chain reaction at all times:**

1. To **facilitate normal operations and power maneuvers,**
2. To **compensate for changes in the fissile and fission product inventories over time,**
3. To **shutdown the system** for maintenance of both the primary and secondary systems.



The control rods in typical systems have a **variety of complicated physical designs**, and these discrete control configurations are often referred to as **black absorbers** -- that is, **once a neutron enters a black absorber, it does not return.**

## Reactor Control Considerations



**Diffusion theory** does not do a good job in these situations -- and **detailed transport theory models** are often required...

However, there are **several situations where the neutron poison material can be treated as a homogenous absorber** -- which is then **amenable to simple diffusion theory analyses**.

For example, for **fast reactors**, the **poison rods are not as black** (sometimes referred to as “gray”) because the **absorption cross sections are much lower at higher neutron energies** -- so that the **flux depression in and near the rods is considerably reduced**.

Thus, as a first approximation, **control rod worths in fast reactors can often be treated as homogeneous absorbers**.

## Reactor Control Considerations



And, of course, since the **fission product poisons in all reactors** and the **soluble boron distributions in PWRs** are already **physically well distributed throughout the core**, these situations are **quite accurately represented as homogeneous poisons**.

Thus, there are indeed **several applications where a simple approximate homogeneous poison treatment** can shed light on the approximate control/poison worths within the system -- **this is what we do here...**

## Treatment of Homogeneous Poisons

For the case of a **homogeneous poison**, let's first consider a **fast system represented via 1-group diffusion theory**.

In this case, the **multiplication factor** can be written as

$$k = k_{\infty}P_L = \eta f P_L$$

and a **change in reactivity** is given by

$$\rho_w = \rho - \rho_0 = \frac{k-1}{k} - \frac{k_0-1}{k_0} = \frac{k-k_0}{kk_0} \approx \frac{k-k_0}{k}$$

where  $\rho_w$  is the **reactivity worth** of the poison or control material,  $\rho$  is the **reactivity level with control**,  $\rho_0$  is the **reference poison-free reactivity state**, and the **last approximation assumes that the reference state is nearly critical with  $k_0 \approx 1$** .

## Treatment of Homogeneous Poisons

However, **since  $\eta$  and  $P_L$  do not change significantly**, an estimate of the **reactivity worth** is

$$\rho_w = \frac{\eta f P_L - \eta f_0 P_L}{\eta f P_L} = \frac{f - f_0}{f}$$

Note also that the **reactivity worth is often written as a positive value for convenience** and, **if control is being inserted we know that the worth is negative**, and **if it is being removed, then the reactivity change is positive**.

If this convention is followed, then  $\rho_w$  is given by

$$\rho_w = \frac{f_0 - f}{f}$$

since  $f_0 > f$  (recall that  $f_0$  is for the **reference poison-free state**).

## Treatment of Homogeneous Poisons

Now, for a **homogeneous system**, appropriate expressions for the fuel utilization can be inserted into this expression to give

$$\rho_w = \frac{\frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}} - \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aP}}}{\frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aP}}}$$

or

$$\rho_w = \frac{\Sigma_{aF}^2 + \Sigma_{aF}\Sigma_{aM} + \Sigma_{aF}\Sigma_{aP} - \Sigma_{aF}^2 - \Sigma_{aF}\Sigma_{aM}}{\Sigma_{aF}(\Sigma_{aF} + \Sigma_{aM})}$$

or

$$\rho_w = \frac{\Sigma_{aP}}{\Sigma_{aF} + \Sigma_{aM}}$$

worth of a homogeneous poison using 1-group theory

## Treatment of Homogeneous Poisons

**For thermal systems**, the **above general development can be adapted** to approximate the worth of homogeneous poisons (soluble boron and fission products), but **not for control rods**.

In this case, we argue that **the well-distributed poison material mostly affects the thermal utilization within the six-factor formula**, which leads to **essentially the same result** as above (with care taken to use the average thermal cross sections), or

$$\rho_w = \frac{\bar{\Sigma}_{aP}}{\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}}$$

worth of a homogeneous poison using 2-group theory

In **thermal systems**, **several alternate forms are also frequently used...**

## Treatment of Homogeneous Poisons

For example, recalling that  $f_o = \bar{\Sigma}_{aF} / (\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM})$ , the above expression can be written as

$$\rho_w = \bar{\Sigma}_{aF} f_o / \bar{\Sigma}_{aF}$$

Also, if we make the assumption that the **un-poisoned system is critical**, then

$$k_o = \eta_T f_o p \epsilon P_T P_F = \frac{v \bar{\Sigma}_f}{\bar{\Sigma}_{aF}} f_o p \epsilon P_T P_F = 1$$

and

$$f_o = \frac{\bar{\Sigma}_{aF}}{v \bar{\Sigma}_f p \epsilon P_T P_F}$$

Thus, the worth can be written as

$$\rho_w = \frac{\bar{\Sigma}_{aP} / \bar{\Sigma}_f}{v p \epsilon P_T P_F}$$

this form is particularly useful for estimating the worth of fission product poisons

## Treatment of Homogeneous Poisons

Another **useful expression for  $\rho_w$**  can also be developed by systematically eliminating any reference to the absorption rate in the fuel, as follows:

$$\rho_w = \frac{\bar{\Sigma}_{aP}}{\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}} = \frac{\bar{\Sigma}_{aP} / \bar{\Sigma}_{aM}}{\bar{\Sigma}_{aF} / \bar{\Sigma}_{aM} + 1}$$

but

$$\frac{\bar{\Sigma}_{aF}}{\bar{\Sigma}_{aM}} = \frac{\bar{\Sigma}_{aF} / (\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM})}{\bar{\Sigma}_{aM} / (\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM})} = \frac{f_o}{1 - f_o}$$

Thus,

$$\rho_w = \frac{\bar{\Sigma}_{aP}}{\bar{\Sigma}_{aM}} \left( \frac{1}{\frac{f_o}{1 - f_o} + 1} \right) = \frac{\bar{\Sigma}_{aP}}{\bar{\Sigma}_{aM}} \frac{1 - f_o}{f_o + 1 - f_o}$$

or

$$\rho_w = (1 - f_o) \frac{\bar{\Sigma}_{aP}}{\bar{\Sigma}_{aM}}$$

this form is used to estimate the worth of boric acid in PWRs

## Treatment of Homogeneous Poisons

As a specific application, this last form can be used to determine an approximate value for the **soluble boron worth in a PWR system**.

In particular, **boric acid ( $H_3BO_3$ )** is soluble in water and this homogeneous poison can be used to help **override the initial excess reactivity** of the fuel and to **compensate for fuel depletion and fission product buildup** -- and, since it is **distributed evenly throughout the coolant**, it **influences the reactivity without significantly affecting the flux and power profiles**.

Once dissolved in the water, the concentration of the boron within the system is usually given in **parts per million**, where **1 ppm implies 1 gram of boron per  $10^6$  grams of water**.

## Treatment of Homogeneous Poisons

If we let **C be the soluble boron concentration in ppm**, then the **ratio of the boron atom density to the moderator (water) atom density** is given by

$$\frac{N_B}{N_M} = \frac{m_B \frac{N_A \text{ atoms of B}}{10.8 \text{ g of B}}}{m_W \frac{N_A \text{ molecules of H}_2\text{O}}{18 \text{ g of H}_2\text{O}}} = \frac{18}{10.8} \frac{m_B}{m_W} = \frac{18 \times 10^{-6}}{10.8} C$$

where  **$m_B$  is the mass of boron** and  **$m_W$  is the mass of water**.

If we also **include estimates of the microscopic absorption cross sections** (from Lamarsh), we have

$$\frac{\bar{\Sigma}_{aB}}{\bar{\Sigma}_{aM}} = \frac{N_B}{N_M} \frac{\bar{\sigma}_{aB}}{\bar{\sigma}_{aM}} = \frac{18 \times 10^{-6}}{10.8} C \left( \frac{759}{0.66} \right) = 1.92 \times 10^{-3} C$$

## Treatment of Homogeneous Poisons

Finally, putting this last expressions into the  $\rho_w$  formula, gives

$$\rho_w = 1.92 \times 10^{-3} (1 - f_o) C$$

as the approximate **worth of C ppm of soluble boron** distributed evenly throughout a PWR core.

As a **numerical example**, a typical PWR with 3 – 5 w/o enriched fuel will usually have  $f_o$  in the range of 0.90 to 0.95.

Putting these values into the above formula gives

$$f_o = 0.90: \rho_w = 1.92 \times 10^{-3} (0.10)(1) = 0.0192 \text{ } \Delta k/k \text{ per ppm} \approx 0.02 \text{ } \Delta k/k \text{ per ppm}$$

$$f_o = 0.95: \rho_w = 1.92 \times 10^{-3} (0.05)(1) = 0.0096 \text{ } \Delta k/k \text{ per ppm} \approx 0.01 \text{ } \Delta k/k \text{ per ppm}$$

And, since a common “**unit of reactivity**” is the **pcm** (where **1 pcm stands for percent milli  $\rho$**  or, **1 pcm =  $10^{-5} \Delta k/k$** ), this simple example indicates that the **soluble boron worth in PWRs is usually about 10 – 20 pcm/ppm** (a rough “**rule-of-thumb**”).

## Worth of a Partially Inserted Rod

To complete our **brief discussion of control considerations** in both fast and thermal systems, we should address the **worth versus position of a partially inserted discrete control rod or blade**.

Although **diffusion theory does not allow an accurate treatment of the control rod worth**, it does permit a **good qualitative picture of the worth distribution versus insertion depth**.

In particular, using **Perturbation Theory Methods** (see brief overview in the Lecture Notes), it can be shown that the “**worth of a material inserted to an axial depth z within the reactor is proportional to the product of the forward and adjoint fluxes integrated over the perturbed domain**”.

The “**adjoint flux**” is related to the importance of a neutron to the system multiplication factor,  $k_{\text{eff}}$



## Worth of a Partially Inserted Rod

In particular, assuming 1-group theory and that movement of the control rod only perturbs the absorption cross section, we have

$$\rho_w(z) = \alpha \int_0^z \phi^*(z') \Delta \Sigma_a(z') \phi(z') dz'$$

However, since the 1-group diffusion equation is self-adjoint, the adjoint and forward fluxes are identical, which gives

$$\rho_w(z) = \alpha \int_0^z \phi^2(z') \Delta \Sigma_a(z') dz'$$

Now, for a bare 1-D homogeneous critical reactor of total height  $H$ , the flux profile is given by

$$\phi(z) = A \sin Bz \quad \text{with} \quad B^2 = \left( \frac{\pi}{H} \right)^2$$

where  $z$  is measured from the top of the reactor (for simplicity, we have ignored the extrapolation distance,  $d$ , here).

## Worth of a Partially Inserted Rod

Finally, if the rod absorption cross section is constant, then combining the flux profile for a homogeneous system with the equation for  $\rho_w(z)$  gives

$$\rho_w(z) = C \int_0^z \sin^2 \frac{\pi z'}{H} dz' = C \left[ \frac{z'}{2} - \frac{H}{4\pi} \sin \frac{2\pi z'}{H} \right]_0^z = C \frac{H}{2} \left( \frac{z}{H} - \frac{1}{2\pi} \sin \frac{2\pi z}{H} \right)$$

where  $C$  is just a new proportionality constant.

To evaluate this constant, we let  $\rho_w(z)|_{z=H} = \rho_w(H)$ , which is the total rod worth.

With this constraint we have

$$\rho_w(H) = C \frac{H}{2} (1 - 0) \quad \text{or} \quad C = \frac{2}{H} \rho_w(H)$$

## Worth of a Partially Inserted Rod

Thus, the so-called **ideal integral worth distribution** becomes

$$\rho_w(z) = \rho_w(H) \left( \frac{z}{H} - \frac{1}{2\pi} \sin \frac{2\pi z}{H} \right)$$

ideal integral worth curve

where  $\rho_w(z)$  is the worth of a partially inserted rod to depth  $z$ .

Also of interest is the **rate of change of  $\rho_w(z)$  per unit distance**.

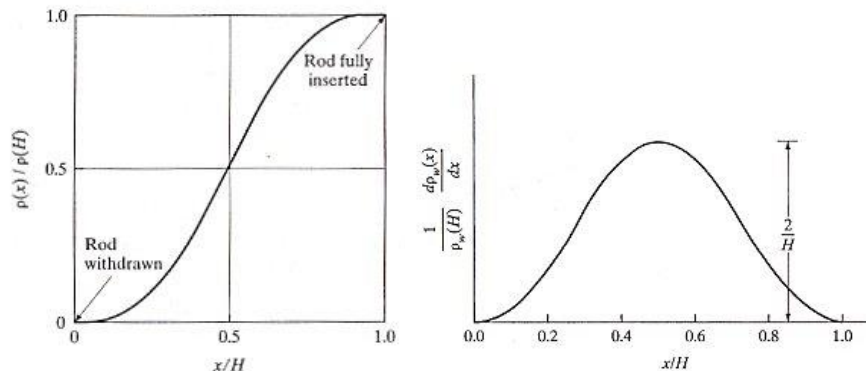
This **ideal differential worth profile** can easily be obtained by differentiation of the ideal integral worth expression, or

$$\frac{d}{dz} \rho_w(z) = \frac{\rho_w(H)}{H} \left( 1 - \cos \frac{2\pi z}{H} \right)$$

ideal differential worth curve

## Worth of a Partially Inserted Rod

Finally, if **one plots the integral and differential worth profiles**, the **ideal S-shaped normalized integral rod worth curve** and the **familiar bell-shaped differential rod worth curve** are obtained (here  $x = z$  from our development)...



## Worth of a Partially Inserted Rod

In practice, of course, the **integral and differential worth curves for real reactor systems differ somewhat from the ideal curves shown here** (note that these were developed using **first-order perturbation theory for a bare homogeneous 1-group system -- a pretty idealized situation indeed...**).

However, they **do give a good qualitative view** of what to expect for a real system:

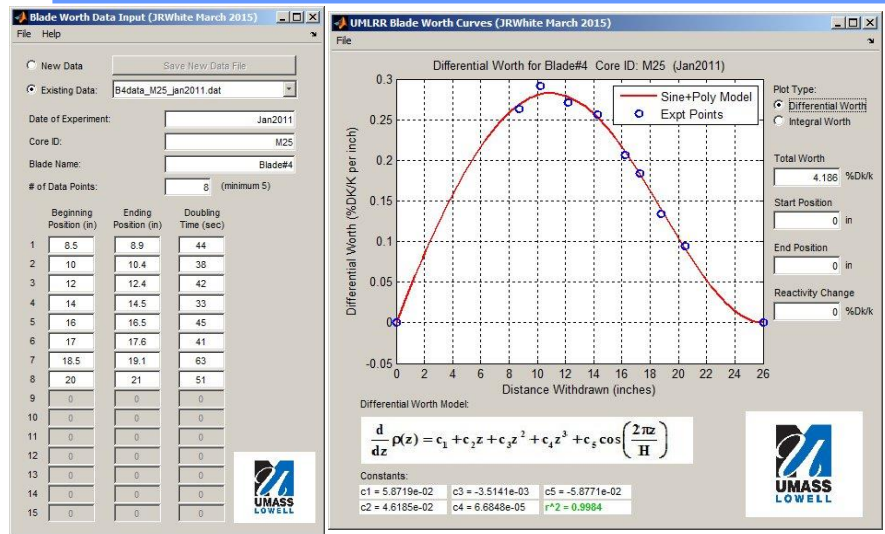
1. They show a **low differential worth near the upper and lower boundaries** -- **where the flux and importance functions are relatively low.**
2. They show a **peak differential worth near the core center** -- **where we expect the highest flux and the largest neutron importance.**

See the `blade_worth_gui` code for measured curves generated specifically for the UMLRR

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## blade\_worth\_gui Interface



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## UMLRR Blade Worth Profiles



The `blade_worth_gui` code shows, using real measured data, that the differential worth curves within the UMLRR tend to follow a slightly bottom-skewed bell-shaped curve:

The bell-shaped profile is due to the higher neutron flux at core center and that neutrons in this central region contribute more to the system's criticality than neutrons near the ends of the core.

The slight downward skew is associated with the remaining control blades that are partially inserted into the upper portion of the core to offset the available excess fuel reactivity.

This partial insertion causes a slightly bottom-peaked flux distribution and differential blade worth profile.

Thus, the worth of a partially inserted UMLRR control blade behaves qualitatively as expected from simple theory -- but for quantitative evaluation, real measured data are always required...

## Lesson 11 Summary



In this Lesson we have briefly discussed the following subjects:

The term reactivity coefficient and why this must be negative at hot conditions within any reactor.

The fact that the total reactivity coefficient is simply the sum of the coefficients associated with individual effects.

The basic concepts associated with justifying the sign of the fuel temperature or Doppler coefficient.

The competing effects that are often associated with establishing the sign of the moderator/coolant temperature coefficient.

The development of several expressions for the reactivity worth of a homogeneous poison within both fast or thermal systems.

The shape of typical integral and differential rod worth curves.

How to use the `blade_worth_gui` program and why the measured differential blade worth curves for the UMLRR is bottom-peaked.