24.536 Reactor Experiments and 407.403 Advanced Nuclear Lab

HW #8: "Integral Blade Worth Curves" Pre-lab Exercises -- Solutions

Problem 1: Briefly identify the three methods discussed for measuring the integral rod or blade worth curves in a real system. What are the advantages/disadvantages of each method?

From the Lecture Notes:

The three experimental methods for generating the desired worth profiles are the **Stable Period Method**, the **Inverse Count Rate Method**, and the **Inverse Kinetics Method**.

Stable Period Method:

Procedure:

1. From near full insertion with the reactor at a low-power critical state, move the blade of interest (BOI) out a small Δz.
2. After a short transient time, measure the doubling time and reactor period and use the reactivity equation to convert this to a reactivity change, $\Delta\rho$.
3. This gives one data point, $\Delta\rho/\Delta z$, at the midpoint of the blade position for the given interval.
4. Use the remaining blades to return to a just critical condition at the new reference location for the BOI (this step is time consuming).

Repeat the above sequence to get good coverage of the full blade traverse, and then fit the full data set to a mathematical model for the differential worth distribution.

Advantages/Disadvantages:

Although the basic approach for the Stable Period Method is quite straightforward, it does pose some operational difficulties. First the method is actually rather time consuming in that it takes a minimum of 20-30 minutes to obtain the needed information for a single data point -- and typically, 10-12 data points should be taken to get a good fit. Thus, it takes 3-4 hours to obtain a good blade worth curve for each control device.

In addition, since the technique involves reactivity balancing at critical, it is sometimes difficult to get good coverage over the full range of the blade traverse. In particular, for the current UMLRR configuration, the excess reactivity is under 3 %Δk/k, but the total individual worths of Blades 3 and 4 are well over this amount, so the reactor cannot be made critical if either Blade 3 or 4 is fully inserted into the core -- which means that it is not possible to get experimental data at the lower end of the blade worth curve (i.e. the near full-insertion region). On the other side of the curve, when one of the high-worth blades is nearly fully withdrawn, it is also hard to balance the reactivity swing, since this situation leads to relatively large radial and axial flux tilts -- which are also undesirable. Thus, the Stable Period Method is certainly not ideal, but it has indeed served the reactor staff sufficiently well for many years...

Inverse Count Rate Method:

Procedure:

1. Record the detector count rate with the blade in its fully inserted position with the reactor at a subcritical state -- C_{in}.

2. Move the blade of interest (BOI) out a small Δz.
3. After a short transient time, measure the steady-state count rate, C_z, for the current blade position z.

Repeat Steps 2 and 3 to get good coverage of the full blade traverse, and then fit the full data set to a mathematical model for the normalized integral worth distribution.

Advantages/Disadvantages:

The biggest advantage associated with this method is that it is done at subcritical conditions, where the time to reach stable operation between data points is quite reasonable (relative to the stable period method that requires returning to critical by rebalancing the blades after each point). Thus, the Inverse Count Rate method is attractive since it can be performed in a timely manner.

However, it has several disadvantages, the most important being that it only gives the relative profile, not the absolute worth. In addition, the method is only approximate because of the assumption that the configuration-dependent proportionality factors cancel. Although the assumption that the count rate ratio is equal to the inverse of the subcritical reactivity ratio [i.e. $C(z)/C_{in} = \rho_{in}/\rho(z)$] is often quite good between neighboring configurations that involve small changes (i.e. between configuration i and i+1), it does often introduce some error from the first to last arrangement (that is, assuming that $\alpha_{in} \approx \alpha_{out}$ is probably much less accurate than saying $\alpha_{i+1} \approx \alpha_i$). Finally, within the UMLRR, we know that the startup counter is rather noisy, and this also introduces issues in using this method to determine accurate blade worth curves for the system.

Inverse Kinetics Method:

In the inverse kinetics problem, the usual signal flow path is reversed -- that is, given the observed power vs. time behavior, P(t), as the known "input", we want to compute the "output" $\rho(t)$ -- and this fits in quite nicely with our goal here of measuring blade worth curves.

One complication, however, is that at low reactivity levels, there is a "drift" in the $\rho(t)$ prediction due to gamma interference within the power detectors within the UMLRR -- and this dictates that the deviation from critical should be held within about $\pm$ 0.4 $\%\Delta k/k$ and the power swing, especially on the low side, should not be much greater that a factor of 10-20 below the reference critical value. Thus, for applications involving the measurement of the full blade worth curve, the negative reactivity addition associated with the blade of interest being inserted into the core must be balanced by the other blades in the system.

Procedure:

Perform a series of blade movements at low power as follows:

1. Ramp the BOI in a small amount, wait a few seconds, then remove the other blades as needed to over-compensate the negative insertion and to bring the power back up into the top half of the given range (usually 1 – 15 kW).
2. Then, as quickly as possible, repeat this sequence as many times as necessary to get the BOI fully inserted.

Advantages/Disadvantages:

The clear advantage here is that this procedure can be done rather rapidly (usually around 30 minutes or less for one blade). The only real disadvantage is associated with the "reactivity drift" noted above, but this has already been accounted for within the actual procedure used within the UMLRR.

Problem 2: The theoretical development of the "ideal integral rod worth curve" uses some concepts from First Order Perturbation Theory (FOPT). As part of our brief introduction to the topic of FOPT, we introduced the concept of an "adjoint operator". What is the adjoint to the second derivative operator -- that is, if $H = d^2/dx^2$, what is H* assuming that the "boundary terms" vanish?

Directly from the Lecture Notes:

From the basic definition of our bracket notation for $H = \partial^2/\partial x^2$, we have

$$\langle \phi^* H\phi \rangle = \int_a^b \phi^*(x)\frac{\partial^2}{\partial x^2}\phi(x)dx \qquad (A.4)$$

Integrating by parts as above, we have

$$u = \phi^* \quad \text{and} \quad dv = \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)dx \qquad \text{with} \qquad du = \frac{\partial \phi^*}{\partial x}dx \quad \text{and} \quad v = \frac{\partial \phi}{\partial x}$$

and, upon substitution, this gives

$$\langle \phi^* H\phi \rangle = \phi^*\frac{\partial \phi}{\partial x}\bigg|_a^b - \int_a^b \frac{\partial \phi}{\partial x}\frac{\partial \phi^*}{\partial x}dx$$

Since we desire a form of the integral where the operator is operating completely on ϕ^*, let's try integration by parts once again. This time we have

$$u = \frac{\partial \phi^*}{\partial x} \quad \text{and} \quad dv = \frac{\partial \phi}{\partial x}dx \qquad \text{with} \qquad du = \frac{\partial}{\partial x}\left(\frac{\partial \phi^*}{\partial x}\right)dx \quad \text{and} \quad v = \phi$$

and, after substitution, we have

$$\langle \phi^* H\phi \rangle = \phi^*\frac{\partial \phi}{\partial x}\bigg|_a^b - \phi\frac{\partial \phi^*}{dx}\bigg|_a^b + \int_a^b \phi\frac{\partial^2 \phi^*}{\partial x^2}dx = BT + \langle \phi H^* \phi^* \rangle \qquad (A.5)$$

Thus, by comparing eqns. (A.4) and (A.5), we see that for $H = \partial^2/\partial x^2$, the adjoint operator becomes $H^* = H = \partial^2/\partial x^2$. When this occurs (i.e. when $H^* = H$), the operator is said to be ***self-adjoint***. Also note that, in general, we can write this relationship in terms of the multidimensional Laplacian,

$$H = \vec{\nabla}\cdot\vec{\nabla} = \nabla^2, \qquad H^* = H = \vec{\nabla}\cdot\vec{\nabla} = \nabla^2 \qquad (A.6)$$

and also, for the special case of the leakage term within the neutron diffusion equation, we have

$$H = -\vec{\nabla}\cdot D\vec{\nabla}, \qquad H^* = H = -\vec{\nabla}\cdot D\vec{\nabla} \qquad (A.7)$$

Problem 3: Continuing with the subject of FOPT, write out both the forward and adjoint diffusion equations for the usual 2-group thermal reactor problem (i.e. with no upscatter and $\chi_1 = 1.0$ and $\chi_2 = 0.0$), and identify the key differences in these sets of equations. Also identify what is meant by the term "self adjoint" and determine if the 2-group equations satisfy this condition -- and be sure to explain why or why not.

Directly from the Lecture Notes:

Specializing the general equations to the case of the usual 2-group thermal reactor problem (i.e. with no upscatter and $\chi_1 = 1.0$ and $\chi_2 = 0.0$), the expressions for the **forward equations** become

$$\begin{aligned} -\vec{\nabla}\cdot D_1\vec{\nabla}\phi_1 + (\Sigma_{a1}+\Sigma_{1\to 2})\phi_1 - \lambda(\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2) &= 0 \\ -\vec{\nabla}\cdot D_2\vec{\nabla}\phi_2 + \Sigma_{a2}\phi_2 - \Sigma_{1\to 2}\phi_1 &= 0 \end{aligned} \qquad \text{(B.5)}$$

and the appropriate **two-group adjoint equations** are

$$\begin{aligned} -\vec{\nabla}\cdot D_1\vec{\nabla}\phi_1^* + (\Sigma_{a1}+\Sigma_{1\to 2})\phi_1^* - \Sigma_{1\to 2}\phi_2^* - \lambda\nu\Sigma_{f1}\phi_1^* &= 0 \\ -\vec{\nabla}\cdot D_2\vec{\nabla}\phi_2^* + \Sigma_{a2}\phi_2^* - \lambda\nu\Sigma_{f2}\phi_1^* &= 0 \end{aligned} \qquad \text{(B.6)}$$

As mentioned above, one should take special note of the key differences in these equations, primarily in the inscatter and the fission source terms. In fact, the adjoint equation is fundamentally different in the sense that the "inscatter source" for group 1 requires knowledge of the group 2 fluxes. In essence, "neutron importance" scatters (or transfers) from group 2 to group 1, whereas physical neutrons scatter from group 1 to group 2. Also note that the adjoint equations contain a fission source term in each group (both of which are functions of ϕ_1^*).

In practice, since scattering in the adjoint case is from group 2 to group 1, the energy equations are usually solved in reverse order. During the inner iteration of a finite difference calculation for the adjoint equation, for example, the energy loop goes from low to high energy (instead of the standard high to low scheme for the forward solution). Other than this change, and the fact that the scattering and fission source components are "transposed", the numerical solution scheme for the adjoint equation is identical to that for the forward equation. Thus, many deterministic reactor physics code systems solve both forms of the balance equation with little interaction on the part of the user. In fact, in most systems, the choice of a forward/adjoint option switch is all that is required.

Finally we emphasize that the inscatter and fission source operators in the 2-group equations are NOT the same (the matrix operators are the transpose of each other). Thus, these operators are NOT self-adjoint and the forward and adjoint 2-group equations give quite different solutions for the forward and adjoint fluxes in the system. The forward equations give the usual neutron flux distribution for the fast and thermal groups, and the adjoint equations give an indication of the neutron importance to reactivity.

Problem 4: In the theoretical development of the "ideal integral rod worth curve" we showed that, assuming "1-group theory for a bare 1-D homogeneous critical reactor of height H", the ideal integral worth distribution is given by

$$\rho_w(z) = \rho_w(H)\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right)$$

Your job here is to **formally derive this result** showing all the details and assumptions involved. In doing this you can start with the statement "that the worth of a material inserted to an axial depth z within the reactor is proportional to the product of the forward and adjoint fluxes integrated over the perturbed domain" and, for 1-group theory, this statement can be written mathematically as

$$\rho_w(z) = \alpha\int_0^z \phi^*(z')\Delta\Sigma_a(z')\phi(z')\,dz'$$

where α is the proportionality constant and ϕ^* is the adjoint flux.

Directly from the Lecture Notes:

Using ***Perturbation Theory Methods***, it can be shown that the worth of a material inserted to an axial depth z within the reactor is proportional to the product of the forward and adjoint fluxes integrated over the perturbed domain. In particular, assuming 1-group theory and that movement of the control rod only perturbs the absorption cross section, we have

$$\rho_w(z) = \alpha\int_0^z \phi^*(z')\Delta\Sigma_a(z')\phi(z')\,dz' \qquad (1)$$

where α is a proportionality constant and ϕ^* is known as the adjoint flux or importance function.

However, since the 1-group diffusion equation is self-adjoint, the adjoint and forward fluxes are identical, and eqn. (1) becomes

$$\rho_w(z) = \alpha\int_0^z \phi^2(z')\Delta\Sigma_a(z')\,dz' \qquad (2)$$

Now, for a bare 1-D homogeneous critical reactor of total height H, the flux profile is given by

$$\phi(z) = A\sin Bz \qquad \text{with} \quad B^2 = \left(\frac{\pi}{H}\right)^2$$

where z is measured from the top of the reactor (for simplicity, we have ignored the small extrapolation distance in this simple development). Finally, if the rod absorption cross section is constant, then combining the flux profile for a homogeneous system with eqn. (2) gives

$$\rho_w(z) = C\int_0^z \sin^2\frac{\pi z'}{H}\,dz' = C\left[\frac{z'}{2} - \frac{H}{4\pi}\sin\frac{2\pi z'}{H}\right]_0^z = C\frac{H}{2}\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right)$$

where C is just a new proportionality constant.

To evaluate this constant, we let $\rho_w(z)|_{z=H} = \rho_w(H) = \rho_{tot}$, which is the total rod worth. With this constraint we have

$$\rho_w(H) = C\frac{H}{2}(1-0) \qquad \text{or} \qquad C = \frac{2}{H}\rho_w(H)$$

and the so-called ***ideal integral worth distribution*** becomes

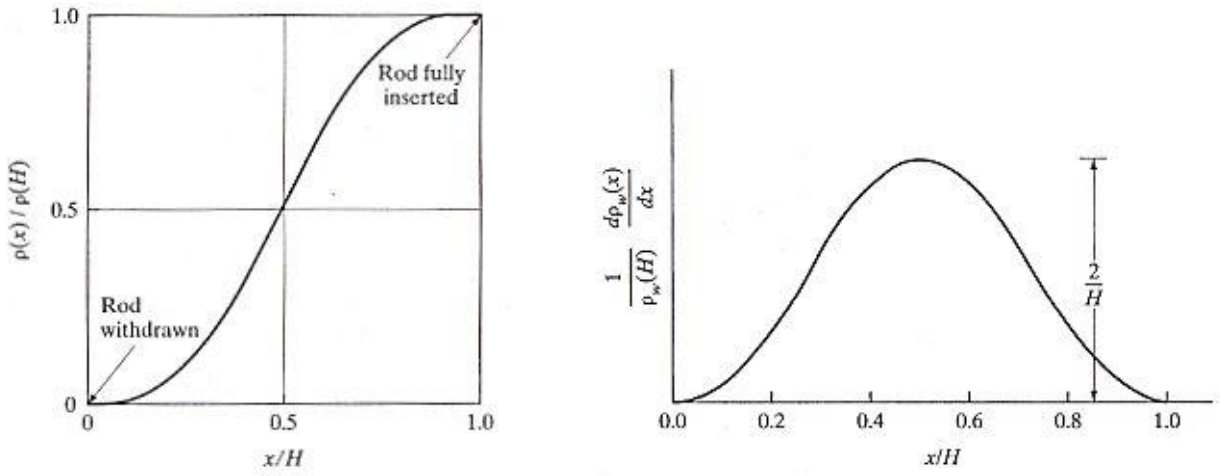
$$\rho_w(z) = \rho_w(H)\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right) \tag{3}$$

where $\rho_w(z)$ is the worth of a partially inserted rod to depth z. Finally, if one plots the relationship $\rho_w(z)/\rho_w(H)$, the ideal S-shaped normalized integral rod worth curve is obtained (as shown in the sketch below from Ref. 8).

Also of interest is the rate of change of $\rho_w(z)$ per unit distance. This differential worth can easily be obtained by differentiation of eqn. (3), or

$$\frac{d}{dz}\rho_w(z) = \frac{\rho_w(H)}{H}\left(1 - \cos\frac{2\pi z}{H}\right) \tag{4}$$

This function, when plotted, gives the familiar differential rod worth curve (as shown below in the sketch from Ref. 8).



In practice, of course, the integral and differential worth curves for real reactor systems differ somewhat from the ideal curves shown above (note that these were developed using first-order perturbation theory for a bare homogeneous 1-group system -- a pretty idealized situation indeed). However, they do give a good qualitative view of what to expect for a real system, with low differential worth near the upper and lower boundaries (where the flux and importance functions are relatively low) and a peak differential worth near the core center (where we expect the highest flux and the largest neutron importance). In a real reactor, if the control rods are inserted from the top, then the worth distribution often tends to be slightly bottom peaked, and it is slightly top peaked if the rods are inserted from the bottom (assuming, of course, that everything else is axially symmetric). However, to a rough first approximation, eqns. (3) and (4) and the sketches given above should help establish a reasonable set of expectations for the measurement of blade or rod worth curves for most real systems -- with the additional expectation that some asymmetry may be observed.

Problem 5: Briefly discuss what is meant by the term "inverse problem" and relate this concept to the solution of a typical reactor kinetics problem. Note that this should be a general discussion -- that is, no formal equation or model development is needed here!!! I just want a general overview of the topic…

Directly from the Lecture Notes:

When modeling the behavior of a system, one can often think of the physical system as acting on the independent inputs to produce a particular output response that is related to the input and to the inherent dynamics for the system of interest. This rather abstract view is illustrated for the general case of input u(t) and output y(t) on the left side of Fig. 1, and a specific application of this input-output perspective as applied to the point kinetics equations is given on the right side, where the externally-applied reactivity, ρ(t), is the input and the transient reactor power, P(t), is the output of interest. This is the usual situation that is encountered in modeling most systems and, in particular, when simulating the dynamic behavior of nuclear reactor cores. For example, a routine power maneuver can be simulated by inserting some small positive reactivity [or by moving a control rod or blade outward, which is then converted into an appropriate ρ(t)] and, once the desired power level is achieved, one simply returns the applied reactivity (or control device) back to its original value and allows the system to approach steady state at the new desired power level. The key element in this "usual" for "forward" view is that the reactivity is the driving function or system input and the power level is the output response -- this is the usual input-output view of the system.

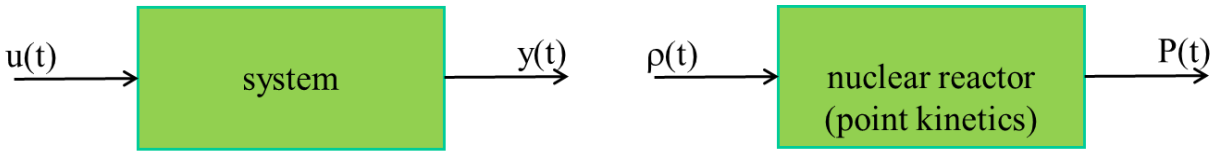


Fig. 1 Usual input-output view of a generic system and a specific reactor dynamics model.

Now, let's reverse our perspective somewhat and ask the question "Given some observed output y(t), what was the input u(t) that caused this output response?". This perspective is referred to as the **inverse problem** -- that is, trying to determine the driving function u(t) that led to some observed system behavior, y(t). And, in the context of reactor dynamics, this is called **inverse kinetics**, where the goal is to determine ρ(t) by observing the P(t) behavior. This "inverse" view of the system is illustrated in Fig. 2 for the reactor dynamics problem, which simply reverses the arrows and the input-output relationships relative to the sketch given in Fig. 1. Thus, in the inverse problem, the signal flow is reversed -- that is, given the observed power vs. time behavior, P(t), as the known "input", we want to compute the "output" ρ(t). This perspective is quite different in that we put on our "detective hat" and by observing some measurable system behavior, we try to determine what actually caused the observed response. This is the goal of all inverse problems...

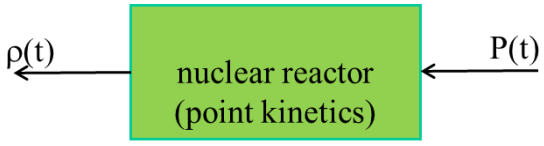


Fig. 2 Input-output view for the inverse reactor dynamics problem.

Problem 6: In discussing the practical implementation of the Inverse Kinetics Methods within the UMLRR, a problem with "reactivity drift" was mentioned. Briefly identify the cause of this issue and the limitations it imposes on the application of the method.

Directly from the Lecture Notes:

For implementation of the inverse kinetics equations for use within the UMass-Lowell research reactor (UMLRR), Thomas Michaud, as part of his MS thesis[3], concluded that, for near critical operation, an average of the linear power 1 and 2 channels gave the best P(t) signal to use within eqns. (4) and (5) for evaluating $\rho(t)$. In addition, as part of his work, he also discovered that, at low reactivity levels, there was a "drift" in the reactivity prediction due to gamma interference within the power detectors. In particular, the power data measured by the three power channels (the Linear 1 and 2 channels and the LogPower signal) are not solely related to the neutron signal, but rather they represent a combination of the neutron and the gamma signals. At near critical operation at relatively high power levels (above 500 W), this is not an issue since the neutron signal dominates. However, for fast negative transients, the neutron level drops much faster than the gamma level because of the longer-lived fission product gammas and, during the transient, the assumption that the detector signal is simply proportional to the neutron level may no longer be valid -- and this can lead to significant discrepancies with the inverse kinetics method (i.e. the observed reactivity "drift" noted in Ref. 3). Thus, for practical implementation within the UMLRR for near critical operation, the deviation from critical should be held within about $\pm$ 0.4 $\%\Delta k/k$ and the power swing, especially on the low side, should not be much greater that a factor of 10-20 below the reference critical value. Within these rough limits, the inverse kinetics method proved to be an excellent technique for measuring the dynamic reactivity within the UMLRR (see Ref. 3 for several example test scenarios).

Finally, we note that, although the inverse kinetics method should also be applicable within subcritical configurations, the startup counter within the UMLRR is simply too noisy for practical operation with the current detector system. Thus, our use of the inverse kinetics method within the UMLRR is currently limited to the measurement of dynamic reactivity changes from critical, where the power deviations from reference are such that the power channels are still primarily sensitive to the neutron level (i.e. with minimal gamma interference).