Nuclear Reactor Theory

Lesson 10: The Time Dependent Reactor I

Overview and Treatment of Point Kinetics

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Lesson 10 Objectives

List the three main time-dependent phenomena of interest and identify the time scale associated with each area.

Write the fission source term needed for reactor kinetics studies and explain how this differs from the steady state fission source.

Outline the procedure used to convert the 1-group space-time kinetics formulation into the 1-speed point kinetics model.

Identify the primary advantage associated with the Lifetime and/or Generation Time Formulations relative to the standard time dependent diffusion equation representation.

Outline the procedure for solving the Generation Time Formulation of point kinetics for a step change in reactivity.

Lesson 10 Objectives (cont.)

Explain the reactivity equation in some detail: discuss the sign and magnitude of the roots and the time dependent behavior of the power level following a step change in reactivity.

Explain how the reactor period and prompt jump/drop approximations are used to estimate the behavior of the power following a step change in reactivity.

Explain the concept of reactivity feedback and sketch the power profiles associated with a step change in reactivity with and without negative feedback.

Perform a series of simple calculations to quantify the reactor period, prompt jump/drop, power level at various times, etc. for a variety of simple transient scenarios.

Time-Dependent Phenomena

There are a number of aspects of reactor analysis (such as the fuel burnup process, reactor operations during reactor startup and shutdown periods and for various power maneuvers, and transient operations and control during off-normal conditions) that we have not discussed as yet -- and all these processes are inherently time dependent.

Within this context, there are three primary time-dependent phenomena of interest with significantly different time constants, as follows:

Subject	Time Scale of Interest
Reactor Kinetics	seconds → minutes
Fission Product Poisoning	hours → days
Fuel Depletion	months → years

Reactor Kinetics

Reactor Kinetics is the treatment of the time behavior of the neutron level and distribution over short periods of time.

The variations are usually caused by changes in control rod positioning, soluble boron concentration, or the fuel and coolant temperatures.

Changes in these parameters add reactivity (positive or negative) to the core.

This perturbs the critical system so that the multiplication factor, k_{eff}, is no longer unity.

Depending on the perturbation, k_{eff} can be slightly greater than or less than unity, and the neutron density and power level will increase or decrease correspondingly.

Reactor Kinetics (cont.)

The physics of the reactor in these situations is described by the time dependent neutron balance equation.

For space-time reactor kinetics, $\phi(\vec{r},t) \rightarrow \psi(\vec{r},t)T(t)$

For point kinetics, $\phi(\vec{r},t) \rightarrow \psi_o(\vec{r})T(t)$, where the spatial distribution is assumed to be essentially time-independent.

We will focus on the Point Kinetics model.

Fission Product Poisoning

Fission Product Poisoning is another important time dependent phenomenon.

Fission products (FPs) accumulate in a reactor from production via the fission reaction and these intermediate mass nuclides cause parasitic absorption in the core.

This is especially important in thermal reactors, since most absorption cross sections are relatively high at thermal energies -- however, the long term effect of fission product poisoning is important in all systems.

A few fission product nuclides play an especially important role in thermal systems because of their extremely large thermal absorption cross sections and their decay behavior.

Fission Product Poisoning

For example, for Xe-135, $\sigma_a(E_o) = 2.65\times10^6$ barns

for Sm-149, $\sigma_a(E_o) = 41{,}000$ barns

for a typical fission product, $\sigma_a(E_o) \approx$ 40-50 barns

The time constants associated with the dynamics of the important FP chains are on the order of hours to days (not a safety concern).

We will highlight the dynamics of the I-Xe chain.

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Fuel Depletion/Transmutation

A third transient effect that requires consideration is the Fuel Burnup Process (occurs over relatively long periods of time).

Fresh fuel inserted into a reactor is usually free of fission product poisons and the higher actinides.

However, once power operation begins, neutron fission, which produces the FPs, and neutron capture, which produces higher actinides, alter the distribution of nuclides in the system.

This transmutation of the heavy elements and the continuous buildup of FPs certainly affect the instantaneous neutron balance within the system.

To maintain criticality over the design cycle length, considerable excess fuel must be loaded initially, where the initial excess reactivity is balanced by neutron poisons (typically soluble boron and burnable absorbers in a PWR).

Fuel Depletion/Transmutation

As the fuel depletes and the FPs accumulate, the amount of the controlled poisons is reduced and, by definition, the end-of-cycle is reached when the excess reactivity of the fuel is zero with no control in the core.

We will only overview the key aspects of the fuel depletion process.

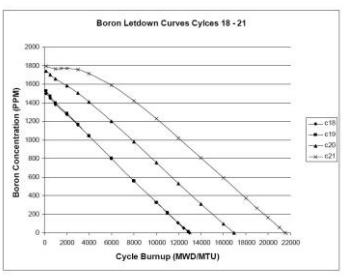

Figure 1: Boron Letdown Curves for Krsko Cycles 18 - 21

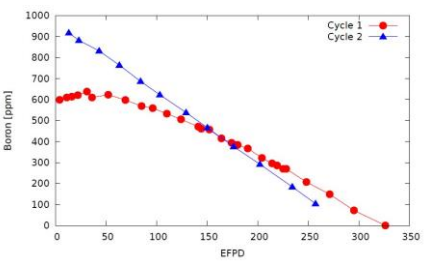

Figure 48: Measured boron letdown curves for two cycles of operation.

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Time Dependent Diffusion Equation

The remainder of this lesson will elaborate on the subject of Reactor Kinetics (with additional discussion of the other topics in future lessons).

The starting point here is the time-dependent diffusion equation that was developed previously.

In words, this equation states that the

$$\begin{matrix}\text{rate of change} \\ \text{of neutron density}\end{matrix} = \begin{matrix}\text{production rate of} \\ \text{neutrons per unit volume}\end{matrix} - \begin{matrix}\text{loss rate of} \\ \text{neutrons per unit volume}\end{matrix}$$

and, for the 1-group or 1-speed diffusion theory approximation, this was written as

$$\frac{\partial}{\partial t} n = \frac{1}{v}\frac{\partial}{\partial t}\phi = \left[Q + \nu\Sigma_f\phi\right] - \left[-\vec{\nabla}\cdot D\vec{\nabla}\phi + \Sigma_a\phi\right]$$

Time Dependent Diffusion Equation

Although this equation, with the derivative set to zero, is valid for steady state studies, the fission source term is not correct for dynamics studies.

In particular, since both prompt and delayed neutrons are produced, we must take into account the timing associated with these separate components of the fission source.

Recall that delayed neutrons are produced from the decay of certain nuclides (called precursors) that are produced in the fission process.

The delayed neutron precursors are usually grouped into six separate groups with six effective decay constants, λ_i, and yields, β_i.

Time Dependent Diffusion Equation

The **total delayed neutron fraction** is given by $\beta = \sum\beta_i$, where

$$\beta_i = \frac{\nu_i}{\nu_T} = \frac{\text{delayed neutrons from precursor group i per fission}}{\text{total neutrons emitted per fission}}$$

$$= \text{fraction of total neutrons emitted that result from the decay of precursor group i}$$

With these definitions, one has

β = fraction of total neutrons that are **delayed**

$1 - \beta$ = fraction of total neutrons that are **prompt**

Thus, the **obvious choice** for the fission source for the 1-speed case becomes

$$S_{fis}^{total} = S_{fis}^{prompt} + S_{fis}^{delayed}$$
$$= (1-\beta)\nu\Sigma_f\phi + \beta\nu\Sigma_f\phi = \nu\Sigma_f\phi$$

But the timing here is wrong, since this only accounts for the number of neutrons, not the time when they are produced.

Time Dependent Diffusion Equation

Concerning the timing of the neutron production terms, the **prompt term**, $(1-\beta)\nu\Sigma_f\phi$, **accounts for the instantaneous release of prompt neutrons at the time of fission.**

The **delayed term**, $\beta\nu\Sigma_f\phi$, **is not the delayed neutron production term but, instead, it is the instantaneous precursor production rate.**

The **delayed neutrons, in turn, result from the decay of the precursors** (which have characteristic decay constants, λ_i).

Thus, since **each precursor decay produces one delayed neutron**, we have

$$S_{fis_g}^{delayed} = \sum_{i=1}^{6} \chi_{gi}\lambda_i C_i$$

Note that the delayed neutron spectrum is softer than the prompt neutron spectrum.

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Time Dependent Diffusion Equation

Therefore, for the **multigroup case**, the **total fission source** can be written as

$$S_g^{fis} = \chi_g^p \sum_{g'} (1-\beta)\nu\Sigma_{fg'}\phi_{g'} + \sum_i \chi_{gi}^d \lambda_i C_i$$

and the **total fission source for the one energy group case**, becomes

$$S^{fis} = (1-\beta)\nu\Sigma_f\phi + \sum_i \lambda_i C_i$$

This cannot treat the differences in the prompt and delayed spectra.

NOTE: In practice, the use of β_{eff} instead of β is used in the final equations to account for the actual differences in the prompt and delayed neutron spectra…

1-Speed Space-Time Kinetics

In summary, we shall write the complete (and correct) 1-speed space-time kinetics equations as follows:

Neutron Balance

$$\frac{1}{v}\frac{\partial}{\partial t}\phi = \left[(1-\beta)\nu\Sigma_f\phi + \sum_i \lambda_i C_i + Q\right] - \left[-\vec{\nabla}\cdot D\vec{\nabla}\phi + \Sigma_a\phi\right]$$

Precursor Balance

$$\frac{\partial C_i}{\partial t} = \beta_i \nu\Sigma_f\phi - \lambda_i C_i \quad \text{for } i = 1, 2, \cdots 6$$

These equations represent a set of seven coupled PDEs, where the cross sections, fluxes, and source are all functions of both space and time.

Computer codes are available to solve the space-time kinetics problem -- but this subject is outside the scope of this course…

In general, these equations are rather difficult to solve!!!

1-Speed Point Kinetics Model

There are many applications in reactor operations when the spatial flux shape does not change significantly with time.

For these cases, the general space-time description can be reduced to a point model (spatially integrated model) that includes time as the only independent variable.

This procedure reduces the system to seven ordinary differential equations (ODEs) – which are significantly easier to solve.

There is a formal procedure for doing this reduction, during which, the "effective" kinetics parameters are defined precisely.

The most general procedure usually starts with the multigroup neutron balance equation, but the 1-speed approximation allows a more straightforward development that gives identical point kinetics equations -- with slightly less rigor in the definition of some parameters.

1-Speed Point Kinetics Model (cont.)

Since the resultant differences in definition do not affect our present discussion and application of the final equations, we will proceed here with the 1-speed formulation (since the notation is much easier to follow).

Starting with the 1-speed space-time model, we assume that the flux can be separated into a slowly varying (or time independent) spatial distribution and a more rapidly varying amplitude function,

$$\phi(\vec{r},t) = \psi(\vec{r},t)T(t) \approx \psi_o(\vec{r})T(t)$$

where the spatial distribution with the 'o' subscript represents the initial steady state value and T(t) represents the time-dependent amplitude of the neutron flux (or power level).

1-Speed Point Kinetics Model (cont.)

Now, we substitute this approximation into the neutron and precursor balance equations and integrate the resultant equations over the spatial domain of interest to give

$$\frac{1}{v}\langle\psi_o\rangle\frac{dT}{dt}=\left[(1-\beta)\langle\nu\Sigma_f\psi_o\rangle T+\sum_i\lambda_i\langle C_i\rangle+\langle Q\rangle\right]-\left[\left\langle-\vec{\nabla}\cdot D\vec{\nabla}\psi_o\right\rangle+\langle\Sigma_a\psi_o\rangle\right]T$$

$$\frac{d}{dt}\langle C_i\rangle=\beta_i\langle\nu\Sigma_f\psi_o\rangle T-\lambda_i\langle C_i\rangle \qquad \text{for } i=1,2,\cdots 6$$

These represent a set of seven coupled first-order ordinary differential equations (ODEs) -- that is, the Point Kinetics Model.

1-Speed Point Kinetics Model (cont.)

The Point Kinetics model given on the previous slide is usually not used in this form for practical application.

In particular, since the cross sections can be time dependent and under operator control (i.e. movement of a control rod affects Σ_a, etc.), almost every term in these equations can be modified to initiate a transient case.

However, from an operational perspective, the effect of a change in cross section (or material composition) manifests itself as a change in the multiplication factor, k, or in the reactivity, ρ.

Changing the above equations to incorporate k or ρ directly leads to the traditional Lifetime Formulation (uses k) and Generation Time Formulation (uses ρ) of point kinetics.

In these formulations, the multiplication factor, k(t), or reactivity, $\rho(t)$, becomes the driving force for initiating most transient analyses.

The Generation Time Formulation

In this lesson we will focus on the Generation Time Formulation, but the formal Lecture Notes develop both schemes in detail.

One first defines the prompt neutron generation time by arguing that, at steady state, the neutron production rate from fission in a critical system is the total neutron population divided by the neutron generation time.

In equation form, this can be written as

$$\text{production rate} = \frac{\text{neutron population}}{\text{generation time}} \quad \text{or} \quad \text{generation time} = \frac{\text{neutron population}}{\text{production rate}}$$

Defining Λ as the prompt neutron generation time, the one-speed approximation gives

$$\Lambda = \frac{1}{v}\langle \psi_o \rangle \Big/ \langle \nu\Sigma_f \psi_o \rangle$$

The Generation Time Formulation

Note: In the context of the 1-group diffusion equation, k and ρ have the following formal definitions (these are used in subsequent manipulations):

$$k = \frac{\text{neutron production rate from fission}}{\text{loss rate}} = \frac{\text{production}}{\text{loss}}$$

$$k = \frac{\langle \nu\Sigma_f \psi_o \rangle}{\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o \rangle + \langle \Sigma_a \psi_o \rangle}$$

and

$$\rho = \frac{k-1}{k} = \frac{\text{production - loss}}{\text{production}}$$

$$\rho = \frac{\langle \nu\Sigma_f \psi_o \rangle - \left[\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o \rangle + \langle \Sigma_a \psi_o \rangle \right]}{\langle \nu\Sigma_f \psi_o \rangle}$$

Generation Time Formulation (cont.)

Now we divide every term in the point kinetics equation for the neutron level by the neutron production rate from fission to give

$$\frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}\frac{dT}{dt} = (1-\beta)\frac{\langle\nu\Sigma_f\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle - \frac{\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o\rangle + \langle\Sigma_a\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}T$$

and use the definitions of ρ and Λ to simplify to

$$\Lambda\frac{dT}{dt} = \left[\frac{\langle\nu\Sigma_f\psi_o\rangle - \left[\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o\rangle + \langle\Sigma_a\psi_o\rangle\right]}{\langle\nu\Sigma_f\psi_o\rangle} - \beta\right]T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle$$

or $$\Lambda\frac{dT}{dt} = (\rho-\beta)T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle$$

Generation Time Formulation (cont.)

Now, we define the normalized precursor and external source amplitudes as

$$c_i(t) = \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle C_i(t)\rangle \qquad q(t) = \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle Q(t)\rangle$$

When these expressions are substituted into the above equation and we use the definition of the generation time, the final neutron balance equation results

$$\Lambda\frac{dT}{dt} = (\rho-\beta)T + \sum_i \lambda_i \frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}c_i + \frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}q$$

or neutron amplitude $$\frac{dT}{dt} = \left(\frac{\rho-\beta}{\Lambda}\right)T + \sum_i \lambda_i c_i + q$$

Generation Time Formulation (cont.)

Finally, to complete the generation time formulation, we divide the precursor equation by the total neutron population and again use the definition of Λ and the normalized precursor amplitude to give

$$\frac{d}{dt}\frac{\langle C_i\rangle}{\frac{1}{v}\langle\psi_o\rangle} = \beta_i \frac{\langle \nu\Sigma_f\psi_o\rangle}{\frac{1}{v}\langle\psi_o\rangle} T - \lambda_i \frac{\langle C_i\rangle}{\frac{1}{v}\langle\psi_o\rangle} \qquad \text{for } i = 1, 2, \cdots 6$$

or

precursor amplitudes

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} T - \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6$$

The highlighted equations represent the Generation Time Formulation of Point Kinetics.

Normalization Considerations

The solution of the kinetics equations usually leads to relative results -- that is, one computes $T(t)/T_o = n(t)/n_o = P(t)/P_o$ etc., where these represent the time-dependent relative flux amplitude, neutron level, power level, etc.

However, when reactivity feedbacks are important, knowledge of the absolute neutron level or power level becomes essential.

As detailed in the Lecture Notes, one can formally derive a set of point kinetics equations that directly include the actual reactor power level, P(t), in watts and the neutron source level, <Q(t)>, in neutrons/sec. The resultant equations are:

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \quad \text{for } i = 1, 2, \cdots 6$$

These Point Kinetics equations will be highlighted in the remainder of these Lecture Notes...

Solution of the Point Kinetics Eqn.

In general, analytical solution of the point kinetics equations is not easy -- recall that we have a coupled set of seven ODEs!

In most cases, these equations are evaluated for a given $\rho(t)$ using numerical methods (such as Matlab's ode15s solver -- where a stiff equation solver is needed because of the large difference in time constants that results).

For a few specific cases, an analytical solution is possible -- and the resultant solutions give considerable insight into the general behavior of the time dependent neutron balance in real systems.

One common situation that can be solved analytically involves a step change in reactivity in a critical reactor operating at low power ("low power" means that feedback effects are negligible).

The solution of this case allows us to introduce some common terminology, and to gain a good understanding of the expected behavior in several common situations.

Step Change in Reactivity

Starting with the generation time formulation of point kinetics with no external source, we have

$$\frac{dP}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right)P + \sum_i \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda}P - \lambda_i c_i \quad \text{for } i = 1, 2, \cdots 6$$

In most applications of these equations, the kinetics parameters (Λ, β_i, and λ_i) are assumed to be constant, the reactivity is the driving force for the transient, and P(t) and $c_i(t)$ are the dependent variables that vary with time due to some changing $\rho(t)$.

However, for a step change in reactivity, $\rho(t) = \rho$ = constant, and the above equations become a system of seven linear constant coefficient ODEs -- and this falls into a class of problems that we know how to handle analytically.

Step Change in Reactivity (cont.)

The standard approach for solving linear time-invariant systems is to assume a solution of the form of a simple exponential.

Following this technique, we assume that

$$P(t) = A_o e^{\omega t} \qquad \text{and} \qquad c_i(t) = A_i e^{\omega t}$$

Now, we substitute these assumed solutions into the precursor balance equations, to obtain

$$A_i \omega e^{\omega t} = \frac{\beta_i}{\Lambda} A_o e^{\omega t} - \lambda_i A_i e^{\omega t} \quad \Rightarrow \quad A_i (\omega + \lambda_i) = \frac{\beta_i}{\Lambda} A_o$$

or

$$A_i = \frac{\beta_i / \Lambda}{\omega + \lambda_i} A_o$$

Step Change in Reactivity (cont.)

Now, putting the assumed solutions, along with the above result, into the P(t) equation gives

$$A_o \omega e^{\omega t} = \left(\frac{\rho - \beta}{\Lambda} \right) A_o e^{\omega t} + \sum_i \lambda_i A_i e^{\omega t} = \left(\frac{\rho - \beta}{\Lambda} \right) A_o e^{\omega t} + \sum_i \lambda_i \frac{\beta_i / \Lambda}{\omega + \lambda_i} A_o e^{\omega t}$$

Cancelling the common $A_o e^{\omega t}$ factor in each term and multiplication by Λ gives

$$\Lambda \omega = (\rho - \beta) + \sum_i \lambda_i \frac{\beta_i}{\omega + \lambda_i}$$

and solving for ρ gives

$$\rho = \Lambda \omega + \beta - \sum_i \frac{\beta_i \lambda_i}{\omega + \lambda_i}$$

Step Change in Reactivity (cont.)

To put this expression into standard form, note that $\beta = \sum \beta_i$.

Now, using this equality, we have

$$\rho = \Lambda\omega + \sum_i \left(\beta_i - \frac{\beta_i \lambda_i}{\omega + \lambda_i} \right) = \Lambda\omega + \sum_i \left(\frac{\beta_i \omega + \beta_i \lambda_i - \beta_i \lambda_i}{\omega + \lambda_i} \right)$$

or

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$

reactivity equation

This equation is the standard form of the so-called reactivity equation (or inhour equation) obtained from the generation time formulation of point kinetics.

Step Change in Reactivity (cont.)

From a pure mathematical viewpoint, the reactivity equation is simply the characteristic equation associated with the original seven linear constant coefficient ODEs -- and, for a given value of reactivity, the roots of this equation give the values of ω that satisfy the original form of the assumed solution.

Also, since we have seven coupled first-order ODEs, we should expect seven roots (i.e. seven values of ω) that will satisfy the so-called reactivity equation.

Assuming that each ω_j for j = 1, 2, ··· 7 is distinct, the linear super-position principle allows us to write the general solution as a linear combination of the linearly independent individual solutions.

Step Change in Reactivity (cont.)

Thus, we can write a general solution for the time-dependent power level P(t) as

$$P(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \cdots + A_7 e^{\omega_7 t}$$

The A_j coefficients are determined from the ICs.

where the ω_j's are the roots of the reactivity equation and the A_j's are the seven arbitrary coefficients needed for the general solution of a 7th order initial value problem (IVP).

Although the above development establishes a solid mathematical foundation, it really has not shed a lot of insight into the actual behavior of P(t).

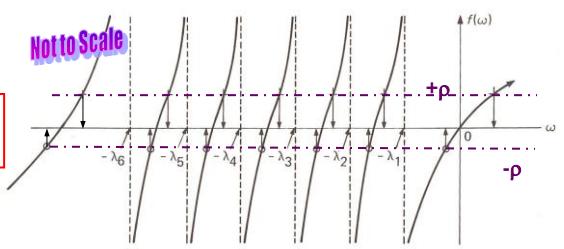
To do this, we must first get a better understanding of the reactivity equation and the values of its roots.

Interpretation of the Reactivity Eqn.

First, we can think of the right hand side (RHS) of the reactivity equation as some function of ω, say f(ω), and simply plot f(ω) vs. ω for a wide range of ω.

Then, if we superimpose the LHS [i.e. ρ(ω) = constant] on the plot, the intersections of the two curves give the desired roots, ω_j, of the reactivity equation.

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$



positive ρ: one positive root and six negative roots

negative ρ: seven negative roots

Interpretation of the Reactivity Eqn.

If we order the roots ω_j from most positive to most negative, then, after a relatively short transient time, the last six terms for P(t) decay away (because $\omega_2, \omega_3, \cdots, \omega_7 < 0$ for both positive and negative reactivity), leaving only the term containing ω_1, or

$$P(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \cdots + A_7 e^{\omega_7 t} \approx P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau}$$

where $\tau = 1/|\omega_1|$ is called the stable reactor period and P_1 is the power level (or flux amplitude) just after the short transient period.

If ρ is positive, $\omega_1 > 0$, and the reactor period is positive -- so P(t) grows indefinitely as $e^{+t/\tau}$ (remember that we assumed no feedbacks up to this point).

Interpretation of the Reactivity Eqn.

For negative reactivity, just the opposite occurs.

That is, $\omega_1 < 0$ and the reactor period is negative -- so the power level decreases towards zero with the form $e^{-t/\tau}$.

Thus, after a short transient time, the dominate behavior of P(t) is simply associated with the most positive root of the reactivity equation and it is represented as a simple growing or decaying exponential.

where all the above descriptions assume no feedbacks

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Typical Solution Profiles

To wrap up our formal discussion of the reactivity equation and the solution of the Generation Time Formulation of Point Kinetics for a step change in reactivity, it makes sense to show the typical P(t) behavior for a specific change in reactivity.

This was accomplished in a simple Matlab code, with and without feedbacks, for the case of both positive and negative reactivity ($\rho/\beta = \pm 0.25$).

To illustrate the stabilizing effect associated with negative feedback, we define a generic power feedback coefficient as

$$\alpha_p = \frac{\partial \rho}{\partial P} = \frac{1}{k^2}\frac{\partial k}{\partial P} \approx \frac{1}{k}\frac{\partial k}{\partial P}$$

Typical Solution Profiles

With this definition, we see that the actual reactivity that enters into the point kinetics equation is a combination of the externally applied reactivity, ρ_{ext} (e.g., due to a change in control rod position) and the feedback reactivity, ρ_f (which is inherently time-dependent due to changes that are not under operator control), or

$$\rho(t) = \rho_{ext} + \rho_f(t) = \rho_{ext} + \alpha_p\left(P(t) - P_o\right)$$

Note that, if the feedback coefficient is positive, the system is inherently unstable and it will quickly destroy itself.

This is readily apparent since, with a positive value of α_p, an increase in reactivity leads to an increase in power, which leads to a further increase in reactivity, which gives another increase in power, and so on -- which leads to a runaway system.

Typical Solution Profiles (cont.)

If α_p is negative, then an increase in power reduces ρ, which decreases P, which increases ρ, etc. until a new steady state condition is realized.

The negative feedback situation is clearly the only reasonable option, and all operating reactors are required to have a negative feedback coefficient under all possible hot conditions!!!

Note that , by definition, criticality is achieved when $\rho = 0$.

Thus, the new steady state power level associated with the negative feedback case will be reached when the feedback reactivity exactly cancels the applied external reactivity,

$$\alpha_p \left(P_{new} - P_o \right) = -\rho_{ext} \qquad \text{or} \qquad P_{new} = P_o - \frac{\rho_{ext}}{\alpha_p}$$

Typical Solution Profiles (cont.)

With this brief background, we can now actually simulate and compare the dynamics of a system with and without feedback.

For the feedback-free case, we set $\alpha_p = 0$, and for the simulation with inherent feedbacks, we set the power feedback coefficient to its appropriate value for the system of interest (for this case, $\alpha_p = -2\times10^{-4}$ $\Delta k/k$ per unit ΔP).

The results of the Matlab simulation for the two cases with and without feedback are shown in the next few slides:

Positive ρ: For ρ_{ext} = +25 cents, we see the expected unbounded exponential increase in the flux or power level for the case of a positive reactivity insertion with no feedback.

However, for the case where the inherent negative feedback is treated, the power level rises less rapidly and it eventually levels off at a new steady state power.

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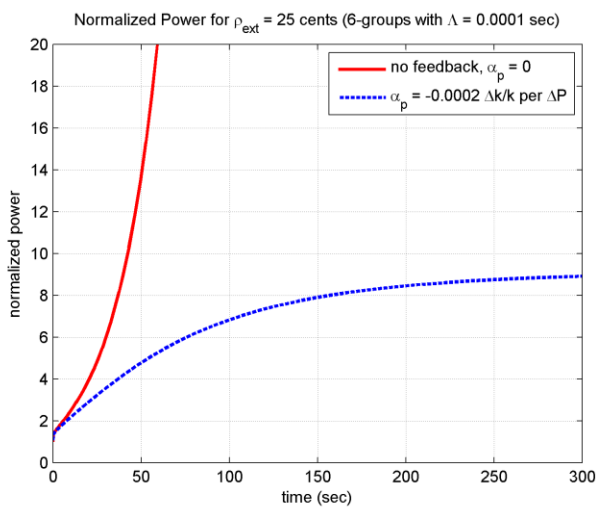
Typical Solution Profiles (cont.)

Negative ρ: For ρ_{ext} = -25 cents, both simulations lead to a decreasing power level.

In the case with inherent feedbacks, the exponential decrease is reduced slightly, but not enough to keep the reactor from complete shutdown.

This is true because the positive reactivity due to the power feedback is not sufficient to overcome the original negative external reactivity added to the system.

Typical Solution Profiles (cont.)



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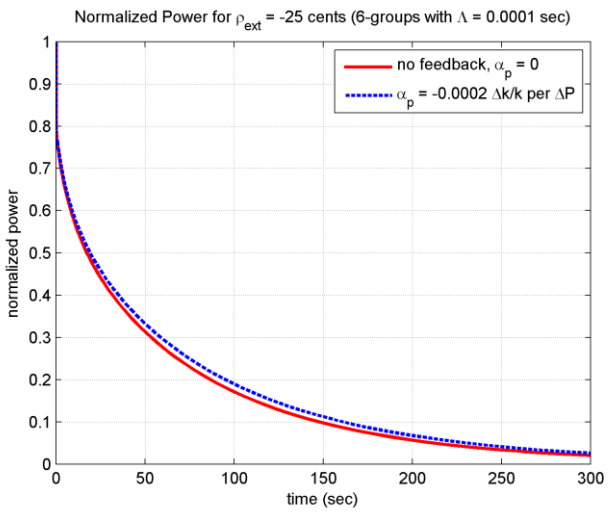
Typical Solution Profiles (cont.)



Numerical vs. Analytical Solution

The main point of the above discussion of the analytical solution method was so we could get a good understanding of the expected behavior and to introduce some important terminology associated with reactor kinetics.

However, actually computing accurate values for all seven roots of the reactivity equations, and then setting up the appropriate equations and solving for the seven coefficients for each transient situation of interest is not really easy to implement.

Also, remember that the analytical solution method can only be applied for ρ = constant -- it does not work for the general case of ρ = ρ(t).

Numerical vs. Analytical Solution

However, even for the simple situation where ρ = constant, the numerical solution of the seven coupled ODEs using an available ODE solver is a much easier path to follow (and this was the technique chosen here to do the actual simulations).

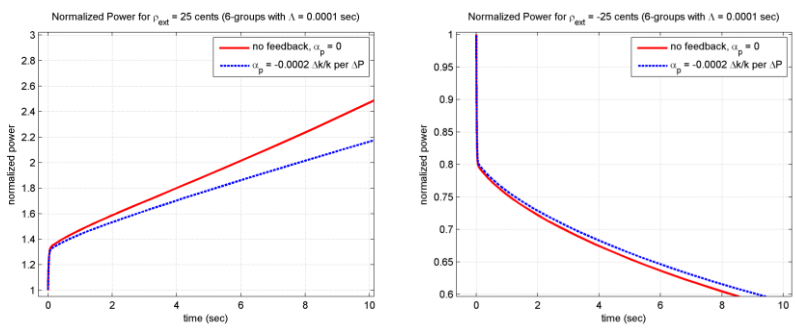
Thus, the numerical approach was selected for two important reasons:

- **The numerical solution is much easier to obtain.**
- **The numerical solution allows the treatment of feedback effects.**

The Prompt Jump/Drop

If we expand the first few seconds of the transient profiles shown previously, we see a nearly instantaneous rise or fall in the normalized power immediately after the step change in reactivity is made.

The rapid change seen here is due to the most negative root, ω_7, of the reactivity equation. Because ω_7 has such a large negative value, this term goes to zero very quickly after initiation of the transient.



The Prompt Jump/Drop (cont.)

Since the prompt jump/drop is an inherent feature of each transient, it would be convenient if we could get a quick and easy-to-use estimate of the magnitude associated with this phenomenon.

In particular, since we have already argued that the $A_1 e^{\omega_1 t}$ term dominates the transient response after a short period (for the no feedback case), if we could determine the normalized power, P_1, just after the prompt jump/drop, we would have a simple way to estimate the complete power profile versus time, or

$$P(t) = \left(\frac{P_1}{P_o}\right) P_o e^{\omega_1 t} \qquad \text{or} \qquad \frac{P(t)}{P_o} = \frac{P_1}{P_o} e^{\pm t/\tau}$$

where P_1/P_o is the desired magnitude of the prompt jump ($P_1/P_o > 1$) or prompt drop ($P_1/P_o < 1$).

The Prompt Jump/Drop (cont.)

To derive an expression for P_1/P_o, let's go back and consider the Generation Time Formulation of Point Kinetics,

$$\frac{dP}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right) P + \sum_i \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} P - \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6$$

Note that, since the phenomenon of interest here occurs very rapidly, we are only interested in the transient state over about 0.5 seconds or less.

Over this short interval, it is very reasonable to assume that the precursor densities do not change significantly.

The Prompt Jump/Drop (cont.)

Thus, over the time scale of interest for the prompt jump/drop, we have $dc_i/dt \approx 0$, or

$$\lambda_i c_{io} = \frac{\beta_i}{\Lambda} P_o \qquad \text{or} \qquad \sum_i \lambda_i c_{io} = \frac{\beta}{\Lambda} P_o$$

where all the quantities are evaluated just prior to the reactivity change (i.e. at t = 0).

With this expression and the same assumption as above, the P(t) equation becomes

$$\frac{dP}{dt} = \left(\frac{\rho - \beta}{\Lambda} \right) P + \frac{\beta}{\Lambda} P_o$$

This is just a simple first order linear ODE that, when written in standard form, gives

$$\frac{dP}{dt} - \frac{(\rho - \beta)}{\Lambda} P = \frac{\beta}{\Lambda} P_o$$

The Prompt Jump/Drop (cont.)

with integrating factor

$$g(t) = e^{\int -\frac{(\rho-\beta)}{\Lambda} dt} = e^{-\frac{(\rho-\beta)}{\Lambda} t}$$

Now, multiplying by the integrating factor, gives

$$e^{-\frac{(\rho-\beta)}{\Lambda} t} \left(\frac{dP}{dt} - \frac{(\rho-\beta)}{\Lambda} P \right) = \frac{d}{dt} \left(e^{-\frac{(\rho-\beta)}{\Lambda} t} P(t) \right) = \frac{\beta}{\Lambda} P_o e^{-\frac{(\rho-\beta)}{\Lambda} t}$$

and multiplication by dt and integration give

$$e^{-\frac{(\rho-\beta)}{\Lambda} t} P(t) = \frac{\beta}{\Lambda} P_o \int e^{-\frac{(\rho-\beta)}{\Lambda} t} dt = \frac{\beta}{\beta - \rho} P_o e^{-\frac{(\rho-\beta)}{\Lambda} t} + C$$

or

$$P(t) = \frac{\beta}{\beta - \rho} P_o + C e^{\frac{(\rho-\beta)}{\Lambda} t}$$ (C = integration constant)

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The Prompt Jump/Drop (cont.)

Now, even without computing the constant C, we can argue that the exponential term containing C will vanish very quickly.

Since $\rho < \beta$, the term $(\rho - \beta)/\Lambda$ is clearly negative.

Also, since the generation time, Λ, is usually quite small, the coefficient in the exponent is usually fairly large, causing this term to decay very quickly.

As an example, let $\Lambda = 0.0001$ sec, $\beta = 0.0065$, and $\rho = 0.25\beta$.

With these values, we have

$$\frac{(\rho-\beta)}{\Lambda} = \frac{(0.25-1)(0.0065)}{0.0001} = -48.75$$

and, in 0.20 sec, we have $e^{-48.75(0.20)} = 5.8\times10^{-5}$

Thus, in about 0.2 seconds, this term is only about 0.006% of its original value.

The Prompt Jump/Drop (cont.)

With the above arguments, it is easy to see that a reasonable approximation for the prompt jump/drop is given by the first term in the P(t) expression, or

$$\boxed{\frac{P_1}{P_o} = \frac{\beta}{\beta-\rho}}$$

where P_1 is the power level just after the prompt jump/drop.

By way of example, for the simulations shown previously, $\beta = 0.0065$ and $\rho = \pm 0.25\beta$.

Now, using the prompt jump/drop approximation, we have

$$\frac{P_1}{P_o} = \frac{\beta}{\beta-\rho} = \frac{1}{1-0.25} = 1.33 \quad \text{(for } \rho = +25 \text{ cents)}$$

and

$$\frac{P_1}{P_o} = \frac{\beta}{\beta-\rho} = \frac{1}{1+0.25} = 0.80 \quad \text{(for } \rho = -25 \text{ cents)}$$

These values agree very nicely with the prompt jump and drop transients seen in the previous figures!!!

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Small Reactivity Values

One last approximation that often simplifies hand calculations, concerns the treatment of small reactivity insertions.

For $\rho \approx 0$ (either positive or negative), the magnitude of the most positive root of the reactivity equation is small compared to the magnitude of all the λ_i values (i.e. $|\omega| << |\lambda_i|$).

With this observation, the reactivity equation becomes

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i} \approx \Lambda\omega + \omega \sum_i \frac{\beta_i}{\lambda_i} = \omega\left(\Lambda + \sum_i \frac{\beta_i}{\lambda_i}\right)$$

and, since the reactor period, τ, is just the inverse of the most positive root, ω_1, we have

$$\tau = \frac{1}{\rho}\left(\Lambda + \sum_i \frac{\beta_i}{\lambda_i}\right) \qquad \text{(for small } \rho\text{)}$$

Small Reactivity Values

Also, in all practical cases, the generation time, Λ, is small compared to the 2nd term inside the brackets.

Thus, we can estimate the reactor period as

$$\tau = \frac{1}{\rho}\sum_i \frac{\beta_i}{\lambda_i} = \frac{t_d \beta}{\rho} \qquad \text{(for small } \rho\text{)}$$

t_d is the mean lifetime of the delayed neutrons (about 12 – 14 seconds)

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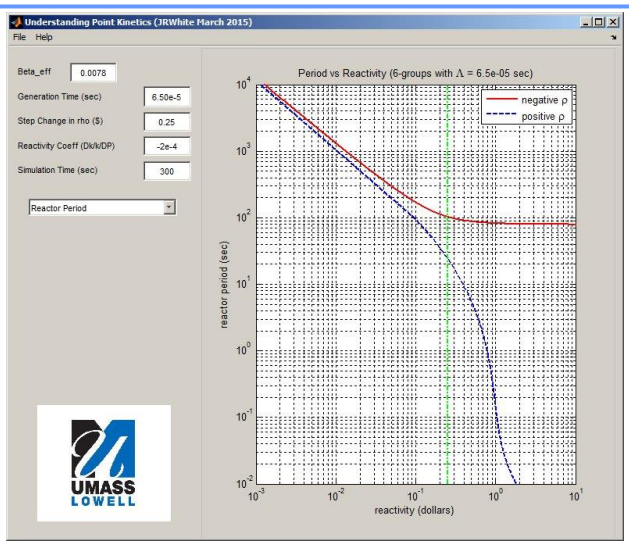
Capabilities of the kinetics_gui Code

Finally, we note that one can simply evaluate the reactivity equation and generate a plot of reactor period vs. reactivity -- this is a very useful operations/design tool.

The capability to do this, as well as plot the reactivity equation and generate the solution profiles for a given ρ has been incorporated into the kinetics_gui code.

The code is very simple to use and it gives a tremendous amount of insight into the workings of point kinetics -- you should give it a test drive!!!

The kinetics_gui Interface



Lesson 10 Summary

In this Lesson we have briefly discussed the following subjects:

The three main time-dependent phenomena of interest and the time scale associated with each area.

The fission source term needed for reactor kinetics studies and how this differs from the steady state fission source.

The procedure used to convert the 1-group space-time kinetics formulation into the 1-speed point kinetics model.

The primary advantage associated with the Lifetime and/or Generation Time Formulations relative to the standard time-dependent diffusion equation representation.

The procedure for solving the Generation Time Formulation of point kinetics for a step change in reactivity.

Lesson 10 Summary (cont.)

The reactivity equation -- including a discussion concerning the sign and magnitude of the roots and the actual time dependent behavior of the power level following a step change in reactivity.

How the reactor period and prompt jump/drop are used to estimate the behavior of the power following a step change in ρ.

The concept of reactivity feedback and the observed power profiles associated with a step change in reactivity with and without negative feedback.

How to perform a series of simple calculations to quantify the reactor period, prompt jump/drop, power level at various times, etc. for a variety of simple transient scenarios.