The solution of the **Generation Time Formulation** of the point kinetics equation with no reactivity feedback and no external source for a **step change** in external reactivity (step change in k) is typically given in terms of the so-called **reactivity equation**. For six delayed neutron groups using the generation time formulation, the reactivity equation can be written as

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$

Now, your job for this problem is to explain, in detail, your understanding of point kinetics via a thorough discussion of the **reactivity equation**. In your discussion you should address such things as:

What does this expression mean and where does this come from (a formal derivation is not required here -- a good explanation of the general process will suffice)?

Discuss its interpretation in terms of the actual time dependent behavior of the neutron density for both positive and negative reactivity insertions.

How many roots are there? What is the sign of the roots for both positive and negative reactivity? What is the significance of the dominant root?

Be sure to introduce the concepts of reactor period and prompt jump/drop in your discussions. Use appropriate sketches as needed.

For 6 delayed neutron groups, the solution to the point kinetics eqns for a step change in ρ gives a solution of the form

$$T(t) = \sum_k A_k e^{\omega_k t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \cdots + A_7 e^{\omega_7 t}$$

This comes about because we have 7 coupled linear constant coeff ODEs — and the general solution is the linear combination of the seven linearly independent individual solutions. Also, since the system is linear and constant coeff, each of the solutions is just a simple exponential $\sim e^{\omega t}$

When the simple form $e^{\omega t}$ is substituted into the point kinetics eqn, we get the characteristic eqn — which here we call the reactivity eqn.

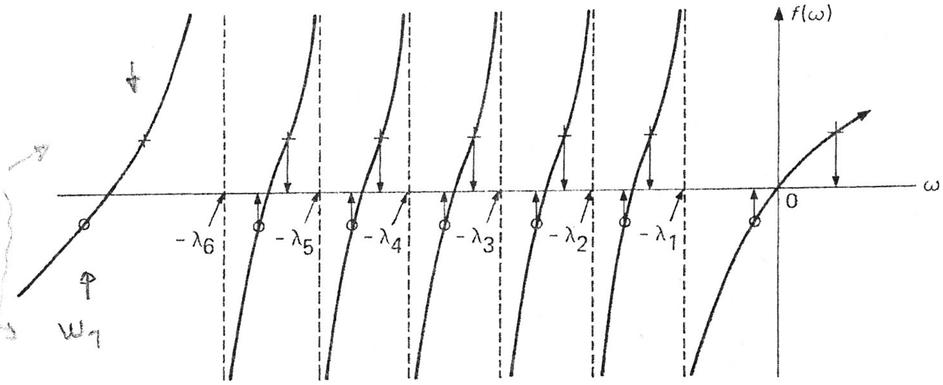
The roots of the reactivity eqn are the desired ω_k values in the above sum for $T(t)$.

A sketch of the RHS of the reactivity eqn can help visualize the location of these values.

(see next page)

not to scale

w_7 is a much larger negative number than all the other w values



Here we see that $f(\omega) \to \pm\infty$ at the values of $-\lambda_k$ and between each asymptote there is a root to the reactivity eqn for either positive or negative reactivity (+ are roots for $+\rho$ and ⊙ are roots for $-\rho$).

The most positive root is for $\omega > -\lambda_1$, and this root is denoted ω_1 and its reciprocal is referred to as the reactor period

$$\tau = \frac{1}{\omega_1}$$

{ ω_1 is the dominant root in that all the others decay away more rapidly

Note that

$$+\rho \Rightarrow \begin{cases} \omega_1 = \text{pos} \\ \omega_2 - \omega_7 = \text{neg} \end{cases} \Rightarrow \text{growing exponential after initial transient}$$

$$-\rho \Rightarrow \begin{cases} \omega_1 = \text{neg} \\ \omega_2 - \omega_7 = \text{neg} \end{cases} \Rightarrow \text{decaying exponential after initial transient}$$

$$P(t) = P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau}$$

where P_1 is the normalized amplitude (or power) after the initial transient and τ is the reactor period (+ for pos ρ and − for neg. ρ).

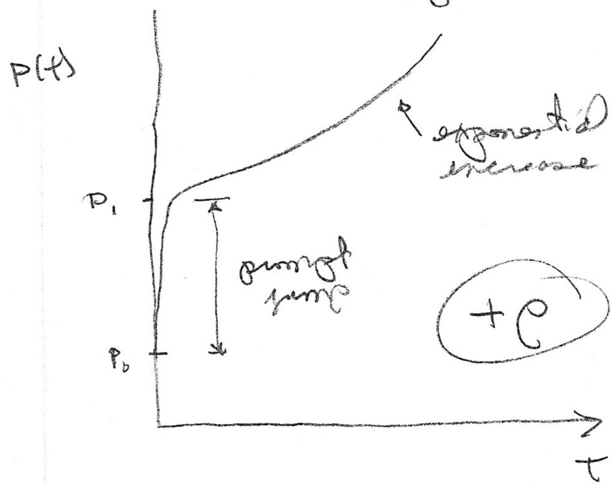
Because ω_7 is such a large negative value, the term containing $e^{\omega_7 t}$ decays to zero very rapidly. This term is associated with the prompt neutrons and this very fast transient is often referred to as the prompt jump/drop.

↑ $+\rho$ ↑ $-\rho$

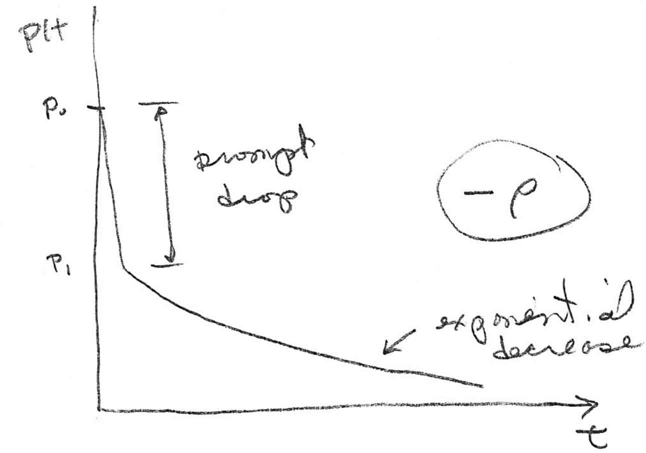
The typical $P(t)/P_0$ profiles for both positive and negative step changes in reactivity are sketched below

→ note the prompt jump / drop } depending on the sign of ρ

→ exponential increase / decay



$$P(t) = \left(\frac{P_1}{P_0}\right) P_0 \, e^{t/\tau}$$



$$P(t) = \left(\frac{P_1}{P_0}\right) P_0 \, e^{-t/\tau}$$

Problem 2: Using the **kinetics_gui** program create a plot of reactor period, τ, vs. reactivity, ρ, for U235 fuel with β = 0.0068 and Λ = 0.1 msec.

a. From this plot, estimate τ for reactivities of +0.14 %Δk/k and -2.0 dollars.

b. If a reactor period of about 5 minutes is observed in a low-power facility, estimate the multiplication factor, k, for this system.

(a) $\rho = \frac{0.0014}{0.0068} = 0.206\,\$ \rightarrow \boxed{\tau \simeq 30\text{ s}}$

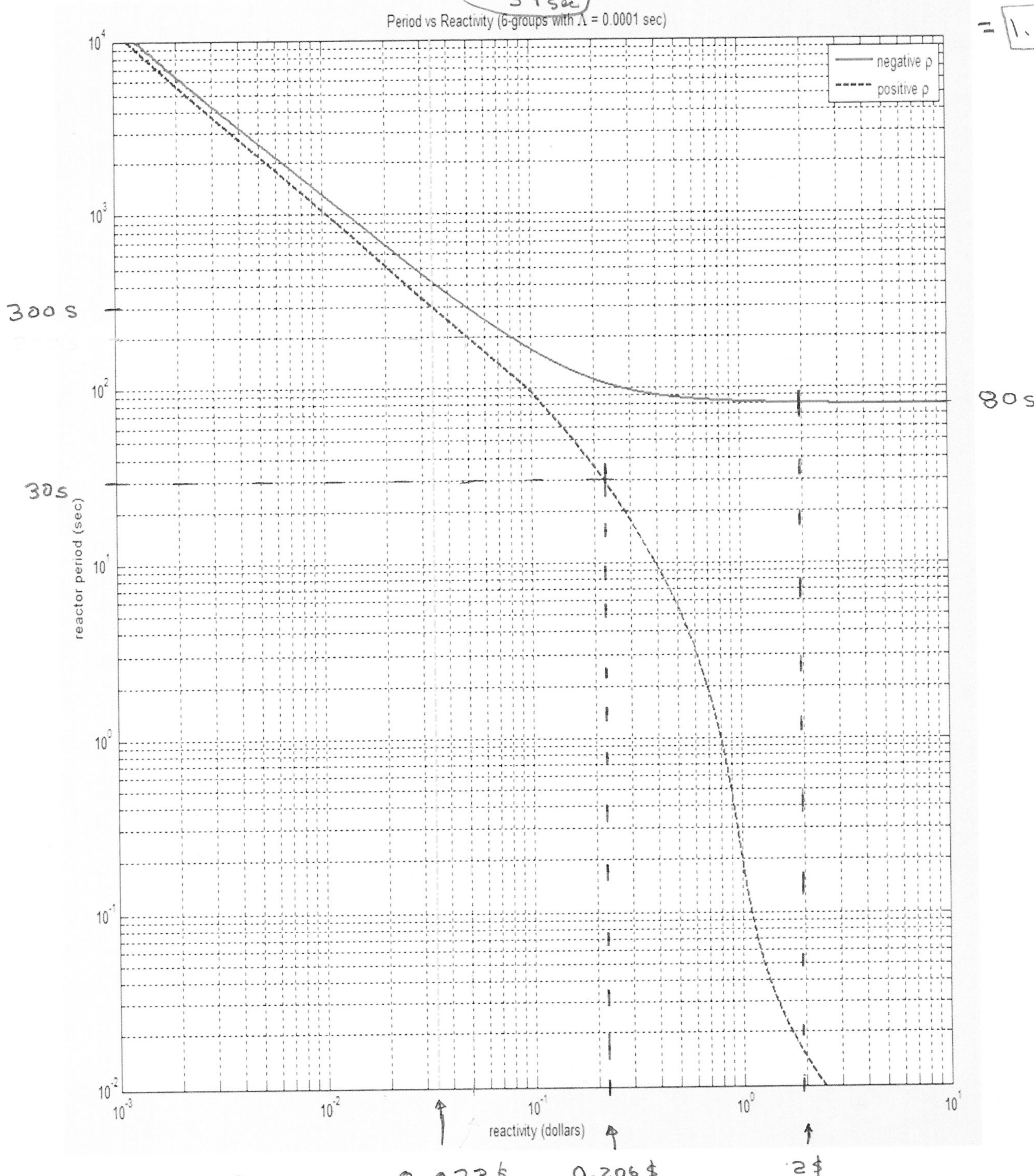
actually closer to 34 sec

$\rho = -2\,\$ \rightarrow \boxed{\tau \simeq -80\text{ s}}$

(b) $T = 5\text{ min} = 300\text{ s}$

$\rho = 0.033\beta = 0.033(0.0068)$

$\boxed{\rho = 0.00022}$ $\quad k = \frac{1}{1-\rho} = \boxed{1.00022}$



Note: Reading values from a log-log plot is very approximate at best ...

Problem 3: A U235-fueled thermal system with $\beta = 0.0068$ and $\Lambda = 0.1$ msec originally operating at a constant power of 500 W is placed on a positive period of 1.5 minutes. Estimate how long it takes for the reactor power level to reach 250 kW? State any assumptions…

Note: From the Kinetics_gui code, a reactor period of 1.5 min = 90 s is associated with a reactivity change of about 0.1\$

the prompt jump for 10¢ of reactivity is pretty small, but we can estimate the effect as follows

$$\frac{P_1}{P_0} = \frac{\beta}{\beta - \rho} = \frac{1}{1-\rho/\beta} = \frac{1}{1-0.1} \doteq 1.11$$

$$\therefore \frac{P(t)}{P_0} = \frac{P_1}{P_0} e^{+t/\tau}$$

$$\frac{P(t)/P_0}{P_1/P_0} = e^{t/\tau}$$

solving for t gives

$$\boxed{t = \tau \ln \frac{P(t)/P_0}{P_1/P_0}}$$

$\therefore$ for $T = 90$ s, the time it takes to go from 500 W = 0.5 kW to 250 kW is

$$t = (90\,s) \ln\left[\frac{(250/0.5)}{1.11}\right]$$

$$= (90)(6.1) = \boxed{550\,s} \simeq \boxed{9.2\text{ min}}$$

Note that if we ignored the prompt jump, then $P_1/P_0 = 1.0$ and the time estimate changes to

$$t = 90 \ln\left[\frac{250/0.5}{1.0}\right]$$

$$= 90\,(6.21) = \boxed{559\,s} \simeq \boxed{9.3\text{ min}}$$

Thus, the prompt jump does not affect this by much…

Problem 4: A U235-fueled system is operating at a steady state fission power level of 5 MW. The system is scrammed by the instantaneous insertion of 3 dollars of negative ρ. Ignoring feedback effects, estimate how long it takes for the reactor fission power level to reach 10 W.

From the kinetics.gui code (with $\beta = 0.0068$ and $\Lambda = 0.1$ msec), $\rho = -3\$$ give $\tau = -80$ s

For this case, the prompt jump is given by

$$\frac{P_1}{P_0} = \frac{\beta}{\beta - \rho} = \frac{1}{1 - \rho/\beta} = \frac{1}{1-(-3)} = \frac{1}{4}$$

$$\therefore \boxed{\frac{P_1}{P_0} = 0.25}$$

$$\therefore \frac{P(t)}{P_0} = \frac{P_1}{P_0} e^{-t/\tau}$$

and solving for t gives

$$\ln\left[\frac{P(t)/P_0}{P_1/P_0}\right] = -t/\tau$$

or $$\boxed{t = -\tau \ln\left[\frac{P(t)/P_0}{P_1/P_0}\right]}$$

here we have $\tau = -80$ s $\quad \frac{P(t)}{P_0} = \frac{10\text{ W}}{5\times10^6\text{ W}} = 2\times10^{-6}$

$$\therefore t = -80 \ln\left(\frac{2\times10^{-6}}{0.25}\right)$$

$$= -80(-11.74) = \boxed{939\text{ s}} = \boxed{15.6\text{ min}}$$

ans

Note here that if we ignore the prompt jump, then

$$t = -80 \ln\left(\frac{2\times10^{-6}}{1}\right)$$

$$= -80(-13.12) = 1050\text{ s} = \boxed{17.5\text{ min}}$$

This is too large

∴ the prompt drop is needed here.

Problem 5: A U235-fueled research reactor is critical with an operating power level of 6 kW. As a demonstration experiment for a reactor theory class, the reactor is put on a positive power transient with the insertion of 10 cents of positive reactivity. As expected, the power level increases for several minutes, and then it eventually stabilizes at a new power level of about 84 kW.

a. With this information, estimate the power coefficient of reactivity for this system.

b. If the maximum licensed power level of the reactor is 200 kW, what is the upper limit for reactivity that can be added in this experiment for the same initial state?

(a) total reactivity = external ρ + feedback ρ

or $\rho_{TOT} = \rho_{ext} + \alpha_P (P - P_o)$

at equilibrium $\Rightarrow$ critical $\Rightarrow$ $\rho_{TOT} = 0$

$\therefore \quad \alpha_P (P_\infty - P_o) = -\rho_{ext}$

or $\boxed{\alpha_P = \frac{-\rho_{ext}}{P_\infty - P_o}}$

here $\rho_{ext} = 10¢ = 0.10\,(0.0068) = 6.8 \times 10^{-4} \frac{\Delta k}{k}$

$P_\infty - P_o = 84\text{ kW} - 6\text{ kW} = 78\text{ kW}$

$\therefore \quad \alpha_P = -\frac{6.8 \times 10^{-4}\ \Delta k/k}{78\text{ kW}}$

$\boxed{\alpha_P = -8.72 \times 10^{-6} \frac{\Delta k/k}{\text{kW}}}$ ans

(b) For max power, $P_\infty = 200\text{ kW}$, then rearranging the above eqn gives

$\rho_{ext}\big|_{max} = -\alpha_P (P_{\infty max} - P_o)$

$= 8.72 \times 10^{-6} \frac{\Delta k/k}{\text{kW}} (200 - 6)\text{ kW}$

$= \boxed{1.69 \times 10^{-3}\ \Delta k/k}$ ans

or $\frac{\rho_{ext}\big|_{max}}{\beta} = \frac{0.00169}{0.0068} = 0.249\ \$$

$\therefore \boxed{\rho_{ext}\big|_{max} \simeq 25¢}$ ← I wouldn't go much above 20¢ to be on the safe side