Xenon Poisoning in Thermal Reactors

Introduction

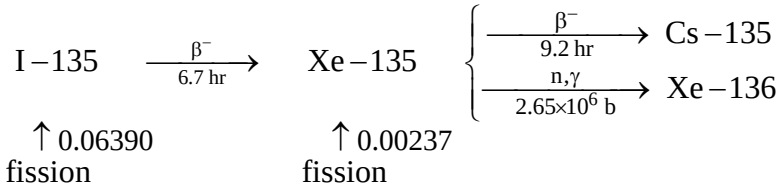
A large variety of fission products (FPs) can accumulate in a reactor from direct production via the fission reaction and from the subsequent transmutations that can occur due to radioactive decay and various neutron absorption reactions. These intermediate mass nuclides cause parasitic absorption of neutrons within the system, especially in thermal reactors, where their thermal absorption cross sections can be quite large. There are several hundred possible fission products, and they all contribute to some degree to the total negative reactivity associated with the FPs. However, there are a few isotopes that have especially large thermal absorption cross sections, and these warrant special attention. In particular, data for three of the more important FP isotopes for thermal systems are given below (data from Ref. 1):

Isotope	Thermal σ_a (barns)	Equilibrium U-235 Yield	Half-Life (hours)
Xe-135	2.65×10^6	0.06627	9.2
Sm-149	41000	0.01071	stable
Gd-157	240000	0.00007	stable

Clearly we see that Xe-135 has the largest 2200 m/s cross section and the largest net equilibrium yield for thermal fission in U-235. In addition, since Xe-135 decays with a half-life of about 9.2 hours, it also has some particularly interesting dynamics. Thus, the full characterization of xenon poisoning in a thermal reactor is quite important, and the goal of this lecture unit is to highlight this topic in some detail. Also, for consistency with many of the other Lecture Notes, we will focus most of our attention in the application section to the specific case of xenon poisoning in the UMass-Lowell research reactor (UMLRR).

Xenon Balance Equations

When writing the appropriate nuclide balance equations for any system, we need to be sure to include all the pertinent production and loss mechanisms. In general, production of an isotope can be from the decay of a parent, from transmutation of a parent in a neutron absorption reaction, or it can be formed directly in the fission process. For stationary fuel, the only loss mechanisms are neutron absorption and radioactive decay. For the present case, Xenon-135 is formed from the radioactive decay of I-135 and also directly from the fission process. Iodine-135 is also formed directly from fission and it has a relatively small absorption cross section. Thus, for the I-Xe chain, the important processes can be summarized as follows:



Based on this sketch, the basic isotope balance equations that define the I-Xe dynamics of interest in thermal systems can be written as (accumulation rate = production rate – loss rate):

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \qquad \text{(I-135 balance equation)} \qquad (1)$$

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \left(\lambda_X + \sigma_{aX}\phi\right) X \qquad \text{(Xe-135 balance equation)} \qquad (2)$$

where

$I(t)$ = average number of I-135 atoms/cm^3 versus time

$X(t)$ = average number of Xe-135 atoms/cm^3 versus time

γ_j = effective fission yield for isotope j (atoms/fission)

λ_j = decay constant for isotope j (decays per unit time where $\lambda = \ln 2 / T_{1/2}$)

σ_{aX} = average Xe-135 thermal absorption cross section (barns, where 1 b = 10^{-24} cm^2)

Σ_f = effective macroscopic fission cross section (1/cm) [see below for explanation]

ϕ = average thermal neutron flux in system (neutrons/cm^2-sec)

In these expressions, the thermal neutron flux is directly related to the operating power level. Thus, a change in power can be treated as a driving force that moves the I-135 and Xe-135 densities from their equilibrium or steady-state values to some new state.

There are several subtle approximations built into eqns. (1) and (2), mostly related to the formal treatment of the space and energy dependence of the cross sections and the neutron flux. Here we have assumed that an averaging process has been applied to the spatial variable, leading to the spatial independence associated with eqns. (1) and (2) and the interpretation of I(t) and X(t) as the average I-135 and Xe-135 concentrations, respectively, within the reactor core. Concerning the energy dependence, if we interpret the $\Sigma_f\phi$ and $\sigma_{aX}\phi$ terms as integrals over energy, then we can accurately account for any energy variation within the system. In practice, however, the Xe-135 cross section is extremely large at thermal energies relative to that at high energy. Thus, ignoring the fast component of the reaction rate is certainly justifiable. Also, since most of the fissions that occur in a thermal system occur at thermal energies, one often assumes that the $\Sigma_f\phi$ term includes only thermal energies. However, if one uses a 2-group approximation, this assumption is not really necessary since it is relatively easy to do the integral over energy, where

$$\Sigma_f \phi \Rightarrow \Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2 = \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi_2 \Rightarrow \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi \qquad (3)$$

Thus, with the notation from above, where ϕ implies the thermal flux, we will let Σ_f represent the effective fission cross section implied by the correspondence in eqn. (3). In this way, we will be able to use the same equations as given in Ref. 1, yet we will not simply ignore the fast fission contribution to the production of the I-135 and Xe-135 fission products. This approach, however, requires that an estimate of the fast to thermal flux ratio be available.

These assumptions, along with the fact that the I-135 absorption rate is small compared to its decay rate, lead to the simplified balance equations given above. These equations can be used to gain a lot of insight into the dynamics of the I-Xe chain and they can predict, with reasonable accuracy, the reactivity poisoning associated with the buildup and loss of Xe-135 within thermal systems.

Reactivity Effects

We can quantify the xenon poisoning effect by looking at the change in the multiplication factor, k, due to the presence of xenon. In particular, using the well known six-factor formula (see Ref. 2), $k = \eta f p \varepsilon P_F P_T$, the addition of a well-distributed neutron poison only affects the thermal utilization, f, to a significant degree. Thus, the change in reactivity can be approximated by

$$\rho = \frac{k - k_o}{k} \approx \frac{f - f_o}{f} \tag{4}$$

where k represents the multiplication factor with xenon present and k_o represents the un-poisoned state.

For a homogeneous 1-region core model, the two thermal utilization terms in eqn. (4) can be written as

$$f_o = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}} = \frac{\Sigma_{aF}}{\Sigma_a} \qquad \text{and} \qquad f = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aP}} = \frac{\Sigma_{aF}}{\Sigma_a + \Sigma_{aP}} \tag{5}$$

where Σ_a is the macroscopic thermal absorption cross section without poison, Σ_{aF} is the thermal absorption cross section associated with only the fuel, and Σ_{aP} is the poison cross section associated with the presence of Xe-135 in the system, etc.. Substituting eqn. (5) into (4) gives

$$\rho(t) = \frac{\Sigma_{aP}}{\Sigma_{aF} + \Sigma_{aM}} = \frac{\Sigma_{aP}(t)}{\Sigma_a} = \frac{X(t)\sigma_{aX}}{\Sigma_a} \tag{6}$$

Thus, we see that knowledge of the time-dependent Xe-135 concentration, X(t), will allow us to estimate the xenon reactivity effect (note that negative reactivity is implied here).

Before addressing the details of finding X(t) for various cases, let's first put eqn. (6) into a more convenient form for evaluation purposes. If we assume that the initial configuration, without xenon, is just critical, we have

$$k_o = \eta f_o p \varepsilon P_F P_T = \frac{\eta \Sigma_{aF} p \varepsilon P_F P_T}{\Sigma_a} = 1.0$$

and, since $\eta = \nu\Sigma_{f2}/\Sigma_{aF}$, solving this equation for Σ_a for the homogeneous critical system gives

$$\Sigma_a = \eta \Sigma_{aF} p \varepsilon P_F P_T = \nu \Sigma_{f2} p \varepsilon P_F P_T$$

where we have written the thermal fission cross section as Σ_{f2} to distinguish it from the effective fission cross section, Σ_f, that appears in many of the other equations. Finally, putting this expression into eqn. (6) gives

$$\rho(t) = \frac{\Sigma_{aP}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T} = \frac{X(t)\sigma_{aX}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T} \tag{7}$$

Note that, for a large system, the fast and thermal non-leakage probabilities, P_F and P_T, approach unity, and eqn. (7) reduces to the familiar relationship given in Lamarsh (Ref. 1). Equation (7) can be used to estimate the reactivity effect associated with Xe-135 in a thermal system.

Equilibrium Xenon

When the production and loss rates for a particular isotope become balanced (i.e. production rate = loss rate), the isotope concentration no longer changes with time, and it is said to be in equilibrium. This condition occurs in a real system when the reactor has been operating at constant power for a long period of time. To determine the reactivity effect for the equilibrium condition, we first set dI/dt and dX/dt to zero in eqns. (1) and (2), respectively. Doing this gives,

$$I_\infty = \frac{\gamma_I \Sigma_f \phi_\infty}{\lambda_I} \qquad \text{and} \qquad X_\infty = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty} \tag{8}$$

Working, in particular, with the expression for X_∞, we can write the macroscopic Xe-135 absorption cross section at equilibrium as

$$\Sigma_{a\infty} = X_\infty \sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty}\sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\phi_X + \phi_\infty} \tag{9}$$

where ϕ_∞ is the average thermal flux at equilibrium conditions and ϕ_X is given by

$$\phi_X = \lambda_X / \sigma_{aX} \tag{10}$$

Finally, putting eqn. (9) into eqn. (7) gives the reactivity effect of equilibrium xenon, or

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu p \varepsilon P_F P_T}\frac{\Sigma_f}{\Sigma_{f2}}\frac{\phi_\infty}{(\phi_X + \phi_\infty)} \tag{11}$$

Before continuing, it is interesting to find an approximate numerical value for ρ_∞. To do this, we need to make a bunch of assumptions. First assume a large U-235 fueled system ($P_F P_T \rightarrow 1.0$) with $p\varepsilon \approx 1.0$ and $\Sigma_f/\Sigma_{f2} \approx 1.0$. With these rough approximations, eqn. (11) becomes

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu}\frac{\phi_\infty}{(\phi_X + \phi_\infty)} \qquad \text{(assumes large system with } p\varepsilon \approx 1.0 \text{ and } \Sigma_f/\Sigma_{f2} \approx 1.0\text{)}$$

Now, the thermally averaged microscopic absorption cross section for Xe-135 at temperature T is given by (see Ref. 1)

$$\sigma_{aX}(T) = \frac{\sqrt{\pi}}{2} g_a(T)\sigma_{aX}(E_o)\left(\frac{T_o}{T}\right)^{1/2} \tag{12}$$

and, at $T = T_o = 293$ K (20 C), this can be evaluated to give

$$\sigma_{aX} = \frac{\sqrt{\pi}}{2}(1.158)\left(2.65\times10^6 \text{ b}\right)\left(10^{-24}\ \frac{\text{cm}^2}{\text{b}}\right) = 2.72\times10^{-18}\ \text{cm}^2$$

Also, the decay constant for Xe-135 is given by

$$\lambda_X = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{9.2\ \text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} = 2.09\times10^{-5}\ \text{s}^{-1}$$

Using these latter two values in eqn. (10) gives

$$\phi_X = \frac{2.09\times10^{-5}\ \text{s}^{-1}}{2.72\times10^{-18}\ \text{cm}^2} = 0.77\times10^{13}\ \text{cm}^{-2}\text{s}^{-1}.$$

where we note that this has the same units as neutron flux (as expected).

Now, we see that if the thermal flux at equilibrium is much greater than ϕ_X, then the ratio $\phi_\infty/(\phi_X + \phi_\infty)$ approaches unity. Thus, for the case of a large thermal flux, the equilibrium xenon reactivity becomes

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu}(1.0) = \frac{0.06627}{2.43} = 2.73\,\%$$

This represents an approximate value for the equilibrium xenon reactivity in thermal systems with a large thermal flux (where $\phi_\infty >> \phi_X$). This corresponds to about 4 dollars of negative reactivity -- which certainly represents an important consideration in the control and operation of thermal systems.

Other Cases

The above development for the equilibrium xenon reactivity worth is only applicable for operation at constant power (and flux) for a relatively long period of time (time enough for the production and loss rates to equilibrate). All the other cases of interest involve working with the full I-Xe dynamics equations. This means that, given some initial condition and the time-dependent thermal flux (or power) as input, we need to solve eqns. (1) and (2) simultaneously to give X(t), which can then be used with eqn. (7) to give $\rho(t)$ for the particular case of interest. For the general case, where a general $\phi(t)$ is used, the easiest way to solve these equations is via numerical integration with a standard ODE solver (such as Matlab's ***ode45*** routine, for example). However, for the case where $\phi(t) = \phi$ = constant, eqns. (1) and (2) represent a set of sequential, linear, constant coefficient ODEs, that can be solved analytically with relative ease (see any good introductory differential equations text). The result for the constant thermal flux case, after suitable manipulation to get it into desired form, is given by the following equations:

$$I(t) = I_o e^{-\lambda_I t} + \frac{\gamma_I \Sigma_f \phi}{\lambda_I}\left(1 - e^{-\lambda_I t}\right) \tag{13}$$

$$X(t) = X_0 e^{-wt} + \frac{(\gamma_I + \gamma_X)\Sigma_f \phi}{w}\left(1 - e^{-wt}\right) + \frac{\gamma_I \Sigma_f \phi - I_0 \lambda_I}{w - \lambda_I}\left(e^{-wt} - e^{-\lambda_I t}\right) \tag{14}$$

where $w = \sigma_{aX}\phi + \lambda_X$ and I_o and X_o represent the initial I-135 and Xe-135 concentrations.

It is important to emphasis that eqns. (13) and (14) are only valid over a time period of constant flux (or power) given by the value of thermal flux, ϕ. However, even with this restriction, these equations can be very useful, since for most situations of interest, the thermal flux (or power) can be approximated as a piecewise constant function. In particular, there are three situations that make interesting case studies:

I. Instantaneous startup to some constant power and flux level, with $I_o = X_o = 0$.

II. Instantaneous shutdown after a long period of full power operation, with $I_o = I_\infty$ and $X_o = X_\infty$ (using the full power thermal flux to determine the equilibrium concentrations).

III. Instantaneous change to some new constant power and flux after previous operation at a different constant power level. Here the initial concentrations for each new time interval are obtained from the conditions at the end of the prior time interval.

Careful study of these three cases can give significant insight into the general behavior of the I-Xe chain, as well as a good understanding of how xenon buildup and decay can affect the reactivity in thermal systems.

Application to the UMLRR

As an illustration of how xenon dynamics affects a real reactor, we now apply the above theoretical development to the 1 MWth UMass-Lowell research reactor (UMLRR). We first use information from some of the previous Lecture Notes to obtain data specific to the UMLRR, and then briefly describe a Matlab program, **umlrr_xenon.m**, that implements the theory from above. Finally, using the **umlrr_xenon** program, we simulate and describe a series of cases that should help the reader gain a good understanding of this subject.

UMLRR Data

The Xe-135 reactivity effect given by eqn. (7) requires information about the particular system of interest. Here we have used the six-factor formula to estimate the reactivity effect because it is relatively easy to approximate the terms within this expression for the multiplication factor, k, using appropriate 2-group cross sections and some geometry and material information for the system under study. In particular, for the UMLRR, we have already computed the parameters needed in eqn. (7) as part of previous work using 2-group data generated in the summer of 2010, and they are summarized as follows:

$\nu = 2.43$	$p = 0.879$	$\varepsilon = 1.067$
$P_F = 0.665$	$P_T = 0.969$	
$\Sigma_{f1} = 1.21\text{x}10^{-3}$ cm^{-1}	$\Sigma_{f2} = 5.04\text{x}10^{-2}$ cm^{-1}	$\phi_1/\phi_2 = 2.75$

These parameters were generated as part of previous HW assignments given on this material.

Also, for the I-Xe dynamics equations, the decay parameters and the Xe-135 absorption cross section have already been given in these notes. Thus, the only parameter left to be specified is the average thermal flux. However, as noted previously, flux and power are related, and we have chosen to use the reactor power as the independent driving force for the I-Xe simulations. A general formula for power in terms of the neutron flux is

$$P = \iint \kappa\Sigma_f(\vec{r}, E)\phi(\vec{r}, E)d\vec{r}dE$$

where κ is an energy per fission conversion factor. With appropriate spatial averaging and a two energy group approximation, this becomes

$$P = \kappa\left(\Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2\right)V_{core} = \kappa\Sigma_f\phi V_{core} \qquad (15)$$

where Σ_f is the effective fission cross section [see eqn. (3)], ϕ is the average thermal flux, and V_{core} is the core volume. Finally, solving eqn. (15) for ϕ gives

$$\phi = \frac{P}{\kappa \Sigma_f V_{core}} = \frac{rp \times P_{full}}{\kappa \Sigma_f V_{core}} \tag{16}$$

where P_{full} = 1 MW = 10^6 W and the relative power, rp, is now the driving force for simulating the I-Xe dynamics within the UMLRR.

For evaluating ϕ for the subsequent simulations, we used a 20 element core where the fuel assembly cross section is 7.7724 cm x 7.7724 cm and the active fuel height is 59.69 cm. These values give a core volume of 7.212×10^4 cm^3.

The umlrr_xenon Program

With all the required data now available, we are ready to actually evaluate the I-Xe dynamics and the Xe-135 reactivity worth in the UMass-Lowell research reactor (UMLRR) for a variety of situations. In particular, capability for simulating the three case studies listed previously was implemented several years ago into a relatively simple Matlab code called **umlrr_xenon.m**. A complete program listing for **umlrr_xenon** is given in Table 1. The code construction is fairly typical of most of the other Matlab programs written as part of the current series of Lecture Notes. In particular, after a brief set of comments to define the program's purpose, we first define all the data needed in the simulations, and then repeat, for the three specific cases described above, the computation and presentation of the I-135 and Xe-135 concentrations and the Xe-135 reactivity versus time.

The actual evaluation of eqn. (7) and eqns. (14)-(16) is performed in the **xenon_eqns.m** function file. As seen in the listing in Table 2, this file simply evaluates the equations developed above for a known constant relative power during time interval dt, with known conditions, I_o and X_o, for the initial I-135 and Xe-135 concentrations. Upon return, the function file passes back a vector of 100 discrete time points within the interval, along with the time-dependent concentrations and reactivity worth, I(t), X(t), and ρ(t), at each discrete point. These data are simply plotted within the main program to help visualize the results of the various simulations.

The first two cases (the buildup of Xe-135 at full power with zero initial conditions, and the buildup and decay of Xe-135 after shutdown from equilibrium) are rather straightforward, with all the values preset within the **umlrr_xenon** code. The third case, however, asks the user to define the piecewise constant relative power and the length of the time interval, dt, within a looping structure, so that a wide variety of simulation cases can be performed. The calculational loop continues until a negative value for dt is encountered and, upon exit from the while loop, summary plots containing I(t), X(t), and ρ(t), and rp(t) over the full range are presented.

Overall, the codes listed in Tables 1 and 2 are easy to use and understand, and they implement the theory exactly as developed in this set of Lecture Notes for the specific case of the UMLRR. Use of this code, especially with the variety of situations allowed with the Case 3 option, should allow a good general understanding of the effects of xenon poisoning in thermal systems.

Table 1 Listing of the umlrr_xenon.m Matlab program.

```
%
%   UMLRR_XENON.M    Simulate Xenon Buildup and Decay and
%                 the Reactivity Effect of Xe-135 in the UMLRR
%
%   This file simulates the I-Xe dynamics equations for several different scenarios.
%   It uses data specific to the UMLRR, so the simulations here are somewhat specific
%   to this system (but the code can be easily modified for use with other systems).
%   The results of the time-dependent simulations, however, are quite generic and
%   careful study should give a good understanding of the dynamics and reactivity
%   effects associated with xenon buildup and decay in any thermal system.
%
%   The analytical solution for the case of piecewise constant thermal flux (or
%   power) is used to estimate the I-135 and Xe-135 concentrations versus time for
%   different cases.  In addition, a simple equation for the Xe-135 reactivity
%   effect is used to estimate rho(t) expected for different situations.
%
%   The theory and specific equations implemented here are documented in the Lecture
%   Notes: Xenon Poisoning in Thermal Reactors.  The specific data for the UMLRR
%   come from a separate analysis that calculates the separate factors within the
%   6-factor formula.
%
%   Note: 2-group data for the conversion of the UMLRR from HEU to LEU fuel was
%   generated in 1999 using SCALE 4.2 and the VITAMIN-B6 199/42 group library.  In
%   the summer of 2010, a new set of 2-group data was generated using SCALE 6.0
%   with the v7n238 library distributed with SCALE.  The results from these two data
%   sets are somewhat different (there apparently was a problem with the SCALE 4.2
%   BONAMI module). We have officially moved to using the new data set in all our
%   computations for the UMLRR, but just for comparison purposes, I have decided to
%   maintain both sets of data in this simulation code.  Thus, the user can select
%   which data to use -- it turns out that there is not a significant difference
%   here (2010 data gives a slightly increased equilibrium worth)...
%
%   This main program calls the xenon_eqns.m function file to actually evaluate the
%   desired equations for a variety of situations.
%
%   File prepared by J. R. White, UMass-Lowell (April 11, 2015)
%


  clear all, close all, nfig = 0;
  global dci dcx yi yx sigax Pfull kappa Vcore denom sigfeff sigf2
%
%   define I-Xe decay parameters (from Lamarsh)
  Thi = 6.7;                % half life for I-135 (hr)
  dci = log(2)/Thi/3600;    % decay constant for I-135 (1/sec)
  Thx = 9.2;                % half life for Xe-135 (hr)
  dcx = log(2)/Thx/3600;    % decay constant for Xe-135 (1/sec)
  yi = 0.0639;              % yield for U235 for I-135 (atoms/fission)
  yx = 0.00237;             % yield for U235 for Xe-135 (atoms/fission)
%
%   set xenon micro xsec (from Lamarsh)
  T = 293;  To = 293;       % absolute temperatures (K)
  ga = 1.158;               % non 1/v factor for Xe-135  (at T = 20 C = 293 K))
  sigaxo = 2.65e+6;         % 2200 m/s abs xsec for Xe-135  (barns)
  cm2pb = 1e-24;            % conversion factor (cm^2/barn)
  sigax = (sqrt(pi)/2)*ga*sigaxo*cm2pb*sqrt(To/T);   % thermal ave abs xs (cm^2)
%
%   data for UMLRR simulations (data from both 1999 and 2010 are available)
  Pfull = 1e+6;                                % UMLRR full power level (W)
  kappa = 200*1.602e-13;                       % energy per fission (J = W-s)
  assydim = 7.7724;                            % assy square dimension (cm)
  fuelht = 59.69;                              % fuel heigth (cm)
  Vcore = 4*5*assydim^2*fuelht;                % core vol for 4x5 array (cm^3)
  idata = menu('Select 2-g data set for the simulations:', ...
               ' Data generated in 1999 using SCALE 4.2  ', ...
               ' Data generated in 2010 using SCALE 6.0  ');
  if idata == 1     % ***  1999 data  ***
    nu = 2.434; p = 0.9272; epp = 1.0541;      % nu & factors of 4-factor formula
    PF = 0.6737; PT = 0.9690;                  % non-leakage probabilities
    sigf1 = 1.017e-3; sigf2 = 5.046e-2;        % macro fission xsecs (1/cm)
```

```
    ratio = 2.6368;                          % fast to thermal flux ratio
  else              % ***  2010 data  ***
    nu = 2.434; p = 0.8794; epp = 1.0674;    % nu & factors of 4-factor formula
    PF = 0.6652; PT = 0.9691;                % non-leakage probabilities
    sigf1 = 1.214e-3; sigf2 = 5.036e-2;      % macro fission xsecs (1/cm)
    ratio = 2.7528;                          % fast to thermal flux ratio
  end
  denom = nu*p*epp*PF*PT;                    % denominator of reactivity eqn.
  sigfeff = sigf1*ratio+sigf2;               % effective fiss xs (1/cm)
  flxfull = Pfull/(kappa*sigfeff*Vcore);     % 1 MW ave thm flux (neuts/cm^2-s)
%
%  Case 1:  Buildup of Xenon in fresh core (zero initial conditions)
%
  dt = 80*3600;  rp = 1.0;
  Io = 0;   Xo = 0;
  [t1,I1,X1,rho1] = xenon_eqns(rp,dt,Io,Xo);
%
  nfig   = nfig+1;   figure(nfig);
  subplot(2,1,1),plot(t1/3600,I1,'r-',t1/3600,X1,'b--','LineWidth',2),grid
  title('UMLRR\_Xenon:  I-Xe Dynamics (1 MW startup from zero ICs)')
  ylabel('Concentration (atom/cm^3)')
  legend('I-135','Xe-135')
  subplot(2,1,2),plot(t1/3600,rho1*100,'g-','LineWidth',2),grid
  xlabel('Time (hr)'),ylabel('Reactivity (%\Deltak/k)')
%
%  Case 2:  Buildup/Decay of Xenon after shutdown (equilibrium initial conditions)
%
  dt = 80*3600;  rp = 0.0;
  Io = yi*sigfeff*flxfull/dci;
  Xo = (yi+yx)*sigfeff*flxfull/(dcx + sigax*flxfull);
  [t2,I2,X2,rho2] = xenon_eqns(rp,dt,Io,Xo);
%
  nfig   = nfig+1;   figure(nfig);
  subplot(2,1,1),plot(t2/3600,I2,'r-',t1/3600,X2,'b--','LineWidth',2),grid
  title('UMLRR\_Xenon:  I-Xe Dynamics (Shutdown from equil at 1 MW)')
  ylabel('Concentration (atom/cm^3)')
  legend('I-135','Xe-135')
  subplot(2,1,2),plot(t2/3600,rho2*100,'g-','LineWidth',2),grid
  xlabel('Time (hr)'),ylabel('Reactivity (%\Deltak/k)')
%
%  Case 3:  Xenon dynamics for several piecewise constant power changes
%
  opt = menu('Select Initial Conditions for First Time Interval', ...
             'Zero initial conditions', ...
             'Equilibrium conditions from full power');
  if opt == 1, Io = 0;  Xo = 0;  end
  if opt == 2,
    Io = yi*sigfeff*flxfull/dci;
    Xo = (yi+yx)*sigfeff*flxfull/(dcx + sigax*flxfull);
  end
  dt = input('Input duration (hours) for first time interval: ');
%
  n = 1;
  while dt > 0
    dt = dt*3600;
    rp = input('Input relative power (fraction of 1 MW) for interval: ');
    [t,I,X,rho] = xenon_eqns(rp,dt,Io,Xo);
    if n == 2;
      t = t + t3(end);
      t3 = [t3 t];  I3 = [I3 I]; X3 = [X3 X];  rho3 = [rho3 rho];
      rp3 = [rp3 rp*ones(size(t))];
      Io = I(end);  Xo = X(end);
    end
    if n == 1;
      t3 = t;  I3 = I; X3 = X;  rho3 = rho;
      rp3 = rp*ones(size(t));  n = 2;
      Io = I(end);  Xo = X(end);
    end
    dt = input('Input duration (hours) for next interval (< 0 to quit): ');
  end
%
```

```
      nfig   = nfig+1;   figure(nfig);
      subplot(2,1,1),plot(t3/3600,I3,'r-',t3/3600,X3,'b--','LineWidth',2),grid
      title('UMLRR\_Xenon:  I-Xe Concentrations for Variable Power Profile')
      ylabel('Concentration (atom/cm^3)')
      legend('I-135','Xe-135')
      subplot(2,1,2),plot(t3/3600,rp3,'m-','LineWidth',2),grid
      xlabel('Time (hr)'),ylabel('Relative Power')
%
      nfig   = nfig+1;   figure(nfig);
      subplot(2,1,1),plot(t3/3600,rho3*100,'g-','LineWidth',2),grid
      title('UMLRR\_Xenon:  Xe-135 Reactivity for Variable Power Profile')
      ylabel('Reactivity (%\Deltak/k)')
      subplot(2,1,2),plot(t3/3600,rp3,'m-','LineWidth',2),grid
      xlabel('Time (hr)'),ylabel('Relative Power')
%
%   end of program
```

Table 2 Listing of the xenon_eqns.m function file.

```
%
%   XENON_EQNS.M    Evaluate the I-Xe equations and Xe Reactivity Equation
%                     (for an interval of constant power or flux)
%
%   The theory and specific equations implemented here are documented in the Lecture
%   Notes: Xenon Poisoning in Thermal Reactors.  Most of the data needed within the
%   equations are passed in via the global command.  This file is called from the main
%   program umlrr_xenon.m, which does a series of xenon dynamics simulations for the
%   UMass-Lowell research reactor (UMLRR).
%
%   Inputs:
%     rp = relative power (fraction of full power)
%     dt = time duration (seconds) for current time interval
%     Io = initial I-135 concentration for interval (atoms/cm^3)
%     Xo = initial Xe-135 concentration for interval (atoms/cm^3)
%
%   Outputs:
%     t = time vector spanning 0 to dt seconds
%     I = I-135 concentration vs time for interval (atoms/cm^3)
%     X = Xe-135 concentration vs time for interval (atoms/cm^3)
%     rho = Xe-135 reactivity vs time for interval (Dk/k)
%
%   File prepared by J. R. White, UMass-Lowell (April 11, 2015)
%

      function [t,I,X,rho] = xenon_eqns(rp,dt,Io,Xo)
      global dci dcx yi yx sigax Pfull kappa Vcore denom sigfeff sigf2
      t = linspace(0,dt,100);
      P = rp*Pfull;   flx = P/(kappa*sigfeff*Vcore);
      w = sigax*flx + dcx;   II = yi*sigfeff*flx/dci;
      XX1 = (yi+yx)*sigfeff*flx/w;  XX2 = (yi*sigfeff*flx - Io*dci)/(w - dci);
      I = Io*exp(-dci*t) + II*(1 - exp(-dci*t));
      X = Xo*exp(-w*t) + XX1*(1 - exp(-w*t)) + XX2*(exp(-w*t) - exp(-dci*t));
      rho = X*sigax/sigf2/denom;

%
%   end of function
```

Some Simulation Results

Some actual results from **umlrr_xenon** are summarized in this subsection. In particular, the Case 1 and Case 2 simulation results are displayed in Figs. 1 and 2, respectively. Recall that Case 1 involves the buildup of xenon in a fresh core when operating at 1 MWth, and that Case 2 models the I-Xe dynamics upon shutdown of the UMLRR after extended (equilibrium) operation at 1 MWth. For the first case, Fig. 1 shows the buildup of I-135 and Xe-135, and the associated

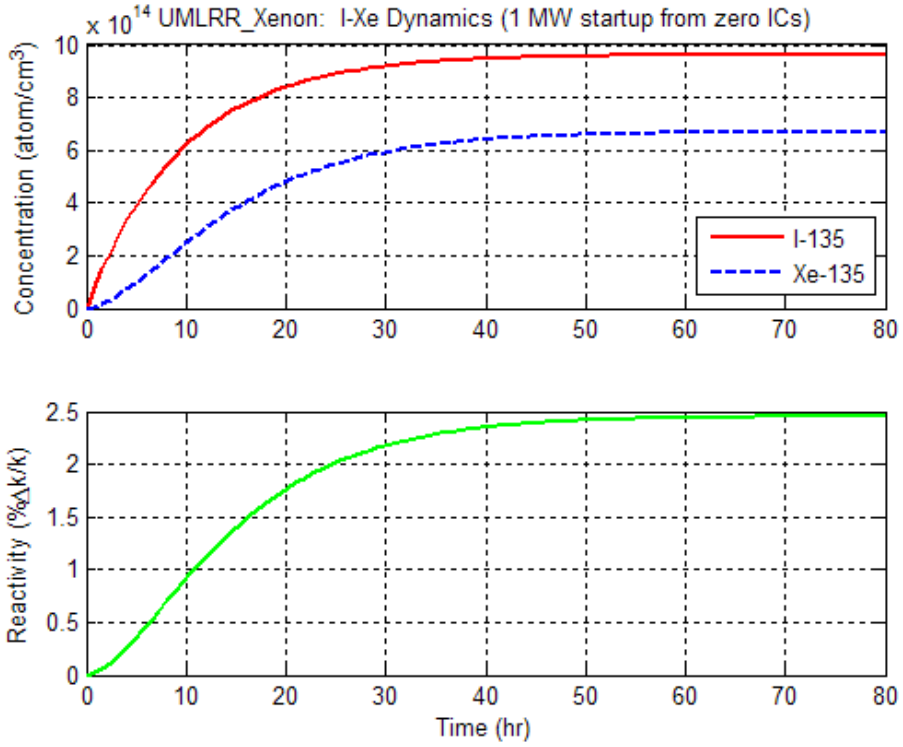


Fig. 1 Simulation results for startup case with zero initial conditions (Case 1).

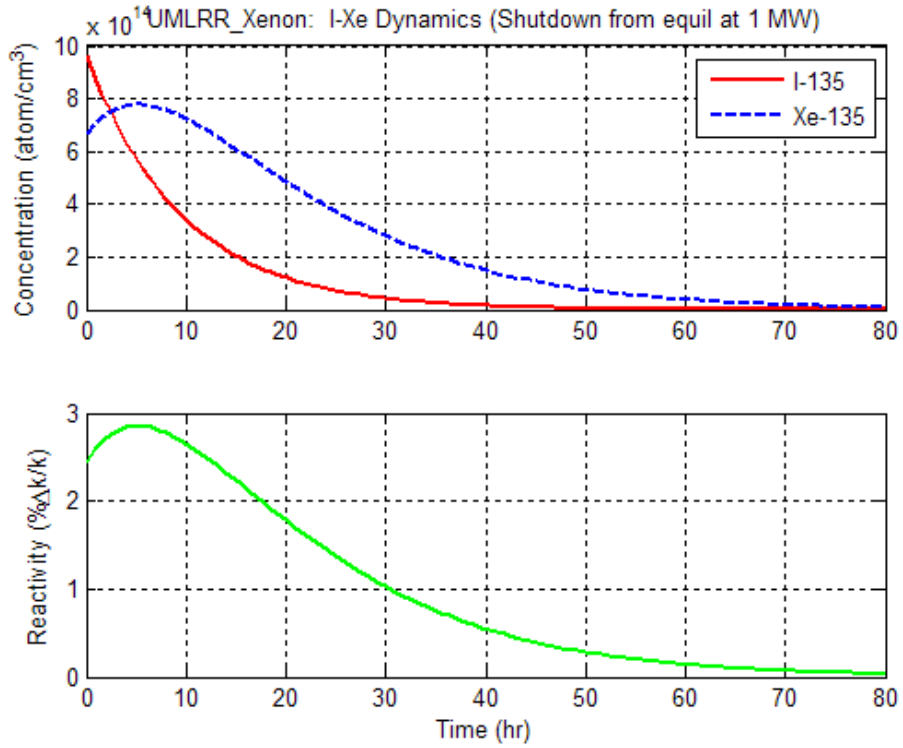


Fig. 2 Simulation results for case of shutdown from equilibrium conditions (Case 2).

Xe-135 reactivity worth, starting at zero and leveling off at their equilibrium values at about 50-60 hours after startup. Even though the concentrations are quite small (10^{14} atoms/cm^3 is very small compared to typical material densities on the order of 10^{22} atoms/cm^3), the reactivity effect due to Xe-135 is quite large -- because of its very large microscopic absorption cross section. As apparent, for the LEU-fueled UMLRR, the equilibrium worth approaches nearly 2.5 %Δ k/k and, with β_{eff} being about 0.0078, this corresponds to more than 3 dollars of negative reactivity.

For the Case 2 simulation, Fig. 2 shows that, as soon as the power level goes to zero, the I-135 concentration starts its exponential decay towards zero. With a 6.7 hours half-life, we expect the I-135 level to be near zero in about 33-34 hours (5 half-lives), and this is exactly what is observed in Fig. 2.

However, the Xe-135 dynamics are quite different. In this case, once the power goes to zero, there is no longer any Xe-135 loss due to absorption. It does decay with a 9.2 hr half-life, so we also expect the Xe-135 to eventually decay to zero. However, as the I-135 decays, it produces Xe-135 and, for the first 5-6 hours after shutdown, this production rate is greater than the Xe-135 decay rate, so the Xe-135 concentration actually increases. Eventually, however, as the I-135 concentration continues to decrease, the production rate of Xe-135 also decreases, and the loss term due to X-135 decay wins out, giving the expected pure exponential decay after about 30 hours from shutdown. Eventually, decay to near zero takes about 70-75 hours (about 3 days).

Note that both Figs. 1 and 2 show the typical behavior that is expected for any thermal system. The equilibrium levels and the magnitude and timing of the peaks after shutdown vary, of course, with the specific system and the actual thermal flux level, but the basic trends as observed here for the UMLRR would be similar for any thermal system.

The Case 1 and Case 2 studies are very important in that they give a lot of basic insight into the physics of the I-Xe dynamics equations, and they also quantify the maximum Xe worth that can be expected in the UMLRR with full-power equilibrium on-off conditions. However, the UMLRR is used as an on-demand training and research facility, and it rarely runs at full rated power for extended periods of time. Instead, the reactor is typically available on an 8 am to 5 pm schedule, five days a week. Thus, as a more reasonable upper limit on the xenon reactivity effect in the UMLRR, we have used the Case 3 simulation capability within the **umlrr_xenon** code to simulate a week-long sequence -- 8 hours at 1 MWth and 16 hours off per day for 5 days, with an additional 48 hour shutdown period for the weekend. Thus, the Case 3 simulation goes from a fresh core with no xenon at 9 am Monday morning to the following Monday at 9 am.

The results of this particular week-long simulation are summarized in Figs. 3 and 4. Figure 3 highlights the time-dependent I-135 and Xe-135 concentrations and Fig. 4 shows the xenon reactivity worth, $\rho(t)$, for a week. Both figures, however, also show the piecewise constant relative power, rp(t) over the week-long simulation. This information was included on each figure so that one can easily correlate the changes observed in the concentration and reactivity profiles with the discrete power on-off sequencing.

Careful study of the resultant profiles in Figs. 3 and 4 is very enlightening. For example, Fig. 3 shows the straightforward buildup and decay of the I-135 concentration during each on-off period, respectively. However, as we saw previously, the Xe-135 concentration continues to build up for several hours after shutdown. Also, at each new startup, the Xe-135 loss rate is increased as the Xe-135 undergoes neutron absorption as well as radioactive decay. This increased loss rate, however, tends to be offset by the production of Xe-135 from the growing

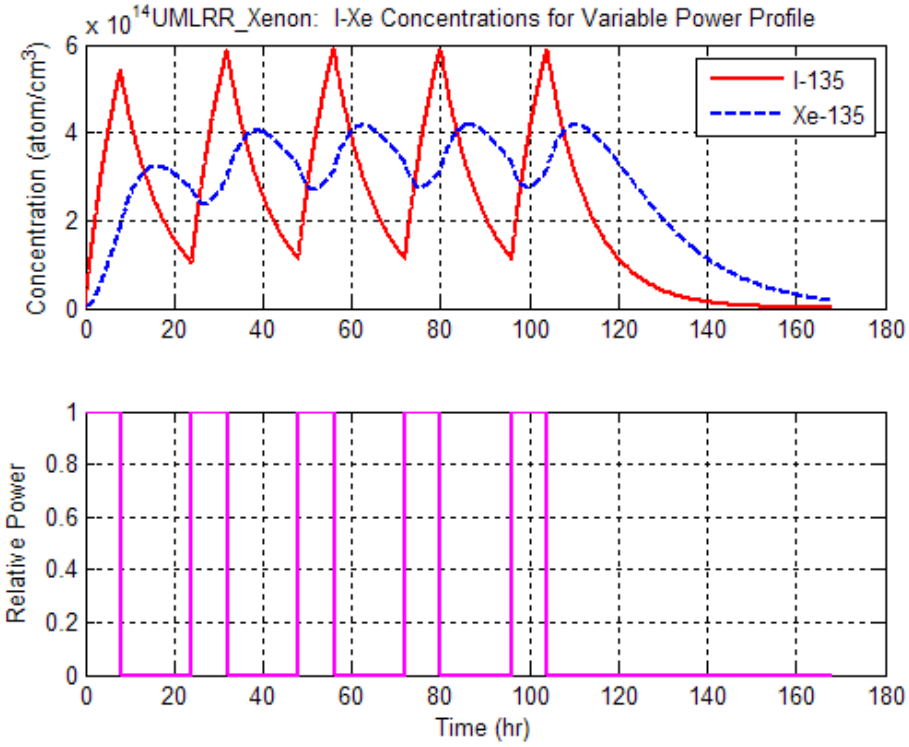


Fig. 3 Concentration results for week-long power on/off sequence (Case 3).

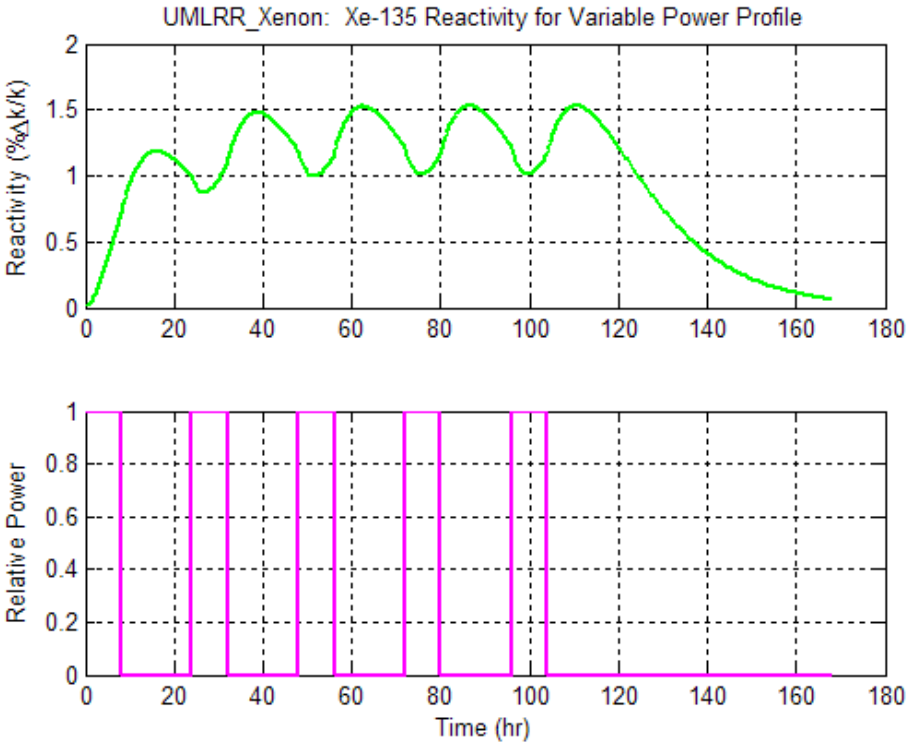


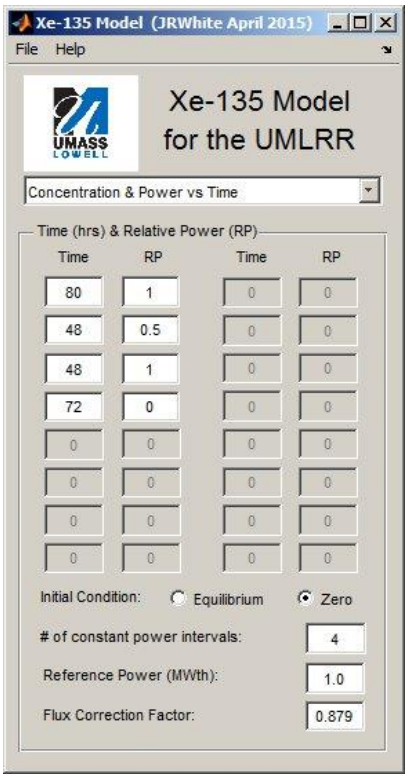
Fig. 4 Reactivity results for week-long power on/off sequence (Case 3).

I-135 concentration. After a few hours, we see that the Xe-135 profile reaches a local minimum, and actually turns around to show a positive growth rate during the latter portion of the power-on cycle. This oscillatory behavior is observed for five cycles to match the Monday through Friday power on-off cycle, and then we see the final shutdown-like behavior during the weekend period.

As expected, the Xe-135 reactivity profile in Fig. 4 simply follows the concentration profile, with the estimated peak reactivity worth just over 1.5 %Δk/k. This value is only about 60 % of the equilibrium value for 1 MWth operation for an extended period. Thus, the capability within the **umlrr_xenon** program should prove useful for predicting the xenon poisoning effect under actual operational conditions in the UMLRR.

The xenon_gui code

More recently, a graphical user interface (GUI) has been added to the **umlrr_xenon.m** code to make the program even easier to use. A snapshot of this new GUI -- called **xenon_gui** -- is displayed to the right. As apparent, the user can specify up to 16 discrete intervals for constant power operation, indicate the choice of initial conditions, set the core reference power level, and identify a flux correction factor to account for various assumptions made in the simple 6-factor formula used here. Note that the default correction factor of 0.879 is an update to the original value of 0.934 that was determined by an actual experiment within the UMLRR (see Ref. 3). The new flux multiplier was needed because a new set of 2-group cross sections were generated in 2010, and a new analysis of the original experimental data suggested that a somewhat different value was needed to better match the measured results.



In addition, the user can select the type of plot to display -- either the I-135 and Xe-135 concentration or the xenon reactivity profiles in the upper subplot, along with the power profile in the lower subplot region. These plots of the key simulation results, as shown in Fig. 5, are very similar in nature to those given in Figs. 1-4. Overall, the new **xenon_gui** code simply adds a little extra flexibility and ease-of-use to the prior code to help the student really understand the basics of typical I-Xe dynamics…

Summary/Conclusions

This unit of Lecture Notes should give the reader a good understanding of the dynamics and inherent negative reactivity effect that Xe-135 has on any thermal reactor. The specific application to the UMass-Lowell research reactor (UMLRR) adds some specificity to the subject and it allows one to get a handle on the magnitude of the reactivity change that can be expected under various scenarios. The **umlrr_xenon** code and the newer GUI version of the program, **xenon_gui**, were written to simulate this xenon effect and to be utilized to study additional cases beyond those given here, and the codes also can be modified quite easily, as needed, to simulate other systems of interest. Overall, the reader should be leaving this unit with a better understanding of the general effects of xenon poisoning in thermal systems and, in addition, have

access to a simple tool for further study of the UMLRR as well as other systems. This increased understanding of the topic was the primary purpose of this unit on **Xenon Poisoning in Thermal Reactors** -- hopefully we were somewhat successful in achieving that goal…

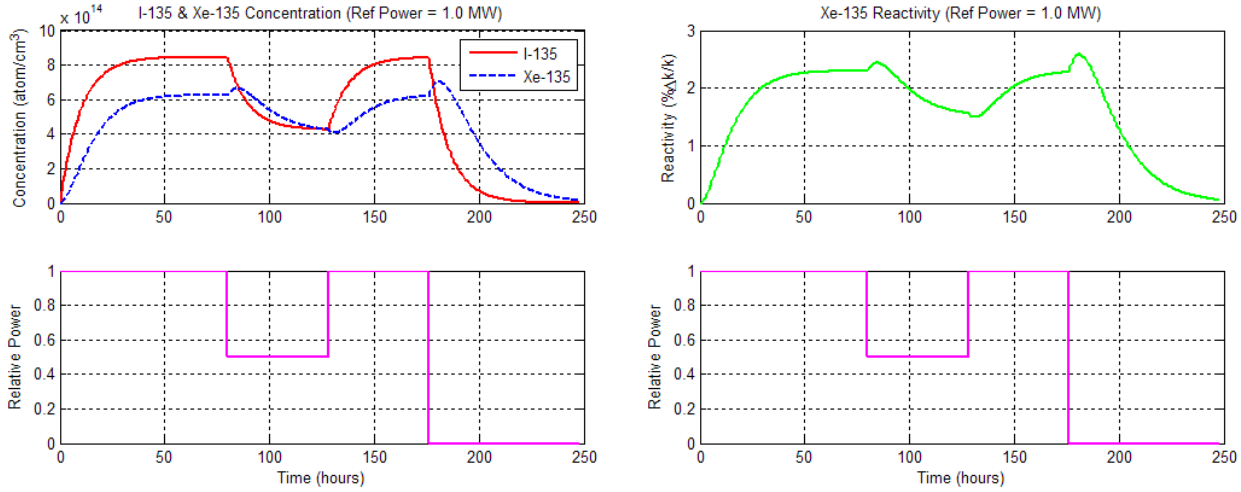


Fig. 5 Typical simulation results from the xenon_gui code.

References

1. J. R. Lamarsh and A. J. Baratta, ***Introduction to Nuclear Engineering***, 3rd Edition, Prentice Hall (2001).
2. J. R. White, "Two-Group Diffusion Theory for Critical Systems," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.
3. J. R. White, "Post-Experimental Analysis of the Xenon Reactivity Experiment Performed on Nov. 23, 2004," part of a series of Demos & Expts. available at www.nuclear101.com.