

Temperature Related Reactivity Coefficients and Feedback Effects

In our previous study of reactor kinetics (see Ref. 1), we gave a brief comparison of the transient behavior of a reactor due to a step change in reactivity for cases with and without negative feedback. In particular, a simulation of a positive reactivity insertion with and without inherent feedbacks is reproduced below in Fig. 1, and clearly the behavior for the two cases is totally different -- with the no feedback case leading to a unbounded (and unsafe) power transient, and the case with inherent feedbacks leveling off at a new steady state power level as the negative feedback compensates for the initial positive reactivity insertion. Clearly, the case with negative feedback is the only situation that can be tolerated in a real system, and being able to quantify these inherent feedbacks is of primary importance in both the design phase and as part of the startup tests performed in real systems. Both fission product poisoning (primarily due to Xe-135) and temperature effects are important here and both these mechanisms need to be studied in more detail. This set of Lecture Notes, in particular, focuses on several aspects of inherent temperature feedbacks, and a companion document deals with the xenon reactivity component (see Ref. 2). Both these subjects are also treated in some detail in a number of reactor theory texts (see Ref. 3, for example).

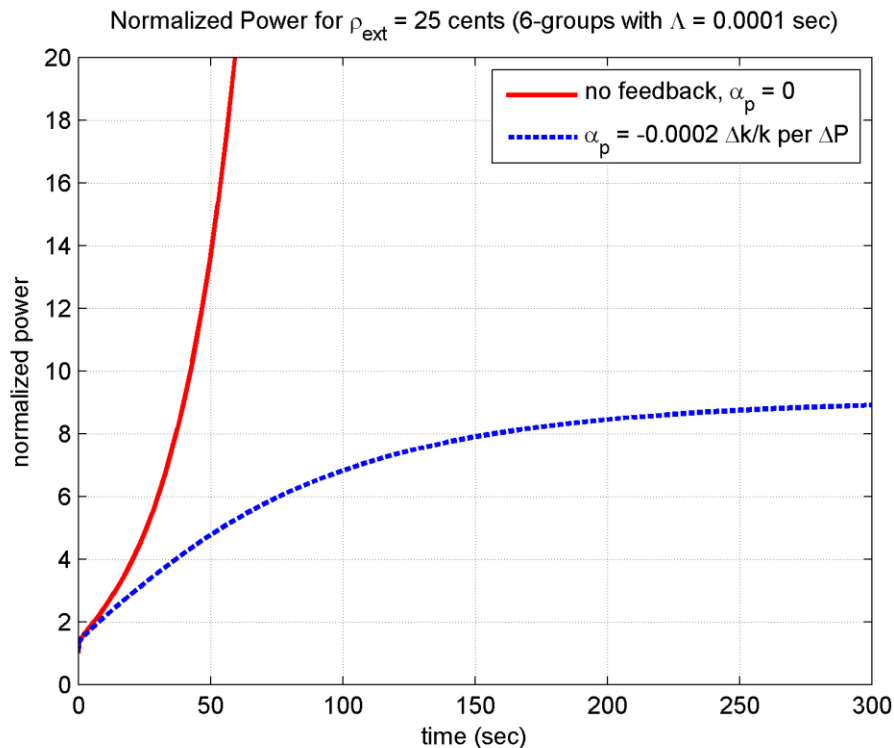


Fig. 1 $P(t)/P_0$ for $\rho_{\text{ext}} = +25$ cents with and without inherent negative feedbacks.

In the simulation comparisons given in Fig. 1, a generic power feedback coefficient, $\alpha_P = \partial\rho/\partial P$, was applied to treat, in a collective fashion, a number of feedback effects, such as to account for the reactivity effect due to changes in the fuel temperature, moderator temperature, and/or

coolant voiding (no xenon reactivity is present in this simple model). In practice, however, the individual temperature coefficients for each separate effect are needed since the time constant associated with each reactivity feedback can be significantly different -- which can indeed be important when considering certain transient situations. For example, in a power excursion, the fuel temperature is the first to respond to an increased fission power, then the coolant temperature, and finally the temperature of the structural components, and the time delay associated with the various heat transport mechanisms can be important when addressing the time dependence of the reactivity feedback within a formal solution of the point kinetics formulation.

Each reactivity coefficient is defined in a similar fashion. For a temperature effect, for example, we simply write the temperature coefficient of reactivity as

$$\alpha_T = \frac{\partial \rho}{\partial T} \quad (1)$$

where the temperature might be associated with the fuel, coolant, or structural materials. Since $\rho = (k - 1)/k$, this can be written as

$$\alpha_T = \frac{\partial \rho}{\partial T} = \frac{\partial}{\partial T} \left(1 - \frac{1}{k} \right) = \frac{1}{k^2} \frac{\partial k}{\partial T} \approx \frac{1}{k} \frac{\partial k}{\partial T} \quad (2)$$

where the last approximation ($k^2 \approx k$) is valid for a near-critical or critical reference state (i.e. $k_{\text{ref}} \approx 1.000$). And, from the basic definition of a reactivity coefficient, α_T , we see that

$$\rho_f(t) = \Delta \rho(t) \approx \frac{\partial \rho}{\partial T} \Delta T(t) = \alpha_T \{T(t) - T_{\text{ref}}\} \quad (3)$$

Thus, once the reactivity coefficients are known, they can be used to approximate the inherent feedback reactivity within the system [as implied by eqn. (3)].

In practice, the temperature coefficients (with units of $\Delta k/k$ per unit temperature) are not really very easy to quantify. Often these are computed using sophisticated computer codes that attempt to model the reactor configuration in as much detail as possible. Usually two discrete temperatures are chosen, and the representative cross sections and atom densities are determined for each temperature. The neutron balance equation is then solved using these data sets to obtain two values of k_{eff} . For example, given the T-k combinations,

$$T_1 = \text{reference temperature} \quad \rightarrow \quad k_1 = \text{reference } k_{\text{eff}}$$

$$T_2 = \text{perturbed temperature} \quad \rightarrow \quad k_2 = \text{perturbed } k_{\text{eff}}$$

the average temperature coefficient over the given temperature range is

$$\bar{\alpha}_T = \frac{\int_{T_1}^{T_2} \alpha_T(T) dT}{\int_{T_1}^{T_2} dT} = \frac{\int_{T_1}^{T_2} \frac{1}{k} \frac{\partial k}{\partial T} dT}{T_2 - T_1} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \frac{dk}{k} = \frac{\ln(k_2/k_1)}{T_2 - T_1} \quad (4)$$

As implied here, the temperature coefficient is a function of temperature, so eqn. (4) may be evaluated over several T-k pairs to develop the rough behavior of $\bar{\alpha}_T$ vs. temperature.

Alternatively, one can plot $\Delta k/k$ vs. T for a set of discrete T - k pairs and, via a curve fit or a finite difference approximation, form $\bar{\alpha}_T$ vs. T from the basic definition given in eqn. (2).

Following the above computational outline, an attempt has been made to compute accurate reactivity coefficients for the UMass-Lowell research reactor (UMLRR). During the early UMLRR HEU to LEU conversion effort, Argonne National Laboratory (ANL) computed these quantities for the UMLRR (see Ref. 4) and these values were used as "reference" for 20+ years. However, the current M-2-5 core configuration is quite different from the one used by ANL, and clearly we should have the base computational capability to do these type of calculations locally at UMass-Lowell. Thus, as part of his MS Thesis⁵, Michael Pike has computed a series of reactivity coefficients for the current M-2-5 core configuration (he used a BOL model since we expect that the small amount of burnup will not have much effect on these coefficients). Most of the calculations were done using 2-group theory within our standard 3-D VENTURE⁶ model of the UMLRR. The cross sections for each temperature considered were generated by a full cross section processing sequence in SCALE⁷ using several different modules to account for both resonance and spatial self shielding effects and to compute the proper fine-group flux weighting spectrum for collapse to two groups. Some energy group sensitivity studies were performed to show that the use of 2-groups was sufficient to get reasonable results. Similarly, a 2-D vs. 3-D comparison showed some slight differences due to axial leakage effects, so all the final results were reported with the 3-D geometry.

The results from the computer calculations by Michael Pike and the "reference" ANL values are given in Table I. These are the reactivity coefficients near room temperature conditions, with the full ρ vs. T plot given in Fig. 2 [note that $\alpha_T(T)$ is the slope of the $\rho(T)$ vs. T curve]. The single combined $\alpha_{T_{tot}}$ from a pool cooldown run within the UMLRR performed in January 2013 is also included.⁸ Comparing this measured value of the total temperature coefficient (including coolant and fuel) for the M-2-5 core at about 50-55 MWD and the calculated value at BOL shows that they differ by nearly a factor of two. As noted above, we certainly acknowledge that these reactivity coefficients are very difficult to compute and to measure, so some differences were expected here -- but not a factor of two, since this kind of difference can have a significant effect on the transient response of the system with feedbacks included! Thus, there is certainly some concern here about the accuracy and interpretation of the computed vs. measured values given in Table I, but this is the best information that is currently available (as of June 2015).

Table I Reactivity Coefficients for several UMLRR Models ($\Delta k/k/^\circ C$) (from Ref. 5)

Component	M-1-3 (BOL)	M-2-5 (BOL)	M-2-5 WPI Fuel	ANL Data	M-2-5 Measured
Water Temp Only	-4.4E-05	-4.2E-05	-1.3E-05	-4.8E-05	---
Water Density Only	-5.3E-05	-5.8E-05	-5.3E-05	-4.6E-05	---
total $T_{coolant}$	-9.7E-05	-1.0E-04	-6.6E-05	-9.4E-05	---
T_{fuel}	-1.7E-05	-1.7E-05	-1.7E-05	-1.5E-05	---
$T_{coolant} + T_{fuel}$	-1.1e-4	-1.2e-4	-8.3e-5	-1.1e-4	-5.9e-5
void ($\Delta k/k/\%void$)	-2.3E-03	-2.6E-03	-2.3E-03	-2.4E-03	---

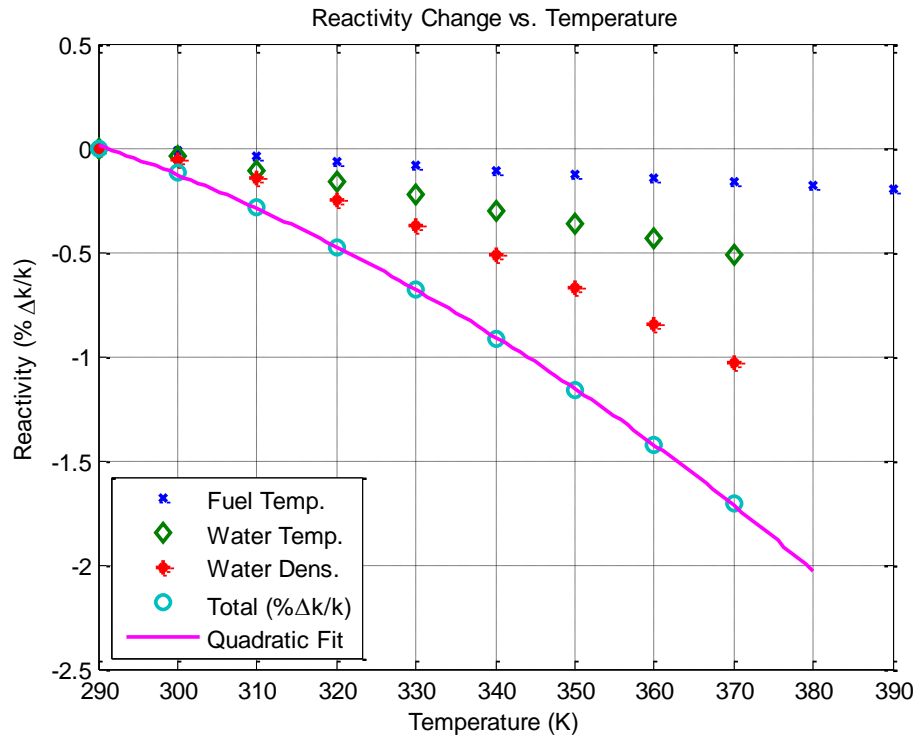


Fig. 2 Calculated reactivity vs. adjusted property for the M-2-5 BOL core (from Ref. 5).

Note also that, for rough qualitative estimates or to simply help physically explain some observed behavior, one can use the 6-factor formula to break α_T into its various subcomponents. Recall that the 6-factor formula⁹ is given by

$$k = k_{\infty} P_T P_F \quad (5)$$

and, taking the natural logarithm of both sides, gives

$$\ln k = \ln k_{\infty} + \ln P_T + \ln P_F$$

Now, taking the partial derivative with respect to temperature (holding all other variables constant), gives

$$\alpha_T = \frac{1}{k} \frac{\partial k}{\partial T} = \frac{1}{k_{\infty}} \frac{\partial k_{\infty}}{\partial T} + \frac{1}{P_T} \frac{\partial P_T}{\partial T} + \frac{1}{P_F} \frac{\partial P_F}{\partial T}$$

or

$$\alpha_T = \alpha_T|_{k_{\infty}} + \alpha_T|_{P_T} + \alpha_T|_{P_F} \quad (6)$$

Note also that, via the same type of argument, the k_{∞} component can be broken into the four individual sub-factors associated with the elements of the 4-factor formula, or

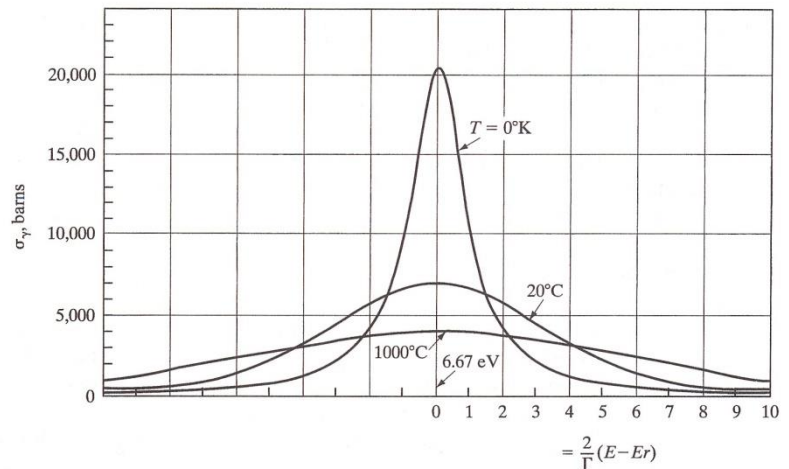
$$\alpha_T|_{k_{\infty}} = \alpha_T|_{\eta_T} + \alpha_T|_f + \alpha_T|_p + \alpha_T|_{\epsilon} \quad (7)$$

Thus, the fuel temperature coefficient, for example, involves summing the individual effects associated with how a change in T_f affects the neutron reproduction factor, the fuel utilization, the resonance escape probability, etc...

Lamarsh (Ref. 3) does a nice job discussing the importance of these individual terms to the overall temperature coefficient of reactivity -- with a focus on establishing/justifying the sign (positive, negative, or essentially zero) of the separate components. In particular, when treating the fuel temperature coefficient -- which is often referred to as the prompt temperature coefficient (or Doppler coefficient) since the fuel temperature feedback usually has the fastest response time -- Lamarsh argues that the dominate contribution here is due to the change in the resonance escape probability. To see this, we note that in thermal systems using low enriched uranium, there is a significant amount of U238 present and a single particularly large resonance at about 6.67 eV plays a dominant role in the overall inherent safety of these systems.

In particular, as shown in the sketch (from Ref. 3) of the U238 capture cross section in the vicinity of the 6.67 eV resonance,

the peak cross section tends to decrease and the wings of the resonance tend to broaden as the temperature is increased. This broadening is due to the increased relative motion of the U238 nuclei as the temperature and average kinetic energy increase (this is often referred to as Doppler broadening). Note, however, that although the shape of the resonance changes, the integral under the $\sigma_c(E)$ curve remains constant.



Thus, the absorption rate associated with this single resonance can be written as

$$F_a = \int N\sigma_c(E)\phi(E)dE = \phi_{\text{ave}} \int \Sigma_c(E)dE = \text{constant} \times \phi_{\text{ave}}$$

where

$$\phi_{\text{ave}} = \frac{\int \Sigma_c(E)\phi(E)dE}{\int \Sigma_c(E)dE}$$

Now, the key observation here is that, as the resonance peak decreases, we see less of a dip in the local flux, $\phi(E)$, within the resonance (i.e. less resonance self shielding), so the effective average flux defined above increases -- which, in turn, increases the overall absorption rate in the resonance. Thus, for an increase in the fuel temperature, we see a broadening of the resonance, which increases the average flux and overall absorption rate, ultimately resulting in a decreased resonance escape probability and decreased reactivity, or

$$\alpha_{T_f} = \frac{1}{k} \frac{\partial k}{\partial T_f} \approx \frac{1}{p} \frac{\partial p}{\partial T_f} < 0$$

Thus, the prompt temperature coefficient or Doppler coefficient is negative in low enriched uranium-fueled thermal systems and this is indeed a practical and essential inherent safety feature in all systems of this type.

For fast reactors, things are a little more complicated because there are both fission and capture resonances in the fuel material (U235, U238, Pu239, etc.) and important capture resonances in the structure and coolant (Fe and Na, for example). The broadening of the fission resonances represents a positive reactivity addition and, as discussed above, the capture resonances represent a negative effect. Although the capture (loss) component tends to dominate, all the resonances need to be treated carefully and the positive and negative contributions need to be added appropriately to give the composite reactivity coefficient.

Although of lesser immediate consequence than α_{prompt} , the coolant/moderator temperature coefficient is also very important and, along with the Doppler coefficient, it tends to drive the ultimate behavior of the system over slightly longer periods of time. In particular, in water-cooled and water-moderated systems, a change in the moderator temperature, which either increases or decreases the water density, affects the multiplication factor in several ways -- it can change the value of the thermal utilization, $f = \Sigma_{aF}/(\Sigma_{aF} + \Sigma_{aM})$, by changing the relative absorption rates of the fuel and non-fuel (moderator) materials, it can change the resonance escape probability, $p = \Sigma_{1 \rightarrow 2}/(\Sigma_{a1} + \Sigma_{1 \rightarrow 2})$, by changing the relative distribution between the fast absorption rate and downscatter rate, and it can change the overall non-leakage probability, $P_F P_T \approx 1/(1 + B^2 M_T^2)$, since neutrons diffuse more readily through less dense materials. Thus, we can write the moderator temperature coefficient in terms of these components as

$$\alpha_{T_m} = \alpha_{T|f} + \alpha_{T|p} + \alpha_{T|P_F} + \alpha_{T|P_T}$$

For the fuel utilization component in water-moderated systems, an increase in moderator temperature leads to a decrease in density which tends to reduce the number of absorptions in the moderator. Thus, f tends to increase with an increase in moderator temperature, and $\alpha_{T|f}$ is positive.

However, for the resonance escape and non-leakage probabilities, just the opposite is true. For example, an increase in moderator temperature decreases the moderator density, which decreases $\Sigma_{1 \rightarrow 2}$, with a subsequent decrease in the resonance escape probability, p . Similarly, this same decrease in density increases the neutron leakage and decreases the P_F and P_T non-leakage probabilities. Thus, the remaining three components, $\alpha_{T|p} + \alpha_{T|P_F} + \alpha_{T|P_T}$, of the overall temperature coefficient, α_{T_m} , are all negative.

In deciding the ultimate sign of α_{T_m} , one must balance the positive fuel utilization component with the other three negative terms. Here, the question becomes whether the “moderator absorbs more than it moderates” or “moderates more than it absorbs”. For the usual case, the second option is true, since the main purpose of the moderator is to slow down neutrons while minimizing parasitic absorption -- thus, here $\alpha_{T_m} < 0$. However, in a PWR with a high soluble boron loading (especially at the beginning of a new fuel cycle), this situation may be reversed (i.e. the moderator and soluble poison material may absorb more than it moderates) and α_{T_m} can become slightly positive. Clearly this situation needs to be avoided under full power operational conditions -- thus there is usually an upper limit on the soluble boron loading to assure that α_{T_m}

remains negative under most conditions [note that a small positive temperature coefficient is allowed under very restricted conditions during reactor startup at the beginning of cycle (BOC)].

Other reactivity coefficients are also important -- see the discussion in Lamarsh, for example, on the void coefficient for both water-cooled and sodium-cooled systems -- and, collectively, these parameters are extremely important to the routine operation and inherent safety of all nuclear systems. Clearly, special care and effort must be taken to assure that all the important reactivity coefficients are properly bounded (as required) and that the systems are always operated within strict guidelines for the safe operation of the facility.

Summary

The brief overview here is intended to simply introduce the terminology, importance, and basic understanding of temperature coefficients in reactor systems. The specific numerical data for the UMLRR are also useful as a general reference, for comparison to measured values, and for use in simulations for the UMLRR. Every nuclear system must be designed to be inherently stable relative to power and temperature changes in the system and a strong negative temperature coefficient under all operating conditions is a prerequisite for meeting this design criterion. This condition must not only be satisfied during the design phase, but it must also be verified via measurement within the physical system -- there can be no compromise here, since a system without sufficient stability (i.e. a strong negative α_T) will simply not be allowed to operate...

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