

## Reactivity Measurement Techniques

### Introduction

One of the most important parameters of interest for proper operation of both critical and subcritical facilities is the absolute reactivity level of a given configuration or the change in reactivity between two operating states. In this set of Lecture Notes, we will address the basic theory behind some of the more popular reactivity measurement techniques. In particular, the **Asymptotic Period Technique** and **Rod Drop Method** are discussed within the context of measuring the magnitude of a reactivity insertion within a critical system, and the **Source Jerk Method** and **Subcritical Multiplication Factor Approach** are developed for application within subcritical systems. Each method is developed individually, along with a simple Matlab simulation that illustrates the basic application of the method (using simulated data). In addition, some practical suggestions/hints for applying the methods within a real system are also given, where appropriate.

The starting point for the development of all the methods is the **Generation Time Formulation** of Point Kinetics. From Refs. 1 and 2, the Generation Time Formulation of the kinetics equations can be written as

$$\frac{d}{dt}P(t) = \frac{(\rho - \beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{v} \frac{1}{\Lambda} \langle Q(t) \rangle \quad (1)$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \quad \text{for } i = 1, 2, \dots, 6 \quad (2)$$

where  $P(t)$  is the power level in watts and  $\langle Q(t) \rangle$  represents the total external source strength in neutrons/sec, and the relationship between the usual normalized neutron amplitude,  $T(t)$ , and reactor power,  $P(t)$ , is given by<sup>2</sup>

$$T(t) = \frac{v}{\kappa} P(t) \quad (3)$$

These expressions represent the basis for the theory behind each of the methods discussed in this set of Lecture Notes. If the reader is unfamiliar with this particular representation of Point Kinetics, he or she should definitely consult Refs. 1-3 before continuing with the discussions given below.

### Asymptotic Period Technique

As discussed in some detail in Ref. 3, the solution to the point kinetics equations given in eqns. (1) and (2) for a step change in reactivity in a critical system with no external source is given by the so-called **reactivity equation**. In particular, for the Generation Time Formulation, the reactivity equation is given by

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i} \quad (4)$$

From a pure mathematical viewpoint, this expression is simply the characteristic equation associated with the original seven coupled linear constant coefficient ODEs -- and, for a given

value of reactivity, there will be seven distinct roots, and the root locations dictate the dynamics of the system. For example, we can write a general solution for the time-dependent power level as

$$P(t) = \sum_{j=1}^7 A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \dots + A_7 e^{\omega_7 t} \quad (5)$$

where the  $\omega_j$  are the roots of eqn. (4) and, as explained in Ref. 3, the key observations to be made from studying the reactivity equation are:

**positive  $\rho$ :** reactivity equation has **one positive root** and **six negative roots**

**negative  $\rho$ :** reactivity equation has **seven negative roots**

Now, if we order the roots from most positive to most negative, then, after a relatively short transient time, the last six terms in eqn. (5) decay away (because  $\omega_2, \omega_3, \dots, \omega_7 < 0$  for both positive and negative reactivity), leaving only the term containing  $\omega_1$ , or

$$P(t) = P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau} \quad (\text{after some transient time}) \quad (6)$$

where  $\tau = 1/|\omega_1|$  is called the reactor period and  $P_1$  is the power level after the short transient period. If  $\rho$  is positive,  $\omega_1 > 0$  and the reactor period is positive -- so the power level and neutron population grow indefinitely as  $e^{t/\tau}$  (recall that the development of eqn. (4) assumes no feedbacks). And, for negative reactivity, just the opposite occurs. That is, for  $\omega_1 < 0$ , the reactor period is negative -- so the power level and neutron population decrease indefinitely towards zero with the form  $e^{-t/\tau}$ . Thus, after a short transient time, the dominant behavior of  $P(t)$  is simply associated with the most positive root of the reactivity equation and it is represented as a simple growing or decaying exponential as shown in eqn. (6). This form for  $P(t)$  is referred to as its asymptotic behavior and it is simply characterized by the reactor period  $\tau$  -- which is indeed a very important quantity in reactor operations.

Now, if a small reactivity change is made in a low-power critical system then, after a short transient time, the asymptotic power level will behave approximately as given in eqn. (6) and, from observation of the measured  $P(t)$ , one should be able to measure the reactor period,  $\tau$  -- which, via the reactivity equation given in eqn. (4), gives an indirect measurement of  $\rho$ , the reactivity change that initiated the transient in the first place.

In particular, notice that taking the natural logarithm of eqn. (6) gives a straight line,

$$\ln \frac{P(t)}{P_1} = \pm \frac{1}{\tau} t$$

with a slope  $m = 1/\tau$ , where  $m$  will be positive for  $P(t)/P_1 > 1$  and negative for  $P(t)/P_1 < 1$ . Thus, the best way to implement the asymptotic period method is to take the natural logarithm of the  $P(t)$  data, do a linear fit to determine the slope, and then compute the reactor period as  $\tau = 1/m$ .

Another common approach for obtaining  $\tau$  is to measure the doubling time,  $t_d$ . That is,  $t_d$  is the time it takes for the reactor power to change by a factor of two after a short transient time following the initial reactivity change. Mathematically, this can be written as

$$\frac{P(t_d)}{P_1} = 2 = e^{t_d/\tau} \quad \text{and this gives} \quad \tau = \frac{t_d}{\ln 2} \quad (7)$$

Thus, the desired asymptotic period can be easily obtained with a simple measurement of the doubling time in the system following a reactivity change. Note also that, for a negative reactivity, the same expression is obtained if we associate  $t_d$  with the time it takes the flux amplitude or power to decrease by a factor of two. Finally we note that, although the doubling time method is easy to use, the noisy detector signals that are characteristic of most real systems can lead to some uncertainty in selecting  $t_d$  and in the measurement of  $\tau$ . If the detector signal is quite noisy, then determining the period from a linear fit to the  $\ln P(t)/P_1$  profile will probably give the best result.

Once the asymptotic period is known, eqn. (4) evaluated with  $\omega = \omega_1 = 1/\tau$  gives

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i\omega}{\omega + \lambda_i} = \frac{\Lambda}{\tau} + \frac{1}{\tau} \sum_i \frac{\beta_i}{\frac{1}{\tau} + \lambda_i} \approx \frac{1}{\tau} \sum_i \frac{\beta_i}{\frac{1}{\tau} + \lambda_i}$$

or

$$\rho = \sum_i \frac{\beta_i}{1 + \lambda_i\tau} \quad (8)$$

This result allows one to convert a measured asymptotic period into a measured reactivity -- and this was our original goal!!!

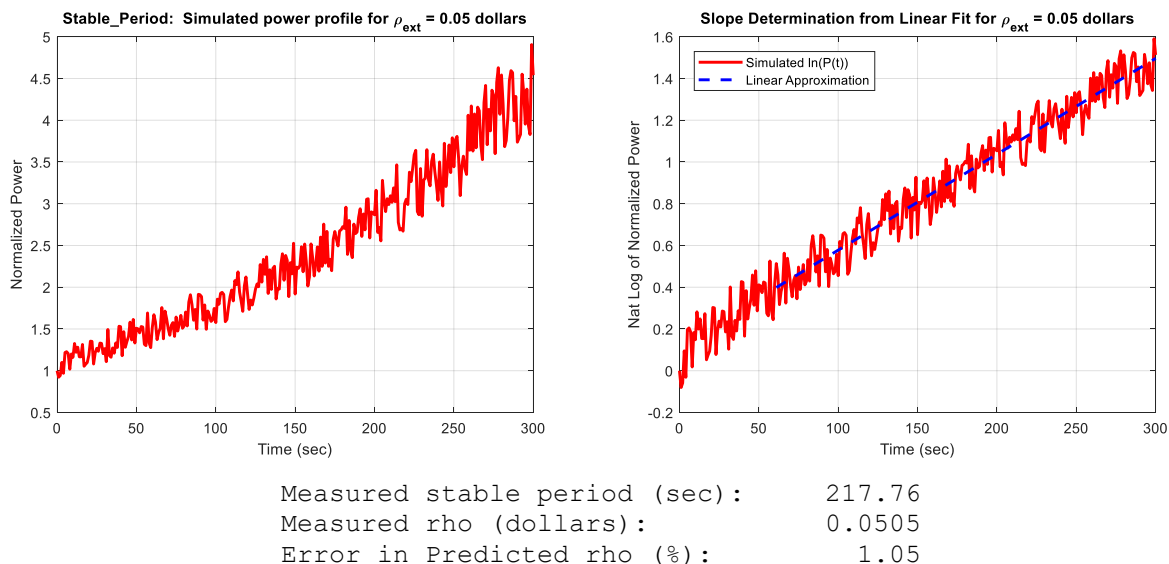
To illustrate how this reactivity measurement procedure might work in practice, a short Matlab code called **rho\_stable\_period.m** was written to simulate  $P(t)$  for a step change in reactivity. A noise component of  $\pm 15\%$  was added to the signal generated via numerical integration of the feedback-free point kinetics equations to simulate the noise that is often observed in real systems. The simulation assumes that the system is initially critical with no source, and either positive or negative reactivity can be inserted. With the noisy simulated  $P(t)$  signal to represent the real reactor power response, the code takes the natural logarithm of the  $P(t)$  data, does a linear fit to the data after a 60 second transient period to determine the slope, computes the reactor period as  $\tau = 1/\text{slope}$ , and then uses eqn. (8) to compute  $\rho$  based on the 'measured' stable period. Finally, a comparison is made between the actual reactivity that was used to generate the simulated data and the one obtained from the simulated experimental data.

The **rho\_stable\_period** code was run for two specific cases with reactivity insertions of positive and negative 0.05 dollars. The results for the positive  $\rho$  case are showed in Fig. 1 and the summary observations for the negative  $\rho$  case are displayed in Fig. 2. Both cases give very good results, with about 1 % error for each situation. However, although these examples illustrate the proper analysis methodology, don't get your expectations too high, since ideal simulated data (even data with a simulated noise component) often gives much better comparisons than can be expected with real reactor data. Nevertheless, the asymptotic period method is usually a good choice when trying to measure small reactivity changes from critical.

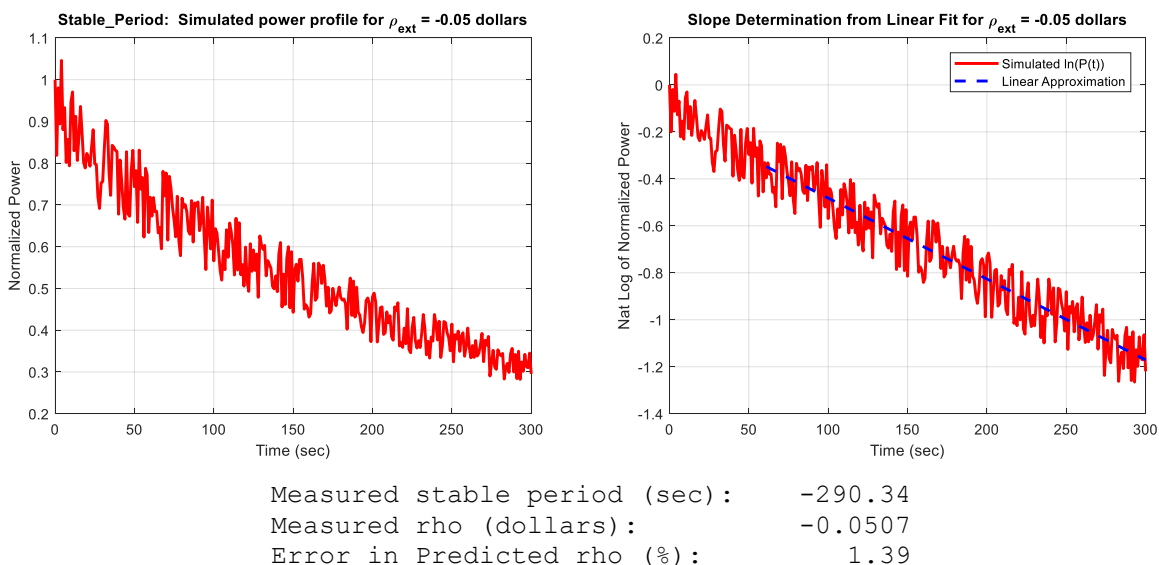
Finally, we should emphasize that, for large reactivity changes, the stable period method breaks down for both positive and negative  $\rho$  -- for quite different reasons. For large positive  $\rho$ , the reactor period becomes too small such that  $P(t)$  increases too rapidly, quickly causing an unsafe reactor condition that clearly must be avoided in all cases. In addition, even for moderate  $+\rho$ , the reactor power often approaches a level where we can no longer assume that the feedbacks are

negligible. Thus, positive reactivity changes beyond about 0.10 dollars simply cannot be measured with the stable period method (actually, there is no simple dynamic method that can be used to measure large positive changes in reactivity).

For large negative  $\rho$ , the situation is quite different. Here the reactor power is decreasing, so safety is not the concern. However, as discussed in Ref. 3,  $\tau$  vs.  $\rho$  approaches a constant for large negative  $\rho$  values -- thus, beyond about -0.10 dollars, it is simply not possible to relate a unique combination of  $\tau$  and  $\rho$  via eqn. (8) [see Fig. 4 in Ref. 3, for example]. Therefore, an alternate method is clearly needed for measuring large negative  $\rho$  (as discussed in the next subsection)...



**Fig. 1 Measured results using simulated data for  $\rho = 0.05$  dollars (positive reactivity).**



**Fig. 2 Measured results using simulated data for  $\rho = -0.05$  dollars (negative reactivity).**

## The Rod Drop Method and Source Jerk Formulations<sup>4</sup>

As noted above, the stable period method does not work for large reactivity changes. In addition, for the asymptotic period method, the system must be initially critical, which eliminates the whole class of subcritical systems that are clearly of interest. Thus, surely, we need to address additional reactivity measurement techniques to expand our capability to measure  $\rho$  for a wider variety of situations. Two techniques that do this for us include the **Rod Drop** and **Source Jerk** methods. Although these methods address different situations, they share a common theory, so we will address both these methods in this subsection. In particular, the Rod Drop Method is used to measure large negative reactivity insertions in critical systems, and the Source Jerk Method is used to determine the absolute reactivity level of subcritical systems -- and both these situations are of interest in practical applications.

As implied by the methods' names, the intent here is to either introduce a large negative step change in reactivity into a critical system (i.e. for the Rod Drop case), or to make a source perturbation to a subcritical system by instantaneously removing the external source (i.e. for the Source Jerk method). Both these instantaneous changes occur at  $t = 0$  and they lead to the following situations

$$\text{for } t > 0^+, \rho = \text{constant} \quad \text{and} \quad \int_{0^+}^{\infty} \langle Q(t) \rangle dt = 0 \quad (9)$$

and both these statements are true for both the Rod Drop and Source Jerk scenarios. For example, for the initial critical case there is no source present and, for the subcritical case, the source is zero for  $t > 0^+$  -- so the second part of eqn. (9) is true in both cases. And, to justify the first statement, we note that this is certainly consistent with the "step insertion" condition associated with the Rod Drop case, and the assumption is made that the instantaneous removal of the source only affects the source strength, not the reactivity level of the system (this, by the way, is only approximately true in practice). Thus, in the following development, we will take the two statements given in eqn. (9) as valid approximations for both the Rod Drop and Source Jerk methods.

Now, to develop the theory for both methods, we start with the generation time formulation of point kinetics as given in eqns. (1) and (2), and integrate both equations from  $0^+$  to  $\infty$ . Doing this gives

$$\int_{0^+}^{\infty} dP = \frac{(\rho - \beta)}{\Lambda} \int_{0^+}^{\infty} P(t) dt + \sum_i \lambda_i \int_{0^+}^{\infty} c_i(t) dt \quad (10)$$

$$\int_{0^+}^{\infty} dc_i = \frac{\beta_i}{\Lambda} \int_{0^+}^{\infty} P(t) dt - \lambda_i \int_{0^+}^{\infty} c_i(t) dt \quad \text{for } i = 1, 2, \dots, 6 \quad (11)$$

where we have already used the relationships in eqn. (9) to simplify eqn. (10).

Now we solve eqn. (11) for the  $\lambda_i \int_{0^+}^{\infty} c_i(t) dt$  term, substitute this result into eqn. (10), and do the indicated sum over all precursor groups, to give

$$\int_{0^+}^{\infty} dP = \frac{\rho}{\Lambda} \int_{0^+}^{\infty} P(t) dt - \frac{\beta}{\Lambda} \int_{0^+}^{\infty} P(t) dt + \frac{\beta}{\Lambda} \int_{0^+}^{\infty} P(t) dt - \sum_i \int_{0^+}^{\infty} dc_i$$

and, with the middle terms cancelling, we have

$$\rho = \frac{\Lambda \left[ \int_{0^+}^{\infty} dP + \sum_i \int_{0^+}^{\infty} dc_i \right]}{\int_{0^+}^{\infty} P(t) dt} \quad (12)$$

The exact derivatives on the right side of eqn. (12) can be written as

$$\int_{0^+}^{\infty} dP = P_{\infty} - P_{0^+} \approx -P_{0^+} \quad \text{and} \quad \int_{0^+}^{\infty} dc_i = c_{i\infty} - c_{i0^+} \approx -c_{i0^+}$$

where the last approximation in each expression assumes that the perturbation made at  $t = 0$  is such that the neutron flux or power and the precursor populations will go to zero as  $t \rightarrow \infty$  -- and clearly this is indeed the case for both the Rod Drop and Source Jerk scenarios described above.

Thus, upon substitution of these results into eqn. (12), the expression of interest for the reactivity,  $\rho$ , becomes

$$\rho = \frac{-\Lambda \left[ P_{0^+} + \sum_i c_{i0^+} \right]}{\int_{0^+}^{\infty} P(t) dt} \quad (13)$$

**Rod Drop Method:** For this case, the system is initially at critical with  $\rho_{0^-} = \rho_0 = 0$  and no source, so the initial conditions are

$$P(0^-) = P_{0^-} \quad \text{and} \quad c_{i0^-} = \frac{1}{\Lambda} \frac{\beta_i}{\lambda_i} P_{0^-} \quad \text{for } i = 1, 2, \dots, 6 \quad (14)$$

where the  $0^-$  notation implies *before* the instantaneous perturbation is made. Also, for this method,  $\rho$  in eqn. (13) represents the worth of the rod (or whatever material) that is instantly inserted into the critical system at  $t = 0$ .

Now, the relationship between quantities evaluated at  $t = 0^-$  and at  $t = 0^+$  is associated with the usual prompt jump/drop phenomena due to the fast response of the prompt neutrons. Thus, because of the very rapid change due to the prompt neutrons, we have  $P_{0^+} \neq P_{0^-}$  but, due to the much slower response of the delayed neutrons, we can argue that  $c_{i0^+} = c_{i0^-}$ . Therefore, it is necessary to carry along the power signal (or neutron amplitude) both before and after the step perturbation in the system (i.e.  $P_{0^-}$  and  $P_{0^+}$ , respectively) but, in contrast,  $c_{i0^+}$  can be replaced with the initial condition expression given in eqn. (14). Making this substitution in the equation for the reactivity gives

$$\left| \frac{\rho}{\beta} \right| = \frac{\frac{\Lambda}{\beta} \left[ P_{0^+} + \sum_i \frac{1}{\Lambda} \frac{\beta_i}{\lambda_i} P_{0^-} \right]}{\int_{0^+}^{\infty} P(t) dt} = \frac{\frac{\Lambda}{\beta} P_{0^+} + \left( \frac{1}{\beta} \sum_i \frac{\beta_i}{\lambda_i} \right) P_{0^-}}{\int_{0^+}^{\infty} P(t) dt} = \frac{\frac{\Lambda}{\beta} P_{0^+} + t_d P_{0^-}}{\int_{0^+}^{\infty} P(t) dt} \quad (15)$$

where

$$t_d = \frac{1}{\beta} \sum_i \frac{\beta_i}{\lambda_i} \quad (16)$$

is the mean lifetime of the delayed precursors and, for convenience, we have chosen to write the reactivity change as a positive quantity in dollars.

As a last step, since  $t_d \gg \Lambda/\beta$ , eqn. (15) reduces to

$$\left| \frac{\rho}{\beta} \right| = \frac{t_d P_{0^-}}{\int_{0^+}^{\infty} P(t) dt} \quad (17)$$

and this is the final expression of interest here. This says that, to measure the worth of the negative step change in reactivity in dollars, we need to measure  $P(t)$ , perform a numerical integration to find its integral, and then simply evaluate the above expression.

Concerning implementation/use of this expression, we note that, within the UMLRR, the average value of the Linear Power 1 and Linear Power 2 channels will be used as the  $P(t)$  signal for application within eqn. (17) for the Rod Drop Method. The reactor will be assumed to be at some steady state power level  $P_{0^-}$  before the negative reactivity insertion, and the  $P(t)$  behavior that results from the perturbation is used to evaluate the denominator of eqn. (17). Also note that, because power,  $P$ , appears in both the numerator and denominator, a relative or normalized power can also be used. Thus, eqn. (17) is also often expressed in terms of the normalized neutron amplitude,  $T(t)$ , as

$$\left| \frac{\rho}{\beta} \right| = \frac{t_d T_{0^-}}{\int_{0^+}^{\infty} T(t) dt} \quad (18)$$

**Source Jerk Method:** For this case, the system is initially subcritical with  $\rho_{0^-} = \rho_0$  being the degree of subcriticality. The initial conditions for this system can be written explicitly as

$$P(0^-) = P_{0^-} = -\frac{1}{\rho_0} \frac{\kappa}{v} \langle Q(0^-) \rangle \quad \text{and} \quad c_{i0^-} = \frac{1}{\Lambda} \frac{\beta_i}{\lambda_i} P_{0^-} \quad \text{for } i = 1, 2, \dots, 6 \quad (19)$$

Here it is important to note that these initial conditions are essentially identical with the Rod Drop case if we simply think of  $P_{0^-}$  as the initial power level and don't dwell on how the steady state neutron level is developed (i.e. critical vs. subcritical system). And, with identical initial conditions, we end up with the identical development and resultant formula as given above to represent the desired system subcriticality, or

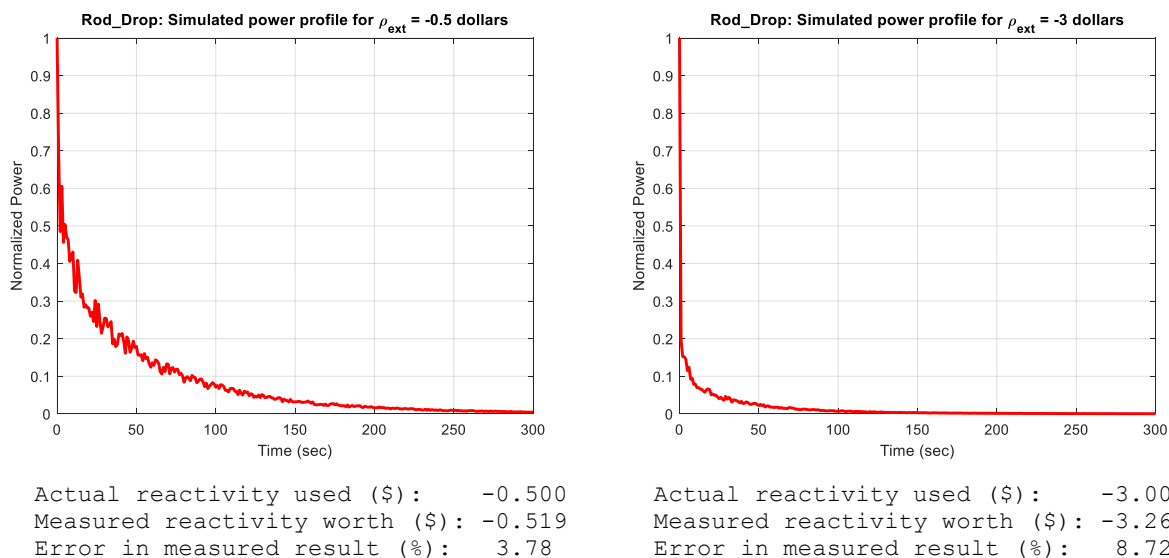
$$\left| \frac{\rho_0}{\beta} \right| = \frac{t_d T_{0^-}}{\int_{0^+}^{\infty} T(t) dt} \quad (20)$$

where  $\rho_0$  is used here so it can be distinguished from the  $\rho$  used in the Rod Drop method. In particular,  $\rho$  in eqn. (18) for the Rod Drop scenario is the worth of the negative reactivity inserted into the critical system (with the initial  $\rho_0$  identically zero). In contrast,  $\rho_0$  in eqn. (20) for the Source Jerk technique is the degree of subcriticality within the subcritical system, where it has been assumed that the instantaneous source removal does not cause any substantial reactivity change -- that is, the only perturbation in the system is the removal of the external

source. Thus, the appropriate equations are essentially identical, only the interpretation of the reactivity term,  $\rho$  vs.  $\rho_o$ , is different for the two scenarios.

Therefore, to measure the degree of subcriticality in a subcritical system, we can rapidly remove the source, measure the neutron amplitude vs. time,  $T(t)$ , perform a numerical integration to find its integral, and then simply apply eqn. (20) to obtain  $\rho_o/\beta$ . As apparent, the basic procedures for the Rod Drop and Source Jerk methods are nearly identical -- the key difference, of course, is that the Rod Drop method starts from a critical system with no source, and the Source Jerk method begins with a steady state subcritical system with an external source present. In addition, for measurement within the UMLRR, we should note that the linear power channels are unresponsive at very low power levels, so the startup detector is used to get the measured  $T(t)$  signal for analysis and experimental determination of the degree of subcriticality within a given subcritical configuration.

An example for each of the two methods has been prepared to illustrate the procedure described above. In particular, the Matlab code `rho_rod_drop.m` has been developed to simulate a noisy  $P(t)$  profile following a large negative insertion of reactivity within a just critical system. The code then uses Matlab's `trapz` function to do the needed numerical integrations, eventually evaluating eqn. (17) for the "measured"  $\rho/\beta$ . Two specific cases, for negative insertions of 0.5 and 3.0 dollars, were simulated and the results are summarized in Fig. 3 -- showing both the resultant noisy  $P(t)$  profiles and the comparisons between the "measured" values and the actual reactivities that initiated the transients. Both comparisons are reasonably good but, because of the fixed sampling time of 1 sec that is used in the simulations (since this is what is used in the UMLRR data acquisition system), the very large and fast prompt drop associated with the larger reactivity insertion leads to somewhat larger prediction errors. Although a smaller  $\Delta t$  between sampled  $P(t)$  data would lead to better results here, a smaller sampling interval is not really practical for use within the real system. Thus, a  $\Delta t = 1$  sec will be used for all the simulations given in these Lecture Notes (the user can easily change this within the codes, if desired).



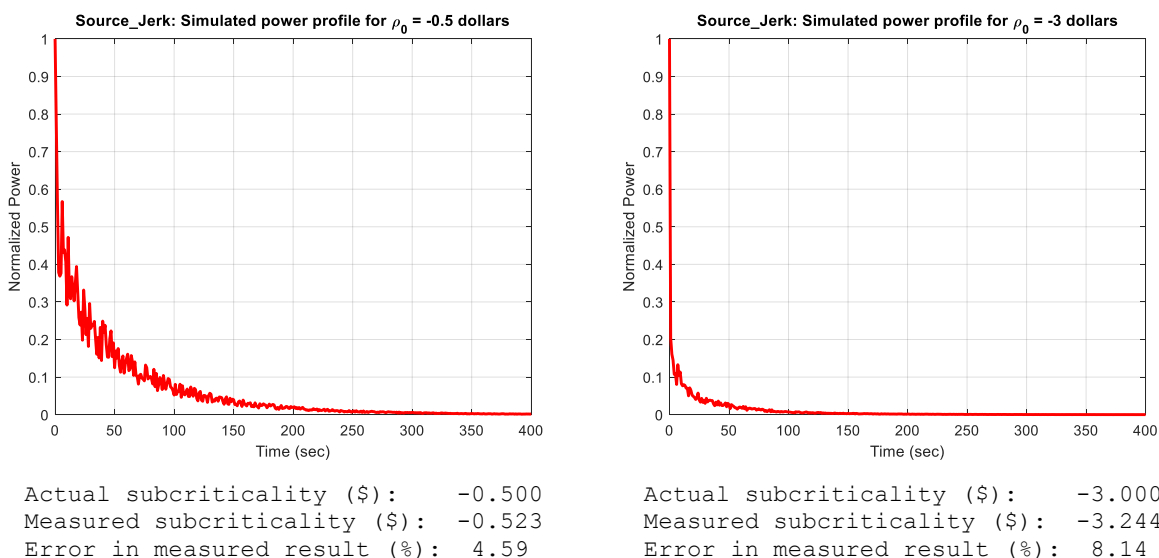
**Fig. 3 Results for the Rod Drop method using simulated data for two different reactivities.**



For the Source Jerk method, a very similar test analysis was performed and the code for this case, `rho_source_jerk.m`, is nearly identical to the `rho_rod_drop` code -- as it should be since the background theory for the two methods is the same. For this simulation, however, we doubled the noise level (from 15% to 30%) since the startup counter within the UMLRR tends to have a somewhat larger noise component relative to the linear power channels. The summary results from the `rho_source_jerk` code are shown in Fig. 4 and, for comparable subcriticality levels relative to the reactivity insertion levels used for the Rod Drop method tests, we see nearly identical  $P(t)$  profiles and "measured" results. Again, this was expected because of the similarity of the methods.

Thus, it appears that both the Rod Drop and Source Jerk methods have the potential to be useful as techniques for measuring reactivity under quite different scenarios. These methods will be tested with real reactor data as part of a subsequent Reactor Experiments lab on Reactivity Measurement Techniques.

**Note:** Much of the theoretical development here for the Rod Drop and Source Jerk methods was based on a similar development done in Ref. 4 which used the Lifetime Formulation of Point Kinetics. Both treatments get the same expressions for  $\rho/\beta$  and  $\rho_o/\beta$  as given in eqns. (18) and (20).



**Fig. 4 Results for the Source Jerk method using simulated data for two different  $\rho_o$  values.**

### Subcritical Multiplication Factor Approach

The methods discussed thus far allow us to measure reactivity changes from critical and to measure the absolute subcriticality level of a particular subcritical configuration, so the only key situation that is missing is to account for reactivity changes while the system is still subcritical. This situation was addressed to some extent in the Approach to Critical Reactor Experiments Lab<sup>5-6</sup> where we introduced the concept of subcritical multiplication and the relative subcritical

multiplication factor,  $M_r$ . In particular, as developed in Ref. 6, this quantity is defined as the ratio of detector count rates for two different subcritical configurations,

$$M_r = \frac{C_1}{C_0} \quad (21)$$

and, with the absolute subcritical multiplication factor for the  $i^{\text{th}}$  state given by  $M_i = 1/(1 - k_i)$ ,  $M_r$  can be written in a number of useful forms as follows:

$$M_r = \frac{C_1}{C_0} = \frac{\alpha_1 M_1 S}{\alpha_0 M_0 S} = \frac{\alpha_1 M_1}{\alpha_0 M_0} = \frac{\alpha_1 (1 - k_0)}{\alpha_0 (1 - k_1)} = \frac{\alpha_1 \left(1 - \frac{1}{1 - \rho_0}\right)}{\alpha_0 \left(1 - \frac{1}{1 - \rho_1}\right)} = \frac{\alpha_1 \rho_0}{\alpha_0 \rho_1} \frac{1 - \rho_1}{1 - \rho_0} \approx \frac{\alpha_1 \rho_0}{\alpha_0 \rho_1} \approx \frac{\rho_0}{\rho_1} \quad (22)$$

where  $\rho_0$  and  $\rho_1$  represent the two subcriticality levels for the two different states.

Now, with these relationships, the change in reactivity from state 0 to state 1 is given by

$$\Delta\rho = \rho_1 - \rho_0 = \rho_0 \left(\frac{\rho_1}{\rho_0} - 1\right) = \rho_0 \left(\frac{1}{M_r} - 1\right) = \rho_0 \left(\frac{1 - M_r}{M_r}\right) = \rho_0 \left(\frac{1 - C_1/C_0}{C_1/C_0}\right) \quad (23)$$

Thus, if we know the subcriticality level for the reference state, then the change in reactivity associated with a perturbation to the subcritical system can be determined by simply comparing the steady state detector count rates before and after the configuration change as given by eqn. (23).

A similar relationship for the reactivity change written in terms of the power level or neutron amplitude at two different subcritical states can also be formally developed quite easily by starting with the point kinetics equations given in eqns. (1) and (2). To see this, we simply set the time derivatives to zero to establish a steady state relationship between the power level and source amplitude for each subcritical state, or

$$P_0 = -\frac{1}{\rho_0} \frac{\kappa}{v} \langle Q_0 \rangle \quad \text{or} \quad \rho_0 = -\frac{1}{P_0} \frac{\kappa}{v} \langle Q_0 \rangle \quad (24a)$$

$$P_1 = -\frac{1}{\rho_1} \frac{\kappa}{v} \langle Q_1 \rangle \quad \text{or} \quad \rho_1 = -\frac{1}{P_1} \frac{\kappa}{v} \langle Q_1 \rangle \quad (24b)$$

where  $\rho_0$ ,  $\langle Q_0 \rangle$ , and  $P_0$  are the subcriticality level, total source amplitude, and power level for the initial reference steady state, and the same quantities with a '1' subscript correspond to the new steady state after some  $\Delta\rho$  has been introduced into state 0. If the source is not modified (i.e.  $\langle Q \rangle = \langle Q_0 \rangle = \langle Q_1 \rangle$ ), then the reactivity change can be written as

$$\Delta\rho = \rho_1 - \rho_0 = \rho_0 \left(\frac{\rho_1}{\rho_0} - 1\right) = \rho_0 \left(\frac{P_0}{P_1} - 1\right)$$

or, with multiplication and division by  $P_1/P_0$ , we have

$$\Delta\rho = \rho_0 \left( \frac{P_0}{P_1} - 1 \right) \frac{P_1/P_0}{P_1/P_0} = \rho_0 \left( \frac{1 - P_1/P_0}{P_1/P_0} \right) = \rho_0 \left( \frac{1 - T_1/T_0}{T_1/T_0} \right) \quad (25)$$

which is clearly the same as eqn. (23) which uses the ratio of count rates,  $C_1/C_0$ , since in all this development (as well as in Refs. 5 and 6) we have been saying that

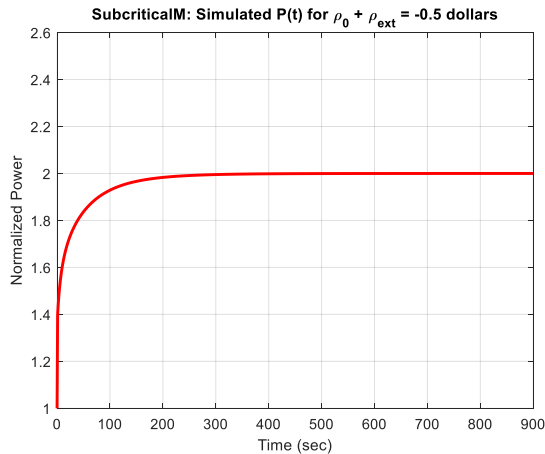
$$M_r = \frac{C_1}{C_0} = \frac{P_1}{P_0} = \frac{T_1}{T_0} \quad (26)$$

Thus, if we know any of these relative quantities (i.e. ratio of count rates, ratio of power levels, or ratio of flux amplitudes) and the reference subcriticality level,  $\rho_0 = \rho_0$ , then obtaining an estimate of  $\Delta\rho$  is a rather trivial task -- we simply need to evaluate eqn. (23) or eqn. (25).

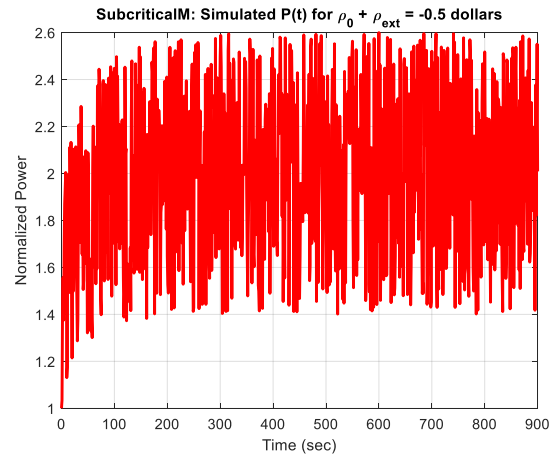
Although not necessary to compute  $\Delta\rho$ , it is also interesting to look at the actual transient that transpires in moving from state 0 to state 1. We can illustrate this transient behavior by solving the point kinetics equation using a numerical ODE solver with a step change in reactivity (either positive or negative) with the system initially at steady state subcritical with subcriticality level  $\rho_0$ . Such a simulation will not only support the steady state conditions given in eqn. (24), but it should also highlight how long it takes to reach the new steady state after the step reactivity change.

In particular, a Matlab code called **rho\_subcriticalM.m** was written to perform the operations described above and a set of typical results are shown in Fig. 5. There are a couple of important comparisons to be highlighted here. First we note that the transient power profile for the cases with and without a noise component give very similar results -- although the real  $P(t)$  signal is very hard to "see" through the  $\pm 30\%$  noise level that was imposed. Note that, for evaluating the new steady state level,  $P_1$  or  $T_1$ , an average value over the last 300 seconds is taken (the system is assumed to be in steady state over this interval -- this can be changed by the user if needed). Concerning the actual prediction of the reactivity change,  $\Delta\rho$ , clearly this was expected to be computed accurately, since the only "error" in the simulation results is due to the noise component (whose average values should be near zero) and the assumption that the last 300 seconds represents the real steady state level after the perturbation (as seen in Fig. 5, this is only approximately true for the 2<sup>nd</sup> set of cases with  $\Delta\rho = 0.9$  dollars).

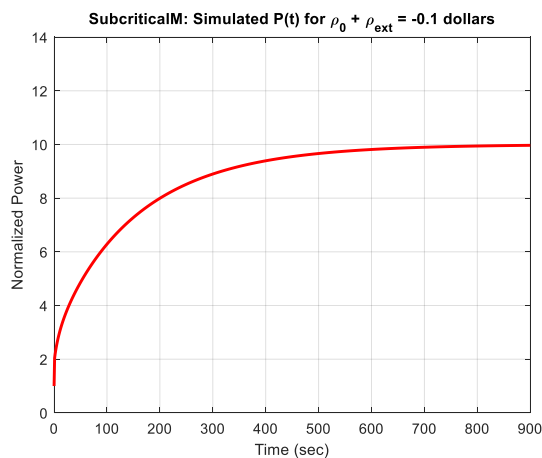
Of primary interest here, however, is the length of the time it takes to stabilize after the step change. As discussed in Refs. 5-6 and during the actual Approach to Critical experiment, the time needed to reach the new steady state gets much larger as one approaches a critical system -- because the delayed neutron effects become more important as one approaches the delayed-critical state. For the two simulation sets shown in Fig. 5, the initial subcriticality level was  $-1.0$  dollars and the reactivity perturbations were 0.5 dollars and 0.9 dollars, respectively, for the top and bottom set of plots. The first simulation which, at the end of the perturbation is still 0.5 dollars subcritical, stabilized completely in under 300 second. However, the second simulation, being only 0.1 dollars subcritical after the perturbation, required more than 600 seconds to reach equilibrium. Finally, we note that, although not illustrated directly here, as  $\rho_1 \rightarrow 0$  (i.e. very close to critical), the stabilization time can become quite long -- and this behavior is fully consistent with expectations and with the experimental observations made during our previous Approach to Critical lab.



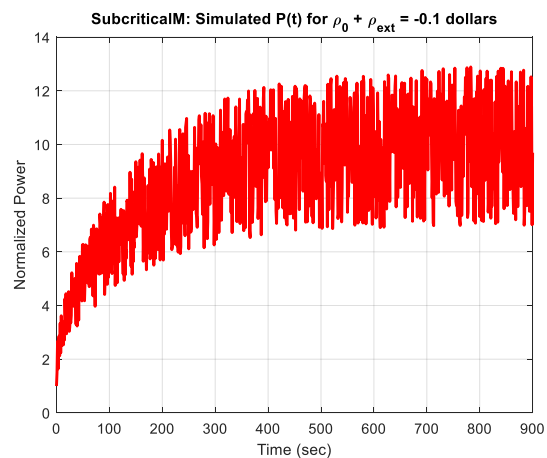
Subcriticality level (\$): -1.000  
 Actual delrho used (\$): 0.500  
 Measured delrho (\$): 0.500  
 Error in measured delrho (%): -0.002



Subcriticality level (\$): -1.000  
 Actual delrho used (\$): 0.500  
 Measured delrho (\$): 0.503  
 Error in measured delrho (%): 0.587



Subcriticality level (\$): -1.000  
 Actual delrho used (\$): 0.900  
 Measured delrho (\$): 0.899  
 Error in measured delrho (%): -0.098



Subcriticality level (\$): -1.000  
 Actual delrho used (\$): 0.900  
 Measured delrho (\$): 0.897  
 Error in measured delrho (%): -0.292

**Fig. 5 Measured results using simulated data for two different reactivity changes with and without a  $\pm 30\%$  noise component.**

## Summary

This set of Lecture Notes summarizes four experimental techniques for measuring reactivity levels or reactivity changes within actual reactor systems. The four methods combined give good coverage of most of the situations that can occur within real systems -- for both critical and subcritical configurations. In addition to the basic theory, actual use of the methods was also illustrated via simulation within a series of short Matlab codes so that the reader could better visualize how to apply these techniques in practical applications. The key take-aways from these

notes should be a good understanding of the various reactivity measurement techniques addressed here, and the confidence and knowledge of how to use these methods within subsequent experiments within the UMLRR...

## References

1. J. R. White, "One-Speed Point Kinetics Equations," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.
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3. J. R. White, "Solution of the Point Kinetics Equations: Step Change in Reactivity," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.
4. R. Skoda, "AGN-201M Exercises - Part 1 (Draft)", Texas A&M University (Dec. 2010).
5. J. R. White, "Understanding Subcritical Multiplication via an Approach to Critical Experiment," one of a series of labs for the Reactor Experiments course at UMass-Lowell.
6. J. R. White, "Subcritical Multiplication," part of a series of Lecture Notes for the Nuclear Engineering Program at UMass-Lowell.