Nuclear Reactor Theory

Lesson 0: Review of Some Key Concepts from Fundamentals of NSE

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Lesson 0 Objectives

- Describe the terms **multiplication factor** and **reactivity**.
- Describe the following **terms** and identify their **units**: **neutron density**, **neutron flux and current**, **microscopic and macroscopic cross sections**, **target atom densities**, and **neutron interaction rates**.
- Explain the **general energy dependence, $\sigma(E)$, of typical cross sections**.
- Write a **formal expression** for an **average multigroup cross section** and define what is meant by the phrase "**the scalar flux for group g**."
- Explain, in general terms, **how all the above items fit together** within the context of **steady-state reactor theory**.

Some Terminology

Multiplication Factor: used to describe the state of the neutron balance in a nuclear system

Rate of Change = Production Rate – Loss Rate

$$k = \frac{\text{fissions in one generation}}{\text{fissions in preceeding generation}}$$

$$= \frac{\text{fission neutrons in one generation}}{\text{fission neutrons in preceeding generation}}$$

$$= \frac{\text{neutron production rate from fission}}{\text{neutron loss rate}}$$

$$k = \frac{\text{production}}{\text{absorption + leakage}}$$

Critical → production = absorption + leakage

Supercritical → production > absorption + leakage

Subcritical → production < absorption + leakage

Some Terminology (cont.)

Reactivity: measure of the deviation from critical

$$\rho = \frac{k-1}{k}$$ or $$\rho = \frac{\text{production - loss}}{\text{production}}$$

When the neutron production and loss rates are in balance, then the neutron population remains constant or, in steady state, we have

for steady state

Rate of Change = Production Rate – Loss Rate = 0

external source
fission source
inscatter source

We need to describe these rates mathematically

leakage
absorption
outscatter

Reaction Rates

Recall that:

n = **neutron density** (neutrons/cm^3)

v = **neutron speed** (cm/sec)

ϕ = **neutron flux** (neutrons/cm^2-sec)

$$\phi = nv \quad \rightarrow \quad \frac{n}{cm^2 - s} = \left(\frac{n}{cm^3}\right)\left(\frac{cm}{s}\right)$$

N = **target atom density** (atoms/cm^3)

σ_x = **microscopic cross section** (barns, where 1 b = 10^{-24} cm^2)

(measure of probability of interaction for reaction x)

R = F = **interaction rate density** (reactions/cm^3-sec)

$$R_x = N\sigma_x\phi \quad \rightarrow \quad \frac{reactions}{cm^3 - s} = \left(\frac{atoms}{cm^3}\right)\left(cm^2\right)\left(\frac{neutrons}{cm^2 - s}\right)$$

Reaction Rates (cont.)

Recall also that:

The term $N\sigma$ occurs so often that it is given the symbol, Σ, and called the **macroscopic cross section**.

For example,

$$\Sigma_t = N\sigma_t, \quad \Sigma_f = N\sigma_f, \quad \text{and} \quad \Sigma_c = N\sigma_c$$

refer to the macroscopic total, fission, and capture cross sections, respectively.

These quantities have units of cm^{-1} -- that is,

$$\Sigma_x = N\sigma_x \quad \rightarrow \quad \left(\frac{atoms}{cm^3}\right)\left(cm^2\right) = \left(\frac{atoms}{cm}\right) = cm^{-1}$$

$\mu = N\sigma$ is used for photons (linear interaction coeff.)

Σ (or μ) is the interaction probability per unit path length

Note, however, that the cross sections and neutron flux the are strong functions of neutron energy…

U235 Cross Sections

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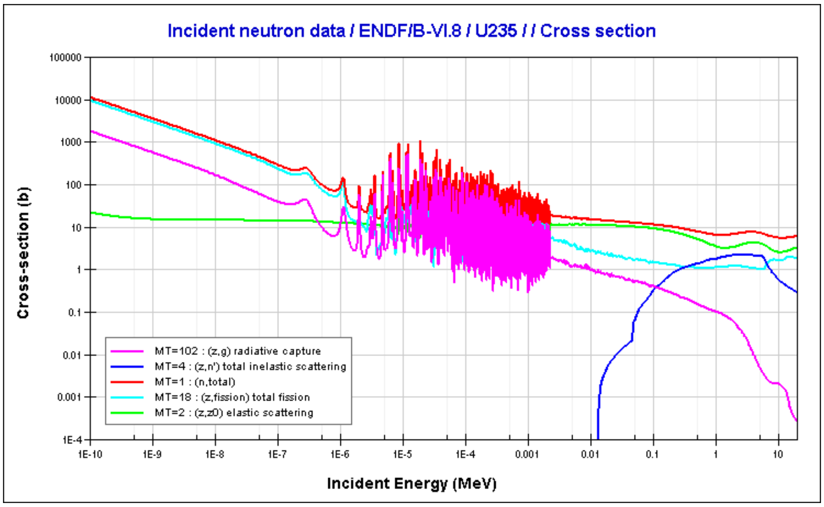

Natural Carbon Cross Sections

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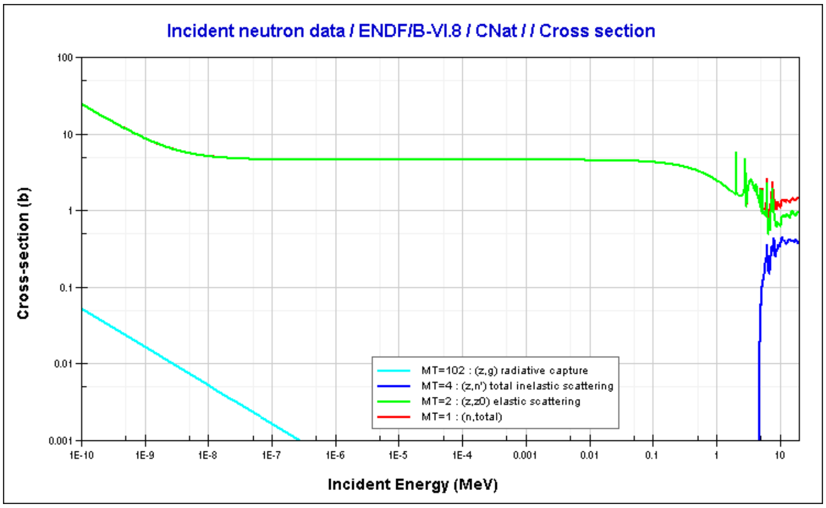

The Scalar Neutron Flux

As part of a practical model reduction process, we seek to simplify the full functional dependence of the angular neutron flux:

$$\phi(\vec{r},\vec{v},t) = n(\vec{r},\vec{v},t)\,v(E)$$

But, the neutron velocity vector, $\vec{v}$, can be written in terms of a magnitude or speed, v, and a unit vector, $\hat{\Omega}$, that points in the direction of neutron travel, or $\vec{v} = v\hat{\Omega}$.

Also, since $E = \frac{1}{2} mv^2$ for neutrons, we can interchange the dependence on v or E as desired.

Thus, the angular flux is usually written as

$$\phi(\vec{r},E,\hat{\Omega},t) = n(\vec{r},E,\hat{\Omega},t)\,v(E)$$

angular neutron flux

And the scalar flux is simply the angular flux integrated over all angles, or

$$\phi(\vec{r},E,t) = \int_{\substack{\text{all}\\ \text{angles}}} \phi(\vec{r},E,\hat{\Omega},t)\,d\hat{\Omega}$$

scalar neutron flux

The Net Neutron Current

We will also need to discuss the flow of neutrons across a surface -- and the neutron current, $\vec{J}$, is used in these situations.

The angular neutron current is a vector quantity and it is defined in a similar fashion as the neutron flux, or

note the vector notation

$$\vec{J}(\vec{r},E,\hat{\Omega},t) = n(\vec{r},E,\hat{\Omega},t)\,\vec{v}(E)$$

angular neutron current

But, $\vec{v} = v\hat{\Omega}$ and ϕ = nv, so the angular neutron current can be written as

$$\vec{J}(\vec{r},E,\hat{\Omega},t) = \hat{\Omega}\phi(\vec{r},E,\hat{\Omega},t)$$

And the net neutron current is simply the angular current integrated over all angles, or

$$\vec{J}(\vec{r},E,t) = \int_{\substack{\text{all}\\ \text{angles}}} \phi(\vec{r},E,\hat{\Omega},t)\,\hat{\Omega}\,d\hat{\Omega} = \int_{\substack{\text{all}\\ \text{angles}}} \vec{J}(\vec{r},E,\hat{\Omega},t)\,d\hat{\Omega}$$

net neutron current

ϕ and $\vec{J}$ have the same units!!!

Multigroup Cross Sections & Flux

Now, in the context of reactor physics applications, we are interested in calculating average multigroup cross sections for use in solving the multigroup diffusion equation (to be derived shortly) for determining $\phi_g(\vec{r})$.

If we denote Σ_g as the average value of $\Sigma(E)$ over energy interval ΔE_g, then Σ_g is given by

$$\Sigma_g = \frac{\int_{\Delta E_g} \Sigma(E)\phi(E)dE}{\int_{\Delta E_g} \phi(E)dE} = \frac{1}{\phi_g}\int_{\Delta E_g} \Sigma(E)\phi(E)dE$$

average cross section in group g

where the group flux , ϕ_g, is given by

$$\phi_g = \int_{\Delta E_g} \phi(E)dE$$

scalar flux for group g

Putting It All Together

With an estimate of $\phi(E)$, we can compute Σ_g and, with knowledge of the multigroup neutron balance equation, we can solve for $\phi_g(\vec{r})$ which allows determination of the reaction rates, k_{eff}, etc.

Thus, in the first part of this course -- steady state reactor theory (removes the time dependence) -- we will focus on the following:

1. Description of the flux spectrum in thermal systems. (Lesson #1)
2. Development of the multigroup balance equation. (Lesson #2)
3. Solution of the diffusion equation for several typical subcritical and critical systems. (Lessons #3 - #8)
4. Relationship of simple analytical treatment addressed here to real-world computational reactor physics design and analysis of actual reactor systems. (Lesson #9)

So let's do it…

Lesson 0 Summary

In this Lesson we have briefly reviewed the following subjects:

The terms **multiplication factor** and **reactivity**.

The following **terms** and their **units**: **neutron density**, **neutron flux and current**, **microscopic and macroscopic cross sections**, **target atom densities**, and **neutron interaction rates**.

The **general energy dependence, $\sigma(E)$, of typical cross sections**.

How to write a **formal expression** for an **average multigroup cross section** and what is meant by the phrase "**the scalar flux for group g**."

How all the **above items fit together** within the study of **steady-state reactor theory**.