Normalization of the Generation Time Formulation of the One-Speed Point Kinetics Equations

Introduction

In theoretical studies involving the point kinetics equations, one often considers the “low-power” case, where temperature or xenon reactivity feedbacks are negligible. In addition, for subcritical situations, the external source term usually represents a “normalized” source that is related to the physical source via some formal integral normalization terms. In both cases, solution of the kinetics equations usually leads to relative results -- that is, one computes $P(t)/P_o$ or $n(t)/n_o$ or $T(t)/T_o$ etc., where these represent the time-dependent change in the power, neutron level, flux amplitude, etc., all relative to their reference initial state. For these situations, the time dependence of these ratios give a lot of insight into the behavior of the system under a variety of transient conditions, and they represent sufficient information for obtaining a good understanding of the basic fundamentals of reactor kinetics.

However, in many situations, such as when reactivity feedbacks are important or the transients under investigation involve sources in critical systems, for example, knowledge of the absolute neutron level or power level, and the absolute source strength, become essential. Unfortunately, conversion of the normalized kinetics equations to include absolute quantities is not obvious to many students, which often introduces unnecessary confusion in the practical application of point kinetics in real reactor operations. Thus, the whole point of this short set of Lecture Notes is to formally derive the equations needed for simulation of real power operations -- that is, to develop a form of the point kinetics equations that includes absolute power, P(t), in watts and absolute source strength, <Q(t)>, in neutrons per second. The kinetics equations written in this form can be applied to address both low-power and high-power situations and can be used to simulate situations involving operations when both the external source and the fission source terms are important.

Model Development

To accomplish our goal, we will focus our development on the one-speed generation time formulation of point kinetics using the specific notation defined in Ref. 1. In particular, the pertinent equations from Ref. 1 for the **Generation Time Formulation** are summarized below:

$$\frac{dT}{dt} = \frac{(\rho - \beta)}{\Lambda} T \quad + \quad \sum_i \lambda_i c_i \quad + \quad q \tag{1}$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} T \quad - \quad \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6 \tag{2}$$

where

$$c_i(t) = \frac{1}{\frac{1}{v}\langle \psi_o \rangle} \langle C_i(t) \rangle \qquad \text{and} \qquad q(t) = \frac{1}{\frac{1}{v}\langle \psi_o \rangle} \langle Q(t) \rangle \tag{3}$$

with

$$\Lambda = \frac{\frac{1}{v}\langle \psi_o \rangle}{\langle \nu \Sigma_f \psi_o \rangle} \tag{4}$$

and $\phi(\vec{r},t)=\psi(\vec{r},t)T(t)\approx\psi_o(\vec{r})T(t)$ (5)

In these expressions, $\psi_o(\vec{r})$ represents the initial spatial flux distribution, T(t) represents the time-dependent amplitude of the neutron flux, $c_i(t)$ and q(t) are the normalized precursor magnitudes and source level, respectively, Λ is the one-speed approximation for the prompt neutron generation time -- and all the other variables have their usual meaning (see Ref. 1 for further details).

The only thing that is not defined explicitly in this set of equations is the flux shape term, $\psi_o(\vec{r})$. This remains somewhat vague since, as we know, for the critical case, the steady-state neutron balance equation represents a homogeneous eigenvalue problem, with fundamental mode solutions having an arbitrary normalization. But, of course, the normalization used in most practical cases is determined by the reactor power level, P.

Thus, to be completely explicit, for the initial state, we can write the power level as

$$P_o=\kappa\sum_g\left\langle\Sigma_{fg}\phi_{og}\right\rangle=\kappa\alpha_o\sum_g\left\langle\Sigma_{fg}\psi_{og}\right\rangle \qquad (6)$$

with the absolute flux, ϕ_{og}, shape function, ψ_{og}, and normalization, α_o, defined via the following equations,

$$\phi_{og}=\alpha_o\psi_{og}, \qquad \sum_g\left\langle\nu\Sigma_{fg}\psi_{og}\right\rangle=1, \qquad \text{and} \qquad \alpha_o=\frac{P_o}{\kappa\sum_g\left\langle\Sigma_{fg}\psi_{og}\right\rangle} \qquad (7)$$

where the second expression in eqn. (7) says that the spatial shape is normalized so that there is 1 fission neutron/sec generated. With this notation, $\psi_{og}(\vec{r})$ is the solution to the diffusion equation and its normalization is such that the total fission source is unity. The overall flux normalization, α_o, then represents the absolute flux magnitude so the power produced in the system is P_o.

Now, to simplify eqn. (7), one often defines a single energy and isotope averaged value for the average number of neutrons emitted per fission (which is formally denoted as $\bar{\nu}$), where

$$\bar{\nu}=\frac{\sum_g\left\langle\nu\Sigma_{fg}\psi_{og}\right\rangle}{\sum_g\left\langle\Sigma_{fg}\psi_{og}\right\rangle}=\frac{1}{\sum_g\left\langle\Sigma_{fg}\psi_{og}\right\rangle} \qquad \text{or} \qquad \sum_g\left\langle\Sigma_{fg}\psi_{og}\right\rangle=\frac{1}{\bar{\nu}}$$

and, with this simplification, we have that

$$\alpha_o=\frac{P_o\bar{\nu}}{\kappa}=\frac{P_o\nu}{\kappa} \qquad (8)$$

Note that, in particular, in the last form shown in eqn. (8), we have dropped the over-bar notation (i.e. $\bar{\nu}\to\nu$) for convenience in writing this term in subsequent expressions. Thus, whenever ν is given separate from the $\nu\Sigma_{fg}$ combination, it is assumed to be an appropriately averaged value for the system of interest.

Now, it should be obvious that the flux normalization, α, defined above and the flux magnitude, T, defined in Ref. 1 [and in eqn. (5) above] are indeed the same quantity. Thus, rewriting eqn. (8), we see that $T_o = \nu P_o/\kappa$ or, more generally, we have

$$T(t) = \frac{\nu}{\kappa} P(t) \tag{9}$$

Now, with eqn. (9), we can easily derive the desired form of the generation time formulation of point kinetics that includes direct reference to P(t) and <Q(t)> -- that is, power in watts and neutron source strength in neutrons/sec.

In particular, multiplying eqn. (1) by κ/ν and using eqn. (3), we have

$$\frac{\kappa}{\nu}\left[\frac{d}{dt}T(t) = \frac{(\rho-\beta)}{\Lambda}T(t) \quad + \quad \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\sum_i \lambda_i \langle C_i(t)\rangle \quad + \quad \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle Q(t)\rangle\right]$$

But, from the relationship given in eqn. (9) and the definition of the generation time given in eqn. (4), this reduces to

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda\langle\nu\Sigma_f\psi_o\rangle}\sum_i \lambda_i \langle C_i(t)\rangle \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda\langle\nu\Sigma_f\psi_o\rangle}\langle Q(t)\rangle$$

and, via the fact that we have normalized the shape function to give a fission source of unity, this expression becomes

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda}\sum_i \lambda_i \langle C_i(t)\rangle \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

Finally, since we can't physically measure the precursor level anyway, for simplicity, let's define a new "normalized" precursor magnitude as

$$\tilde{c}_i(t) = \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle C_i(t)\rangle \tag{10}$$

which gives

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) \quad + \quad \sum_i \lambda_i \tilde{c}_i(t) \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle \tag{11}$$

This is the final form of the time-dependent neutron balance equation that we desired -- here P(t) is the power in watts and <Q(t)> represents the total external source strength in neutrons/sec.

Now, multiplying the precursor equation in eqn. (2) by the same κ/ν factor completes the desired formulation,

$$\frac{d}{dt}\tilde{c}_i(t) = \frac{\beta_i}{\Lambda}P(t) \quad - \quad \lambda_i \tilde{c}_i(t) \qquad \text{for } i = 1, 2, \cdots 6 \tag{12}$$

Equations (11) and (12) represent the desired form of the Generation Time Formulation of the point kinetics.

Summary

The goal here was to derive a set of point kinetics equations that directly includes the actual reactor power level, P(t), and neutron source level, <Q(t)> -- and eqns. (11) and (12) give this desired result. However, for convenience in daily use, we will drop the ~ notation over the precursor magnitudes and simply write the new normalized precursor levels as $c_i(t)$. Thus, the actual equations used for future reference are

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) \quad + \quad \sum_i \lambda_i c_i(t) \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle \tag{13}$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) \quad - \quad \lambda_i c_i(t) \qquad \text{for } i = 1, 2, \cdots 6 \tag{14}$$

where $c_i(t)$ is formally defined via eqn. (10) -- although this detail is rarely ever used in practice. Equations (13) and (14) will be used frequently in subsequent reactor modeling discussions and for simulation comparisons, where appropriate, to real reactor data.

References

1. J. R. White, "One-Speed Point Kinetics Equations," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.