24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Inverse Kinetics

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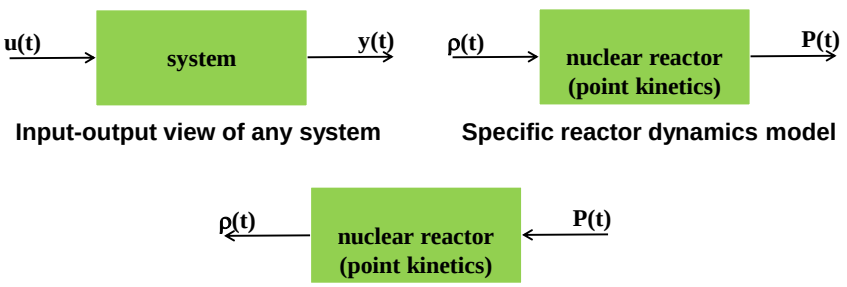
Inverse Point Kinetics



Input-output view of any system **Specific reactor dynamics model**

Input-output view for the inverse reactor dynamics problem

In the inverse problem, the signal flow is reversed -- that is, given the observed power vs. time behavior, P(t), as the known "input", we want to compute the "output" ρ(t). This perspective is quite different in that we put on our "detective hat" and by observing some measureable system behavior, we try to determine what actually caused the observed response. This is the goal of all inverse problems...

Inverse Point Kinetics Equations

Our starting point for the development of Inverse Point Kinetics is the Generation Time Formulation of Point Kinetics,

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \qquad \text{for } i = 1, 2, \cdots 6$$

In the usual forward treatment, time, t, is the independent variable, $\rho(t)$ and $\langle Q(t)\rangle$ are the system inputs, and the power level, P(t) is the desired output.

However, for inverse kinetics, we reverse the roles of $\rho(t)$ and P(t) -- where now P(t) is measured and our goal is to determine the $\rho(t)$ that led to the currently observed P(t) behavior.

Thus, the goal here is to solve this set of seven coupled ODEs for the reactivity, $\rho(t)$, given a measured P(t) profile.

Inverse Point Kinetics Equations (cont.)

To accomplish this goal, we solve for the normalized precursor concentration, then substitute this into the power equation, and eventually solve the resultant expression for $\rho(t)$.

First, rearrange the precursor equation to put it into standard form for solution via the integrating factor method,

$$\frac{d}{dt}c_i(t) + \lambda_i c_i(t) = \frac{\beta_i}{\Lambda}P(t)$$

and multiplication by the integrating factor $e^{\lambda_i t}$ gives

$$e^{\lambda_i t}\left(\frac{d}{dt}c_i(t) + \lambda_i c_i(t)\right) = \frac{d}{dt}\left(e^{\lambda_i t}c_i(t)\right) = \frac{\beta_i}{\Lambda}e^{\lambda_i t}P(t)$$

Inverse Point Kinetics Equations (cont.)

Now, multiply both sides by dt and integrate over discrete time interval t_{j-1} to t_j, to give

$$\int_{t_{j-1}}^{t_j} d\left(e^{\lambda_i t} c_i(t)\right) = \frac{\beta_i}{\Lambda} \int_{t_{j-1}}^{t_j} e^{\lambda_i t} P(t)\, dt$$

or

$$c_i(t_j) = e^{-\lambda_i \Delta t} c_i(t_{j-1}) + \frac{\beta_i}{\Lambda} e^{-\lambda_i t_j} \int_{t_{j-1}}^{t_j} e^{\lambda_i t} P(t)\, dt$$

$\Delta t = t_j - t_{j-1}$ is the sampling time

With respect to the integral in the previous equation, we use Simpson's 1/3 Rule -- which applied over an interval a ≤ x ≤ b is given by

$$\int_a^b f(x)\, dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Inverse Point Kinetics Equations (cont.)

Using this general result for the integral in the c_{ij} equation gives

$$\int_{t_{j-1}}^{t_j} e^{\lambda_i t} P(t)\, dt \approx \frac{\Delta t}{6}\left[e^{\lambda_i t_{j-1}} P_{j-1} + 4\left\{e^{\lambda_i (t_{j-1}+t_j)/2}\left(\frac{P_{j-1}+P_j}{2}\right)\right\} + e^{\lambda_i t_j} P_j\right]$$

Finally, upon substitution into the full equation, we have

$$c_{ij} = e^{-\lambda_i \Delta t} c_{i,j-1} + \frac{\beta_i}{\Lambda}\frac{\Delta t}{6}\left[e^{-\lambda_i \Delta t} P_{j-1} + 2e^{-\lambda_i \Delta t/2}\left(P_{j-1}+P_j\right) + P_j\right]$$

This says that, with measured discrete values for the power vs. time, P_j, we can also easily estimate the time-dependent normalized precursor concentrations, c_{ij}, for each precursor group i.

Inverse Point Kinetics Equations (cont.)

Now, evaluating the power dynamics equation at time point t_j using a central finite difference approximation for dP/dt, gives

$$\left.\frac{dP}{dt}\right|_{t_j} \approx \frac{P_{j+1}-P_{j-1}}{2\Delta t} = \frac{(\rho_j-\beta)}{\Lambda}P_j + \sum_i \lambda_i c_{ij} + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q_j\rangle$$

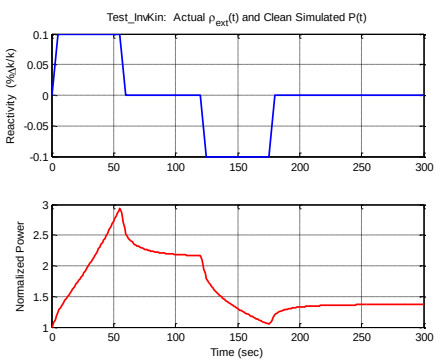
and, solving this expression for the reactivity at the j^{th} time point, gives

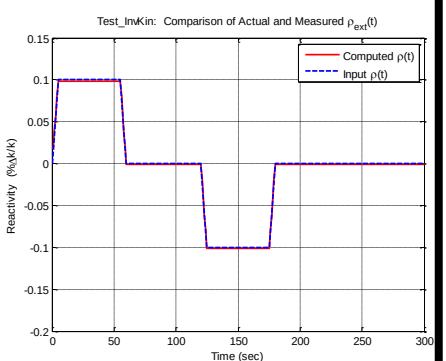
$$\rho_j = \beta + \frac{\Lambda}{P_j}\left[\frac{P_{j+1}-P_{j-1}}{2\Delta t} - \sum_i \lambda_i c_{ij} - \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q_j\rangle\right]$$

This equation, coupled with the expression for the precursor concentrations, c_{ij}, represents the final form of the desired Inverse Point Kinetics equations.

With measured values for P_j, these expressions give the reactivity vs. time profile, ρ_j, that actually caused the observed power vs. time behavior to occur.

Inverse Kinetics Test Case (clean P(t) data)

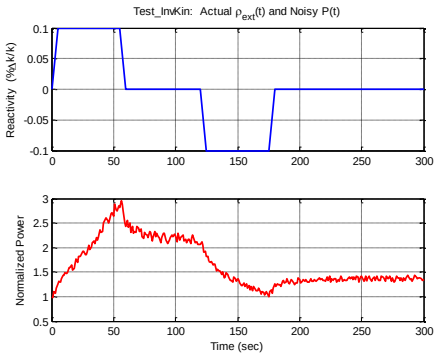


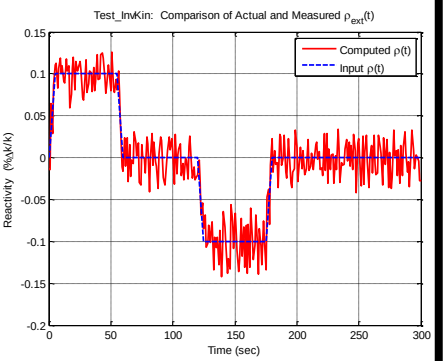


Output from test_invkin.m

(uses invkin_sr.m routine written by T. P. Michaud)

Inverse Kinetics Test Case (noisy P(t) data)





Output from test_invkin.m

(uses invkin_sr.m routine written by T. P. Michaud)

Check out the Matlab code?

Implementation Considerations

For use within the UMLRR, Thomas Michaud found that, for near critical operation, an average of the Linear Power 1 and 2 channels gave the best P(t) signal to use for evaluating ρ(t).

He also discovered that, for negative transients, there was a "drift" in the reactivity prediction due to gamma interference within the power detectors.

In particular, the output signal from the three power channels is not solely related to the neutron level, but rather is a combination of both the neutron and gamma interactions.

At near critical operation above about 500 W, this is not an issue since the neutron signal dominates.

Implementation Considerations (cont.)

However, for fast negative transients, the neutron level drops faster than the gamma level because of the longer-lived fission product gammas -- thus the assumption that the detector signal is simply proportional to the neutron level may no longer be valid.

Thus, for practical implementation within the UMLRR, the deviation from critical should be held within about ± 0.4 %Δk/k and the power swing, especially on the low side, should not be much greater than a factor of 10 below the reference critical value.

Within these rough limits, inverse kinetics proved to be an excellent technique for measuring the dynamic reactivity within the UMLRR.

Implementation Considerations (cont.)

Finally, we note that, although the inverse kinetics method should also be applicable within subcritical configurations, the startup counter within the UMLRR is simply too noisy for practical operation with the current detector system.

Thus, our use of the inverse kinetics method within the UMLRR is currently limited to the measurement of dynamic reactivity changes from critical, where the power deviations from reference are such that the power channels are still primarily sensitive to the neutron level (i.e. with minimal gamma interference).

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