Integral Worth Curves: Theory and Measurement Techniques

Introduction

The worth of the control devices within any reactor is one of the most important quantities that must be measured for each specific system. For the shutdown rods, only the total worth is needed since these are used primarily as safety devices and are either fully inserted (in shutdown mode) or fully withdrawn (during normal operation). However, for the control devices that are utilized for routine reactivity control during daily operations, full rod worth curves (or blade worth curves in the UMLRR) are required -- that is, it is essential that the reactivity worth versus position be established. In particular, the integral worth curve, $\rho_w(z)$ vs. z, represents the worth (i.e. $\Delta\rho$ from reference) of the control device as a function of axial location, where the reference $z = 0$ location can be taken at either the top or bottom of the active region of the core (or some similar location that establishes the fully inserted or fully withdrawn position). Within the UMLRR the four large control blades and the single low-worth regulating blade traverse a distance of about 25-26 inches (varies slightly for each blade), with $z = 0$ at about 0.75 inches below the active fuel. Since the reference position for the UMLRR is at the bottom of the core, the axial position of the control blades is usually given in units of inches withdrawn.

In this set of Lecture Notes, we will study how to develop and interpret the integral blade worth curves for the UMass-Lowell research reactor (UMLRR). The first step is to establish the basic concepts and terminology, and this will be done using 1-group theory for an idealized bare homogeneous reactor model. Although very simplistic, this theoretical treatment allows us to observe a typical worth profile and to establish some insight into what to expect in a real reactor. For real operating systems, however, the rod or blade worth curves are always established by actual measurement, and the real focus here is to discuss typical techniques for doing this. In particular, we will highlight three experimental methods for generating the desired worth profiles: the **Stable Period Method**, the **Inverse Count Rate Method**, and the **Inverse Kinetics Method**.

Within the UMLRR, the Stable Period Method (which is referred to locally as the Doubling Time Method) was the primary tool used by the operations staff for the first 35+ years of operation (from initial criticality in 1975 to late 2012). This basic technique for measuring reactivity is treated in some detail in Ref. 1 and this is also addressed as part of the Reactivity Measurements Lab within the Reactor Experiments course at UMass-Lowell (see Ref. 2). Thus, in the discussions here, we will simply apply this relatively well-established method for measuring a reactivity worth to the specific application of interest -- that is, for establishing the blade worth curves within the UMLRR.

On a similar note, we have also already discussed most of the background needed for the Inverse Count Rate Method, in that the basis of this method deals with subcritical multiplication, which is discussed in some detail in Ref. 3 and also within an experimental context in the first two labs within the Reactor Experiments course (Refs. 2 and 4). Thus, the goal here is to quickly establish the usual notation used (i.e. what is the origin of the "inverse count rate" notation) and the final equations that allow generation of the normalized worth profile, and then focus on providing a specific example of the method. Note that, although new data to test this method will not be generated explicitly as part of the subsequent Measuring Integral Blade Worth Curves... lab,[5] partial data was already generated as part of the Approach to Critical lab (see

Ref. 4) -- so the results from this lab could be used, if desired, to generate a partial blade worth curve profile for the blade of interest from the previous experiment.

Finally, concerning the Inverse Kinetics Method, since this tool is a relatively new technique at UMass-Lowell,[6] we will spend a little more time formally developing this technique and in illustrating how it can be applied within the context of developing the desired blade worth curves for the UMLRR -- and much of the physical lab time will be focused on applying this technique for measuring the actual worth profile of one of the UMLRR control blades.[5]

Theoretical Background

Before measuring any quantity in the lab it is important to have a good idea of the expected result that will be obtained. Thus, to establish this base understanding of what we mean by a "blade worth curve", we will first analytically develop the form of the worth profile for a simple idealized reactor configuration. In particular, using ***Perturbation Theory Methods*** (see brief overview in Ref. 7), it can be shown that the worth of a material inserted to an axial depth z within the reactor is proportional to the product of the forward and adjoint fluxes integrated over the perturbed domain. In particular, assuming 1-group theory and that movement of the control rod only perturbs the absorption cross section, we have

$$\rho_w(z) = \alpha \int_0^z \phi^*(z')\Delta\Sigma_a(z')\phi(z')\,dz' \qquad (1)$$

where α is a proportionality constant and ϕ^* is known as the adjoint flux or importance function.

However, since the 1-group diffusion equation is self-adjoint,[7] the adjoint and forward fluxes are identical, and eqn. (1) becomes

$$\rho_w(z) = \alpha \int_0^z \phi^2(z')\Delta\Sigma_a(z')\,dz' \qquad (2)$$

Now, for a bare 1-D homogeneous critical reactor of total height H, the flux profile is given by[8-9]

$$\phi(z) = A\sin Bz \qquad \text{with} \qquad B^2 = \left(\frac{\pi}{H}\right)^2$$

where z is measured from the top of the reactor (for simplicity, we have ignored the small extrapolation distance in this simple development). Finally, if the rod absorption cross section is constant, then combining the flux profile for a homogeneous system with eqn. (2) gives

$$\rho_w(z) = C\int_0^z \sin^2\frac{\pi z'}{H}\,dz' = C\left[\frac{z'}{2} - \frac{H}{4\pi}\sin\frac{2\pi z'}{H}\right]_0^z = C\frac{H}{2}\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right)$$

where C is just a new proportionality constant.

To evaluate this constant, we let $\rho_w(z)|_{z=H} = \rho_w(H) = \rho_{tot}$, which is the total rod worth. With this constraint we have

$$\rho_w(H) = C\frac{H}{2}(1-0) \qquad \text{or} \qquad C = \frac{2}{H}\rho_w(H)$$

and the so-called ***ideal integral worth distribution*** becomes

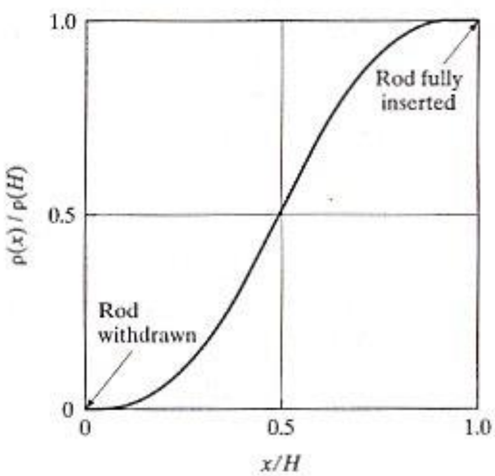
$$\rho_w(z) = \rho_w(H)\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right) \tag{3}$$

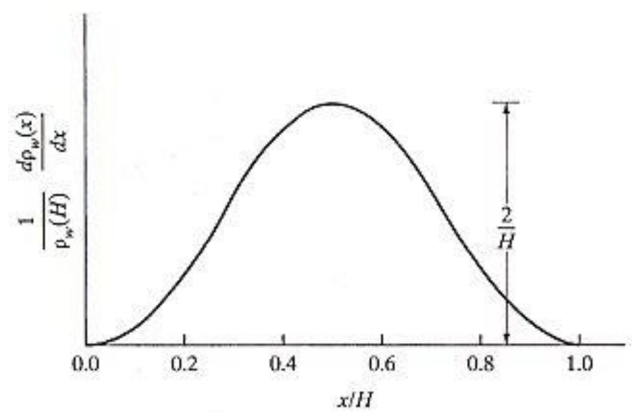
where $\rho_w(z)$ is the worth of a partially inserted rod to depth z [this is the relationship given in Chapter 7 of Lamarsh (i.e. Ref. 8) without much justification]. Finally, if one plots the relationship $\rho_w(z)/\rho_w(H)$, the ideal S-shaped normalized integral rod worth curve is obtained (as shown in the sketch below from Ref. 8).

Also of interest is the rate of change of $\rho_w(z)$ per unit distance. This differential worth can easily be obtained by differentiation of eqn. (3), or

$$\frac{d}{dz}\rho_w(z) = \frac{\rho_w(H)}{H}\left(1 - \cos\frac{2\pi z}{H}\right) \tag{4}$$

This function, when plotted, gives the familiar differential rod worth curve (as shown below in the sketch from Ref. 8).





In practice, of course, the integral and differential worth curves for real reactor systems differ somewhat from the ideal curves shown above (note that these were developed using first-order perturbation theory for a bare homogeneous 1-group system -- a pretty idealized situation indeed). However, they do give a good qualitative view of what to expect for a real system, with low differential worth near the upper and lower boundaries (where the flux and importance functions are relatively low) and a peak differential worth near the core center (where we expect the highest flux and the largest neutron importance). In a real reactor, if the control rods are inserted from the top, then the worth distribution often tends to be slightly bottom peaked, and it is slightly top peaked if the rods are inserted from the bottom (assuming, of course, that everything else is axially symmetric). However, to a rough first approximation, eqns. (3) and (4) and the sketches given above should help establish a reasonable set of expectations for the measurement of blade or rod worth curves for most real systems -- with the additional expectation that some asymmetry may be observed.

Stable Period Method

As noted in the Introduction section, the Stable Period Method has been the primary tool used by the operations staff for many years for measuring the blade worth curves within the UMass-Lowell research reactor (UMLRR). The basic theory behind the method is discussed in Refs. 1-2, an overview of the actual procedure used and a discussion of a Matlab-based **blade_worth** GUI that does the curve fits once the doubling time data are available is given in Ref. 10, and a detailed analysis of the data obtained from a sample experiment for Blade 4 back in 2005 is presented in Ref. 11. Thus, there is ample information and discussion of this method already readily available, so this material will not be repeated here. Instead, the reader is expected to review the references noted here to get a good overview of this experimental technique.

We should note however that, although the basic approach for the Stable Period Method is quite straightforward, it does pose some operational difficulties. First the method is actually rather time consuming in that it takes a minimum of 20-30 minutes to obtain the needed information for a single data point -- and typically, 10-12 data points should be taken to get a good fit. Thus, it takes 3-4 hours to obtain a good blade worth curve for each control device.

In addition, since the technique involves reactivity balancing at critical, it is sometimes difficult to get good coverage over the full range of the blade traverse. In particular, for the current UMLRR configuration, the excess reactivity is under 3 %Δk/k, but the total individual worths of Blades 3 and 4 are well over this amount, so the reactor cannot be made critical if either Blade 3 or 4 is fully inserted into the core -- which means that it is not possible to get experimental data at the lower end of the blade worth curve (i.e. the near full-insertion region). On the other side of the curve, when one of the high-worth blades is nearly fully withdrawn, it is also hard to balance the reactivity swing, since this situation leads to relatively large radial and axial flux tilts -- which are also undesirable. Thus, the Stable Period Method is certainly not ideal, but it has indeed served the reactor staff sufficiently well for many years...

As a simple self-contained illustration of the Stable Period Method (without the limitations associated with a real system as noted above), we will assume a blade worth profile, use this to excite the point kinetics representation of the system, extract doubling time information for several blade movements, compute the differential worths, do a curve fit to the data, and finally integrate this expression to get the integral worth curve to see if we can create the original profile from the simulation data. All this processing is completed within the **bw_stable_period.m** Matlab program and the summary results are illustrated in Fig. 1. Here we see that the differential curve fit matches the simulated data almost perfectly, and that the integral curve generated from this mathematical representation is almost identical to the actual blade worth curve for Blade 3 that was used to generate the simulated data. Thus, this simple test does indeed show that the method works as expected (as was shown previously in Refs. 10 and 11).

Finally we note that the mathematical model for the differential worth curve shown in Fig. 1 is given by

$$\frac{d}{dz}\rho_w(z) = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 \cos\left(\frac{2\pi z}{H}\right) \qquad (5)$$

where z is the distance withdrawn and H is the maximum blade traverse. This distribution allows modeling the slightly bottom peaked differential worth profile that is observed for the UMLRR control blades. Integrating this expression gives the integral worth curve,

$$\rho_w(z) = c_1 z + \frac{1}{2} c_2 z^2 + \frac{1}{3} c_3 z^3 + \frac{1}{4} c_4 z^4 + c_5 \frac{H}{2\pi} \sin\left(\frac{2\pi z}{H}\right) \tag{6}$$

Equations (5) and (6) represent the mathematical models that have been used for many years when working with the doubling time or stable period method within the UMLRR (see Refs. 10 and 11, for example).

However, we should emphasize that, more recently, especially when using the inverse kinetics method (see below), the base formulation for the reactivity versus distance is written with the integral form as the reference model as

$$\rho(z) = c_1 z^4 + c_2 z^3 + c_3 z^2 + c_4 z + c_5 + c_6 \sin\left(\frac{2\pi z}{H}\right) \tag{7}$$

and the actual worth of the integral blade worth is given as

$$\rho_w(z) = \rho(z) - \min\left(\rho(z)\right) \tag{8}$$

which, in most situations, is simply eqn. (7) with $c_5 = 0$. However, when eqn. (7) is fit to noisy experimental data, there are sometimes a few small negative values that occur -- and the combination of eqns. (7) and (8) is one way to remove the negative entries and make the integral worth curve positive everywhere. The reader should be careful not to mix the formulations given by eqns. (5) and (6) and by eqns. (7) and (8), since clearly they represent two completely different reference situations. Equations (5) and (6) have been traditionally used at the UMLRR to represent the so-called sinusoid + polynomial model used with the Doubling Time Method, and eqns. (7) and (8) are used with the relatively new Inverse Kinetics Method (see below) -- thus, the user should be aware that a given set of blade worth coefficients are only valid for the model for which they were generated. Clearly the equation constants generated for eqns. (5) and (6) are not applicable for eqns. (7) and (8). So be careful here...

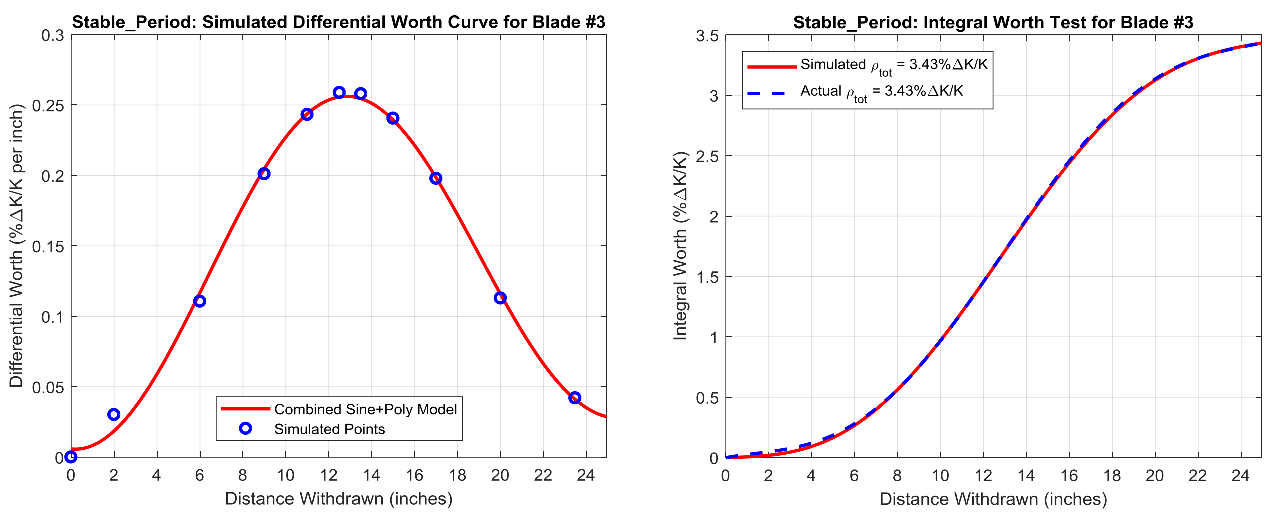


Fig. 1 Simulated differential and integral worths and comparison to actual data.

Inverse Count Rate Method

This method has its basis in the subcritical multiplication factor techniques that have already been discussed in previous labs.[1-4] The method can be developed easily with just a few definitions:

ρ_{in} = subcriticality level with the blade of interest fully inserted

ρ_{out} = subcriticality level with the blade of interest fully withdrawn

$\rho_{tot} = \rho_{out} - \rho_{in}$ = total worth of the control rod or blade

Now, we can define the worth of the blade at any z location as

$$\rho_w(z) = \Delta\rho(z) = \rho(z) - \rho_{in} \tag{9}$$

and multiplying by unity in the form of $\rho_{tot}/(\rho_{out} - \rho_{in})$ gives

$$\rho_w(z) = \left(\rho(z) - \rho_{in}\right)\frac{\rho_{tot}}{\rho_{out} - \rho_{in}} = \rho_{tot}\left(\frac{\rho(z) - \rho_{in}}{\rho_{out} - \rho_{in}}\right) \tag{10}$$

But we know that, with some assumptions about the proportionality constant not changing much from one configuration to another, we have

$$M_r = \frac{C_1}{C_0} \approx \frac{\alpha_1}{\alpha_0}\frac{\rho_0}{\rho_1} \approx \frac{\rho_0}{\rho_1} \qquad \text{or} \qquad \frac{C(z)}{C_{in}} \approx \frac{\rho_{in}}{\rho(z)} \qquad \text{or} \qquad \rho(z) \approx \frac{C_{in}}{C_z}\rho_{in} \tag{11}$$

Thus, eqn. (10) becomes

$$\rho_w(z) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z}\rho_{in} - \rho_{in}}{\frac{C_{in}}{C_{out}}\rho_{in} - \rho_{in}}\right) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z} - 1}{\frac{C_{in}}{C_{out}} - 1}\right) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z} - \frac{C_{in}}{C_{in}}}{\frac{C_{in}}{C_{out}} - \frac{C_{in}}{C_{in}}}\right)$$

or

$$\rho_w(z) = \rho_{tot}\left(\frac{\frac{1}{C_z} - \frac{1}{C_{in}}}{\frac{1}{C_{out}} - \frac{1}{C_{in}}}\right) \tag{12}$$

Therefore, the blade worth profile can be determined by simply measuring the detector count rate for several blade axial locations denoted by the z subscript or, more precisely, in terms of the inverse detector count rate -- and, of course, this is where the name "Inverse Count Rate Method" comes from.

The biggest advantage associated with this method is that it is done at subcritical conditions, where the time to reach stable operation between data points is quite reasonable (relative to the stable period method that requires returning to critical by rebalancing the blades after each point). Thus the Inverse Count Rate method is attractive since it can be performed in a timely manner.

However, it has several disadvantages, the most important being that it only gives the relative profile, not the absolute worth [i.e. ρ_{tot} in eqn. (12) must be available from some other method].

In addition, the method is only approximate because of the assumption that the configuration-dependent proportionality factors cancel. Although the assumptions associated with eqn. (11) are often quite good between neighboring configurations that involve small changes (i.e. between configuration i and i+1), it does often introduce some error from the first to last arrangement (that is, assuming that $\alpha_{in} \approx \alpha_{out}$ is probably much less accurate than saying $\alpha_{i+1} \approx \alpha_i$). Finally, within the UMLRR, we know that the startup counter is rather noisy, and this also introduces issues in using this method to determine accurate blade worth curves for the system.

As an illustration of the method, a simulation of a blade being withdrawn in several distinct steps within a subcritical system has been programmed into the **bw_inverse_rate.m** Matlab code. For each blade withdrawal sequence, the code solves the point kinetics equations at subcritical, stores the power data after stabilization, and then uses eqn. (12) to determine the normalized worth, $\rho_w(z)/\rho_{tot}$, at each discrete z location. Finally, a linear least squares curve fit using the mathematical model given in eqn. (7) is performed and the results are compared to the raw data points and to the actual integral reactivity curve used to initiate the transients and to generate the simulated data.

In particular, the results from a specific case with Blade 3 with an initial subcriticality level of −5 dollars are shown in Fig. 2. The blade position profile, z(t), is shown on the left side of the plot along with the simulated $P(t)/P_o$ or $CR(t)/CR_o$ behavior -- and the trends seen here are exactly as expected for subcritical reactivity changes. The CR data at the end of each interval was stored along with the blade location after step i (after stabilization), and these data were used to compute the normalized reactivity worth vs. position via eqn. (12) [i.e. $\rho(z_i)$ vs. z_i]. The right side of Fig. 2 shows these data points along with the best curve fit using eqn. (7) and the actual blade worth curve used to generate the simulations. Clearly, as apparent from Fig. 2, all three profiles are nearly identical -- demonstrating that the procedure and basic methodology are sound. Recall, however, that even though great results are obtained here (within a simulated environment), the best we can do is to get a normalized profile, since it is not possible to get the absolute $\rho_w(z)$ behavior with the Inverse Count Rate method (i.e. need ρ_{tot} to get the absolute worth curve). Thus, even with a perfect scenario, this method can only give the shape of the integral blade worth curve...

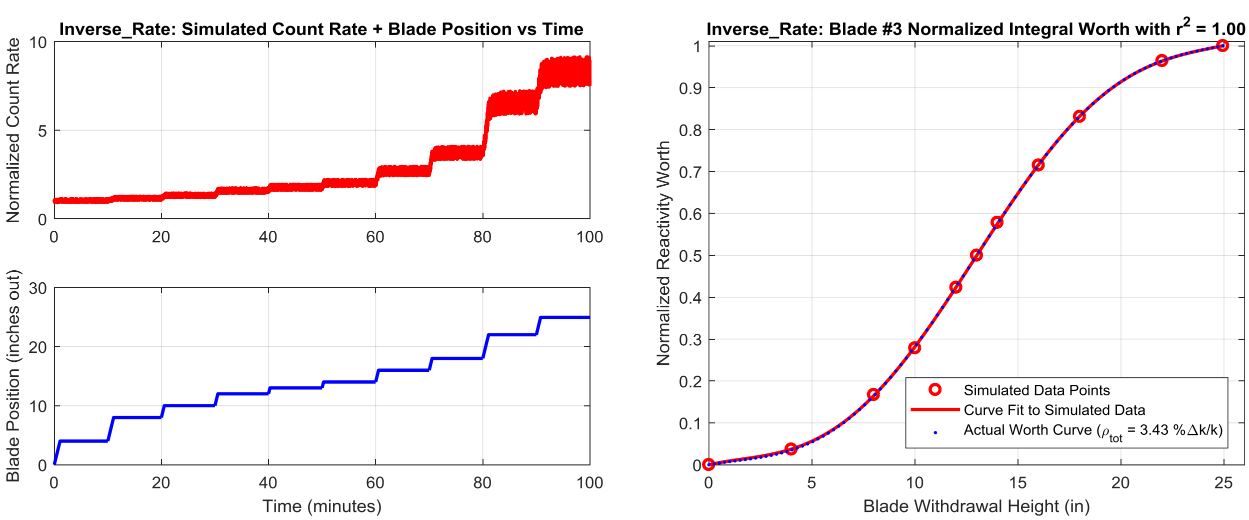


Fig. 2 Results from simulation of the subcritical Inverse Count Rate method.

Inverse Kinetics Method

The details of the Inverse Kinetics Method are given in a separate set of Lecture Notes (see Ref. 12) and the reader is expected to study these notes thoroughly to get a good understanding of the basic methodology. Simply stated, in the inverse kinetics problem, the usual signal flow path is reversed -- that is, given the observed power vs. time behavior, P(t), as the known "input", we want to compute the "output" ρ(t) -- and this fits in quite nicely with our goal here of measuring blade worth curves.

One complication, however, is that at low reactivity levels, there is a "drift" in the ρ(t) prediction due to gamma interference within the power detectors within the UMLRR -- and this dictates that the deviation from critical should be held within about ± 0.4 %Δk/k and the power swing, especially on the low side, should not be much greater that a factor of 10-20 below the reference critical value (see Refs. 6 and 12 for further discussions on this subject). Thus, for applications involving the measurement of the full blade worth curve, the negative reactivity addition associated with the blade of interest being inserted into the core must be balanced by the other blades in the system -- and the lab procedure discussed in Ref. 5 summarizes this process quite nicely.

As a specific example, we illustrate this procedure by simulating the measurement of the integral worth profile for Blade 3 while compensating for the negative reactivity addition as Blade 3 is inserted by moving Blade 4 out of the core. This sequence of inserting Blade 3 and removing Blade 4 is done in several small steps so that the subcriticality level does not violate the above general guidelines of staying within ± 0.4 %Δk/k of critical. The basic procedure for a specific set of Blade 3 inward and Blade 4 outward movements is implemented within the first part of the **bw_inverse_kinetics.m** routine, and the results of this simulation are summarized in Fig. 3.

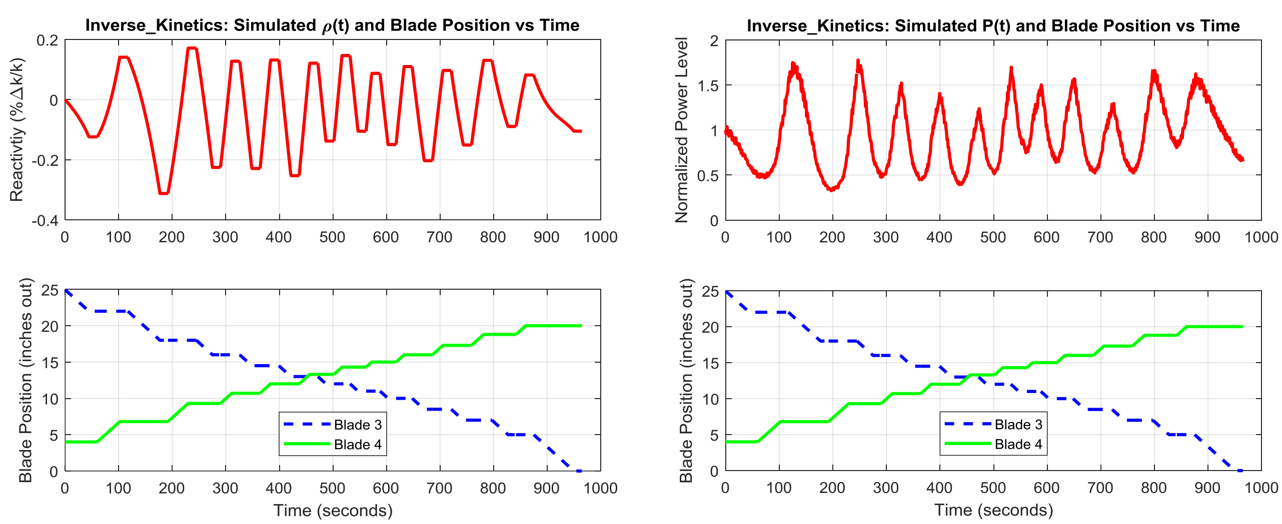


Fig. 3 Reactivity and power vs. time profiles for a specific set of blade movements.

On the left side of the figure is the actual blade positions vs. time and the corresponding reactivity associated with these blade movements, where the ρ(t) profile is simply obtained from the known blade worth curves for the two control devices. With an input z(t) or ρ(t), the point kinetics equations (with no feedbacks) can be solved to produce the power vs. time behavior, P(t), shown on the right side of Fig. 3, where each decrease and increase in power corresponds to the insertion of Blade 3 and removal of Blade 4, respectively. The idea here was to balance the magnitude and timing of the blade movements so that the P(t) profile stays within a reasonable range of the reference P_o and that the absolute reactivity level is maintained within ± 0.4 %Δk/k of critical -- and Fig. 3 clearly shows that these goals have been achieved with the specific blade positions used in the simulation.

Now, with a simulated P(t) signal to represent the real reactor power profile, we simply pass this through the **invkin_sr.m** routine discussed in Ref. 12 to solve for the ρ(t) that produced the observed P(t) behavior -- and this is the whole idea of the Inverse Kinetics Method [that is, by reversing the usual input-output relationship, we can determine the input that caused a specific output]. For this simulation, the computed or predicted reactivity that corresponds to the P(t) profile shown in Fig. 3 is displayed in Fig. 4 along with the actual input ρ(t) that produced the transient simulation. Here we have simulated a noisy P(t) signal by adding a ±5% random noise component and, clearly, the resultant ρ(t) profile reflects (and even magnifies) this noisy behavior. However, even with the noise component, one can clearly see that the "measured" ρ(t) signal follows the "input" reactivity quite nicely -- showing that the inverse kinetics routine has done its job remarkably well.

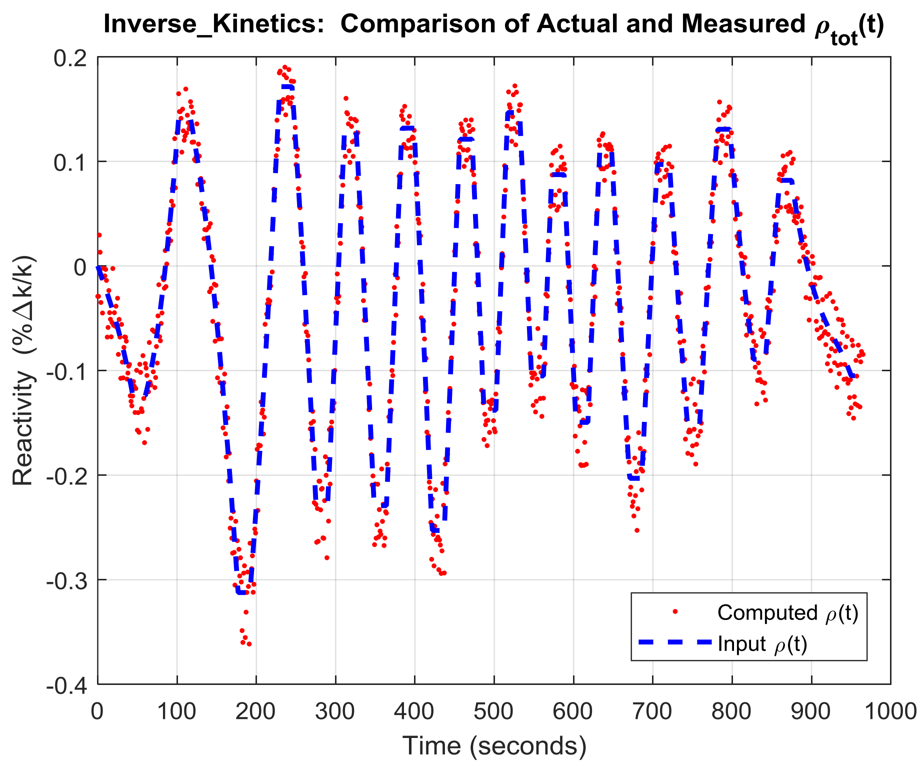


Fig. 4 Comparison of the computed and actual input reactivity from the bw_inverse_kinetics demo.

The $\rho(t)$ profile displayed in Fig. 4 is the total reactivity due to the movement of both Blades 3 and 4. However, of interest in a blade worth calibration is the worth of only the blade of interest (BOI). As detailed in Ref. 6, T. P. Michaud developed a **rebank_adjust** routine that does exactly what is needed here -- that is, to isolate the reactivity change due only to the BOI. The concept here is actually pretty straightforward in that we assume that only one blade moves at a time, and that any reactivity change during that time is associated with the blade that is currently moving. In this way, the **rebank_adjust** routine collects the reactivity changes for the BOI and creates a separate $\rho_{BOI}(t)$ profile that is associated with the $z_{BOI}(t)$ movement. For the current simulation, the total and BOI reactivities are shown in Fig. 5, where the $\rho_{BOI}(t)$ profile is simply a composite of all the negative reactivity insertions for this case (notice that, when the BOI is not moving, $\rho_{BOI}(t)$ is constant).

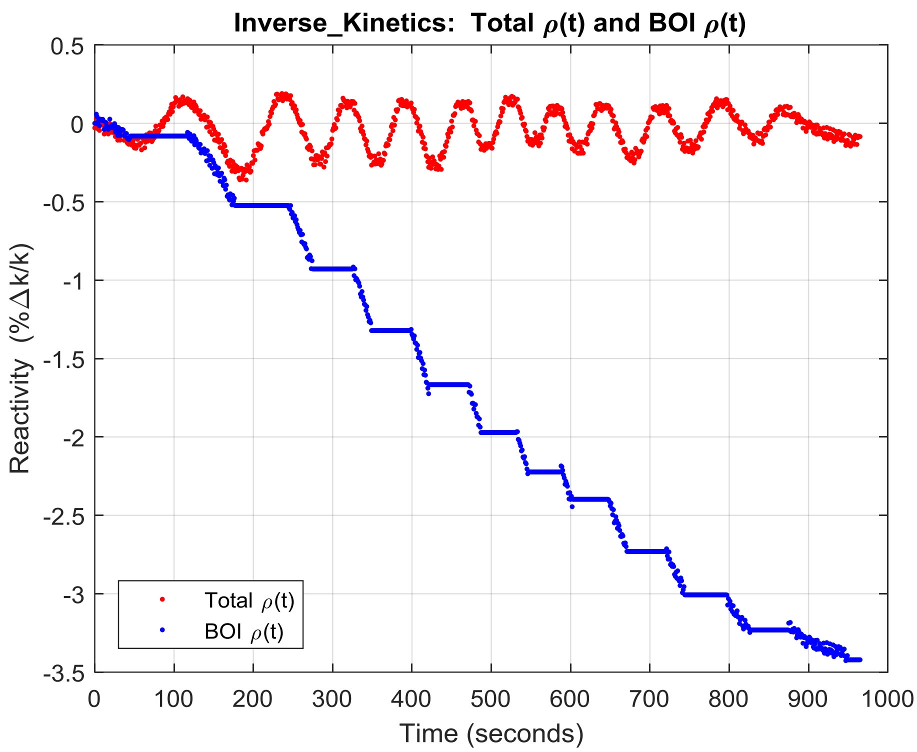


Fig. 5 Reactivities before and after application of the rebank_adjust algorithm.

Well, with the "measured" $\rho_{BOI}(t)$ behavior and $z_{BOI}(t)$ locations, one has all the information needed to generate the desired blade worth curve for the blade of interest (BOI). A little filtering is done to remove the repetitive entries when the BOI is not moving, and then the vectors containing the $\rho(t_i)$ and $z(t_i)$ pairings are sent to a linear least squares curve fit routine, which returns the final $\rho_w(z)$ vs. z integral blade worth curve based on the mathematical model given in eqn. (7) -- and this was the ultimate goal here. The coding to accomplish this final task within the **bw_inverse_kinetics** routine was taken directly from the **umlrr_data** GUI that performs a similar function on the measured reactor data. The result of this final processing step is shown in Fig. 6 along with the actual blade worth curve used in producing the simulated data. Clearly, the two curves are very similar, indicating that the overall procedure is sound and that it has been properly implemented within this example simulation (note that, if the noise level is set to zero, the two blade worth curves are nearly identical). With this validation, this example is complete -- and it has demonstrated, quite clearly, the basic approach that is used to measure the actual blade worth curves within the UMLRR using the Inverse Kinetics Method.

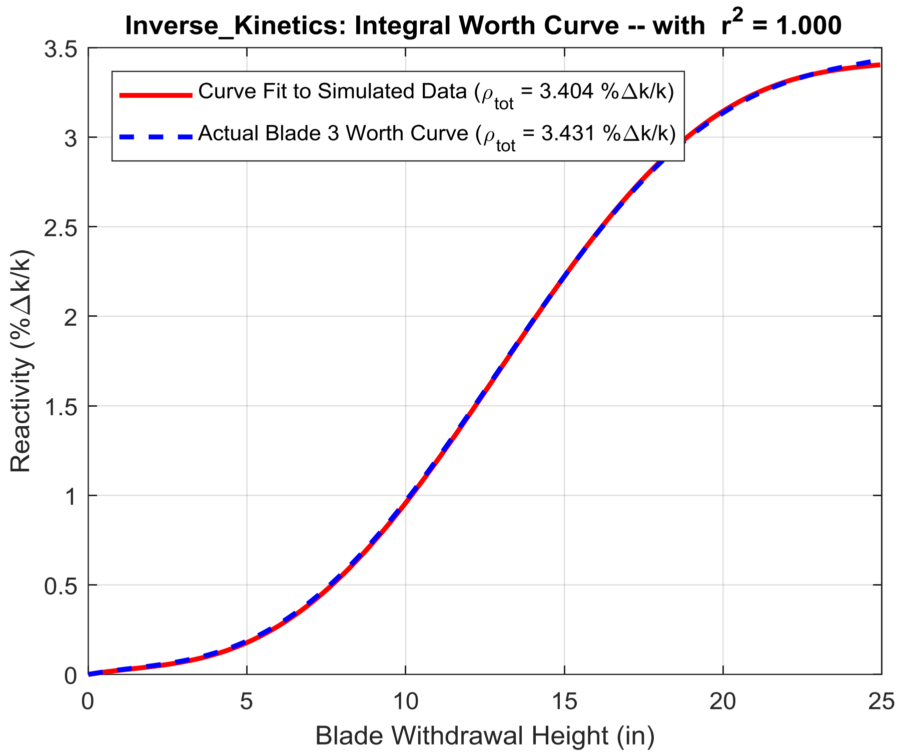


Fig. 6 Comparison of the "measured" and actual Blade 3 worth curves.

Summary

This set of Lecture Notes first introduces the theoretical concepts needed for the discussion of blade worth curves, and it then summarizes three techniques for actually measuring the blade worth profiles within a real system. The three experimental methods -- **Stable Period Method**, **Inverse Count Rate Method**, and **Inverse Kinetics Method** -- are each demonstrated via simulation within a sequence of Matlab codes. These simulations show the basic procedures involved and they validate that each of the methods are theoretically sound and have been implemented correctly. Now, all that remains is to apply these methods using real reactor data (instead of simulated data) -- and this is done within a formal lab as part of the Reactor Experiments course at UMass-Lowell.[5] The key take-aways from the discussion and examples given here should be a good understanding of the theoretical basis for the ideal differential and integral worth curves seen in the literature, and for the various reactivity measurement techniques that can be used to determine these worth profiles in real systems.

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