24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Reactivity Feedback Effects – Part II:
Prediction, Measurement, & Interpretation

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Topic Overview

In the Part I Lecture on temperature feedback effects, we ended by distinguishing between constant power and variable power reactor sequences.

For constant power runs, $\Delta T_f = \Delta T_c$ is a good assumption, and $\Delta\rho_{temp}$ is given by

$$\Delta\rho_{temp} = \alpha_{T_f}\Delta T_f + \alpha_{T_c}\Delta T_c = \left(\alpha_{T_f} + \alpha_{T_c}\right)\Delta T = \alpha_{ITC}\Delta T$$

isothermal temperature coefficient
and
ΔT is related to the measured plenum temperatures

However, for variable power cases, $\Delta T_f \neq \Delta T_c$, so we need a mathematical model to estimate these quantities, since there are no direct measurements for T_f and T_c within a UMLRR fuel assembly.

For this case, an estimate of $\Delta\rho_{temp}$ is given by

$$\Delta\rho_{temp} = \alpha_{T_f}\Delta T_f + \alpha_{T_c}\Delta T_c$$

need estimates of the individual fuel and coolant temperature coefficients
and
a dynamic model for $T_f(t)$ and $T_c(t)$

The Feedback Model

For the feedback reactivity, we have

$$\Delta\rho_f(t) = \alpha_{T_f}\Delta T_f(t) + \alpha_{T_c}(t)\Delta T_c(t) + \Delta\rho_{Xe}(t)$$

Recall that cf = 0.88 for the xenon reactivity model

where the reference values for the temperature coefficients of reactivity are:

α_{ITC} = 5.9e-5 Δk/k-°C (from pool cooldown run in Jan. 2013)

α_{Tfref} = 1.7e-5 Δk/k-°C (from Michael Pike's MS Thesis in June 2013)

$\alpha_{Tcref} = \alpha_{ITC} - \alpha_{Tfref}$ = 4.2e-5 Δk/k-°C

α_{Tcref} (T) = 2.88e-6*T – (1.23e-5 + 1.7e-5) Δk/k-°C (from Jan. 2013 data)

Note: Positive values are listed for convenience but, of course, all the reactivity feedbacks are negative!!!

Note that we will change the value of α_{Tf} shortly…

Two Global Energy Balances

The development of the dynamics model for T_f(t) and T_c(t) is based on two global energy balances -- one for the fuel and one for the coolant.

For the fuel, we have

$$m_f c_f \frac{d}{dt}T_f = P - U_{fc}(T_f - T_c)$$

U_{fc} is the overall HT coefficient between the fuel and coolant

where U_{fc} is the overall HT coefficient between the fuel and coolant, P is the reactor power, m_f is the mass of the fuel, c_f is the heat capacity of the fuel, and T_f and T_c are the average fuel and coolant temperatures, respectively.

Similarly, for the coolant, we have

$$m_c c_c \frac{d}{dt}T_c = U_{fc}(T_f - T_c) + \dot{m}c_c(T_{in} - T_{out})$$

where $\dot{m}$ is the coolant mass flow rate.

Two Global Energy Balances (cont.)

As a simple approximation, we say that the average coolant temperature is roughly the average of the inlet and outlet temperatures,

$$T_c = \frac{T_{in} + T_{out}}{2} \qquad \text{or} \qquad T_{out} = 2T_c - T_{in}$$

and, upon substitution, we have

$$m_c c_c \frac{d}{dt} T_c = U_{fc}(T_f - T_c) + 2\dot{m} c_c (T_{in} - T_c)$$

or

$$\boxed{m_c c_c \frac{d}{dt} T_c = U_{fc}(T_f - T_c) + U_{cc}(T_{in} - T_c)}$$

Recall that

$$U = \frac{1}{R_{th}} \quad \text{or} \quad R_{th} = \frac{1}{U}$$

U_{cc} is like an overall HT coefficient related to the thermal resistance of the flowing fluid

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Transient Thermal Model

To summarize, we simply perform an overall energy balance on a single fuel node and single coolant node.

Doing this gives the following dynamic equations for T_f and T_c:

Fuel Node

$$\boxed{C_f \frac{d}{dt} T_f = P - U_{fc}(T_f - T_c)}$$

Coolant Node

$$m_c c_c \frac{d}{dt} T_c = U_{fc}(T_f - T_c) + \dot{m} c_c (T_{in} - T_{out})$$

$$T_c = \frac{T_{in} + T_{out}}{2} \qquad \text{or} \qquad T_{out} = 2T_c - T_{in}$$

$$m_c c_c \frac{d}{dt} T_c = U_{fc}(T_f - T_c) + 2\dot{m} c_c (T_{in} - T_c)$$

$$\boxed{C_c \frac{d}{dt} T_c = U_{fc}(T_f - T_c) + U_{cc}(T_{in} - T_c)}$$

These coupled equations represent a simple transient Thermal Model for the UMLRR

$C_f = m_f c_f$ & $C_c = m_c c_c$
are the thermal capacitances
and
U_{fc} and U_{cc} are the fuel-coolant and coolant-flow overall heat transfer coefficients for the model

An 11-Equation Dynamics Model

A 1-node kinetics model, thermal model, and I-Xe model, when coupled together, give a complete 11-equation dynamics model for the UMLRR.

Kinetics Model

$$\frac{d}{dt}P(t)=\frac{(\rho-\beta)}{\Lambda}P(t) \quad + \quad \sum_i \lambda_i c_i(t) \quad + \quad \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t)=\frac{\beta_i}{\Lambda}P(t) \quad - \quad \lambda_i c_i(t) \qquad \text{for } i=1,2,\cdots 6$$

Coupling Equations

$$\phi=\frac{cf\times P(t)}{\kappa\Sigma_f V_{core}}$$

$$\rho_{T_f}=\alpha_{T_f}\left(T_f(t)-T_{f_{ref}}\right)$$

$$\rho_{T_c}=\alpha_{T_c}\left(T_c(t)-T_{c_{ref}}\right)$$

$$\rho_{Xe}(t)=\frac{X(t)\sigma_{aX}/\Sigma_{f2}}{\nu p\varepsilon P_F P_T}$$

$$\rho=\rho_{RB}+\rho_{T_f}+\rho_{T_c}+\rho_{Xe}$$

Thermal Model

$$C_f\frac{d}{dt}T_f=P-U_{fc}(T_f-T_c)$$

$$C_c\frac{d}{dt}T_c=U_{fc}(T_f-T_c)+U_{cc}(T_{in}-T_c)$$

I – Xe Model

$$\frac{d}{dt}I=\gamma_I\Sigma_f\phi-\lambda_I I$$

$$\frac{d}{dt}X=\lambda_I I+\gamma_X\Sigma_f\phi-\left(\lambda_X+\sigma_{aX}\phi\right)X$$

also need a HT coefficient model

Coupling through the HT Coeffs.

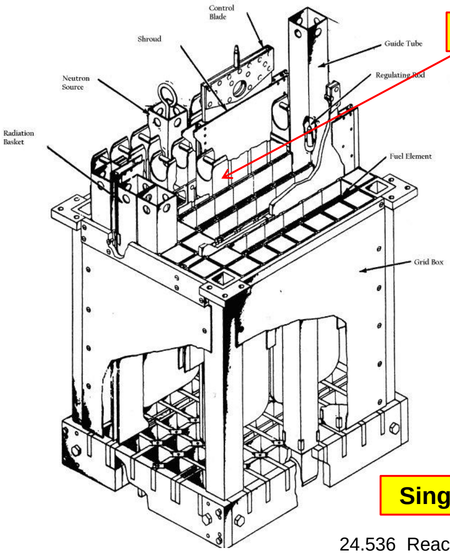
Since the HT coefficients, U_{fc} and U_{cc}, are functions of the fuel, clad, and coolant temperatures and the flow conditions -- these quantities are also directly coupled to the 11-equation dynamics model.

The HT coefficient model within the sstemp_umlrr GUI will be incorporated directly into the 11-equation model to compute $U_{fc}(t)$ ad $U_{cc}(t)$ as an integral part of the full simulation model.

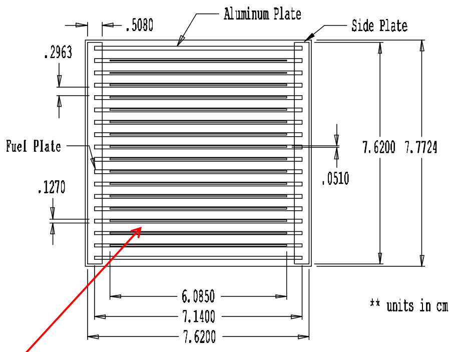
So now, let's discuss a mathematical model to estimate the

Steady State Temperature Profiles in a UMLRR Fuel Channel

Steady State Temperature Profiles in a UMLRR Fuel Channel



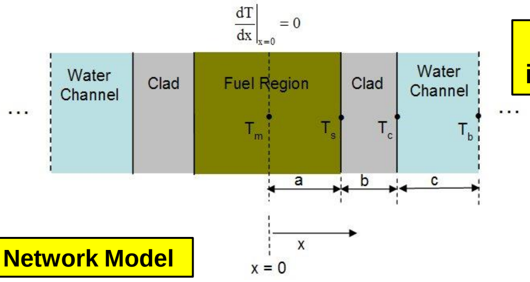
Fuel Assembly



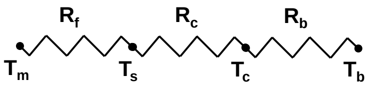
Single Fuel Channel

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)

Heat Transfer in a Fuel Plate



Thermal Network Model



$$q'' = \frac{\Delta T}{R_{th}} = \frac{T_m - T_s}{R_f} = \frac{T_s - T_c}{R_c} = \frac{T_c - T_b}{R_b}$$

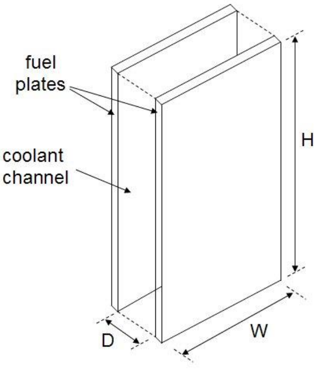
$$R_f = \frac{a}{2k_f} \qquad R_c = \frac{b}{k_c} \qquad R_b = \frac{1}{h}$$

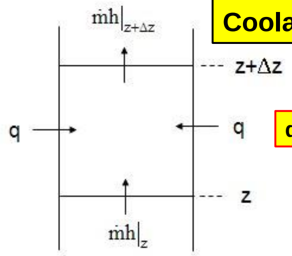
assumes no contact resistance

need to find approximate h

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)

Heat Transfer to the Coolant Channel





$q = q''\cdot 2W\Delta z$

rate of energy into node with flow − rate of energy out of node with flow + rate of energy transferred from fuel to coolant = 0

$$\frac{dT_b(z)}{dz} = \frac{2W}{\dot{m}c_p} q''(z) \qquad \text{with} \quad T_b(0) = T_{in}$$

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)

assume axial symmetry with a simple chopped sinusoidal heat flux profile

$$q''(z) = q''_{max} \sin\left(\frac{\pi(z+\delta)}{H_e}\right)$$

Axial Profile of the Surface Heat Flux

where q''_{max} = peak heat flux (W/m^2) within an average plate

H_e = effective neutronic height of the core (m)

δ = reflector savings (m)

$H_e = H + 2\delta$

The peak heat flux, q''_{max}, is related to the plate power as follows:

$$P_{plate} = 2\int_0^W \int_0^H q''_{max} \sin\left(\frac{\pi(z+\delta)}{H_e}\right) dy\,dz = 2Wq''_{max}\int_0^H \sin\left(\frac{\pi(z+\delta)}{H_e}\right) dz$$

or

$$q''_{max} = \frac{P_{plate}}{2W\int_0^H \sin\left(\frac{\pi(z+\delta)}{H_e}\right) dz}$$

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)

Coolant Temperature Axial Profile

Performing the operations indicated in the previous equation gives

$$q''_{max} = \frac{\pi P_{plate}}{2WH_e} \frac{1}{\cos\left(\frac{\pi\delta}{H_e}\right) - \cos\left(\frac{\pi(H+\delta)}{H_e}\right)}$$

Now, with this result, putting the expression for q''(z) into the balance equation and solving the 1st order ODE leads to

$$\boxed{T_b(z) = T_{in} + \frac{P_{plate}}{\dot{m}c_p}\left[\frac{\cos\left(\frac{\pi\delta}{H_e}\right) - \cos\left(\frac{\pi(z+\delta)}{H_e}\right)}{\cos\left(\frac{\pi\delta}{H_e}\right) - \cos\left(\frac{\pi(H+\delta)}{H_e}\right)}\right]}$$

Thus, with data specific to the UMLRR (P_{plate}, H, W, δ, $\dot{m}$, T_{in}, etc.), we can obtain the desired coolant temperature profile.

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)

The Heat Transfer Coefficient

Even with a known $T_b(z)$, one still needs a reasonable estimate of the heat transfer coefficient, h, to compute the plate surface temperature profile, $T_c(z)$.

This parameter, however, is usually obtained from empirical correlations that are given in terms of the local fluid properties, the type of flow, and the geometry of the system.

Dittus Boelter: $Nu = 0.023\,Re^{0.8}\,Pr^{0.4}$

Sieder-Tate: $Nu = 0.027\,Re^{0.8}\,Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$

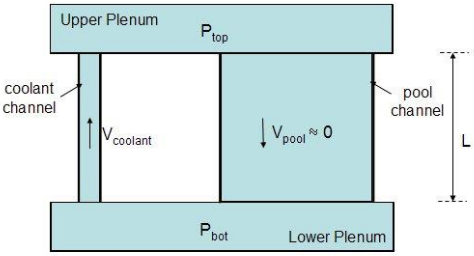
Turbulent Flow

Analytical Result: $Nu = \text{constant} \approx 7.9$

Laminar Flow

where $Nu = \frac{hD_h}{k}$ or $\boxed{h = \frac{Nu\,k}{D_h}}$ and $Re = \frac{\rho V D_w}{\mu}$ $Pr = \frac{\mu c_p}{k}$

Steady State Temperature Profiles in a UMLRR Fuel Channel (cont.)



Natural Convection Considerations

$$h_L = f\frac{L}{D_h}\frac{V^2}{2g} + \left(\sum_i K_i\right)\frac{V^2}{2g}$$

Channel ΔP: $\Delta P = P_{bot} - P_{top} = \rho_{coolant} gL + \rho_{coolant} gh_L$

Pool ΔP: $\Delta P = P_{bot} - P_{top} = \rho_{pool} gL$

These need to match

$$\Delta P_{bouyancy} = \left(\rho_{pool} - \rho_{coolant}\right)gL = \Delta\rho gL = \rho_{coolant} gh_L = \Delta P_{friction}$$

The velocity that develops in natural convection flow will increase until the friction loss in the channel exactly balances the buoyant forces on the channel.

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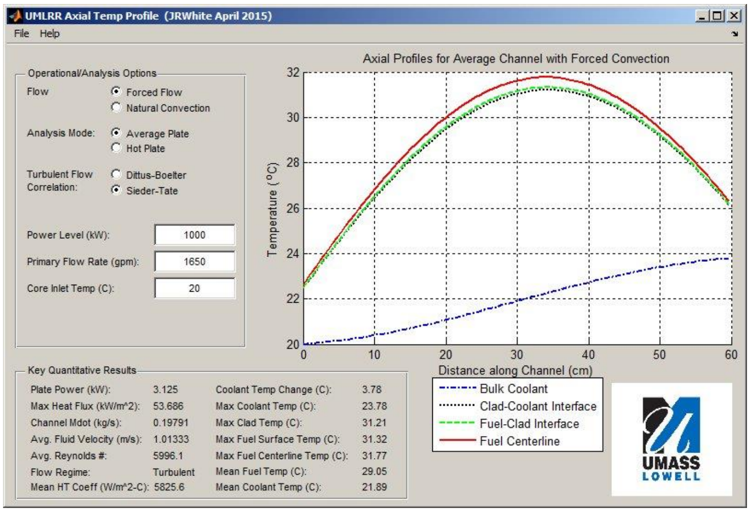
sstemp_umlrr GUI (1 MW Forced Flow)



Operational/Analysis Options

Flow: ● Forced Flow ○ Natural Convection

Analysis Mode: ● Average Plate ○ Hot Plate

Turbulent Flow Correlation: ○ Dittus-Boelter ● Sieder-Tate

Power Level (kW): 1000

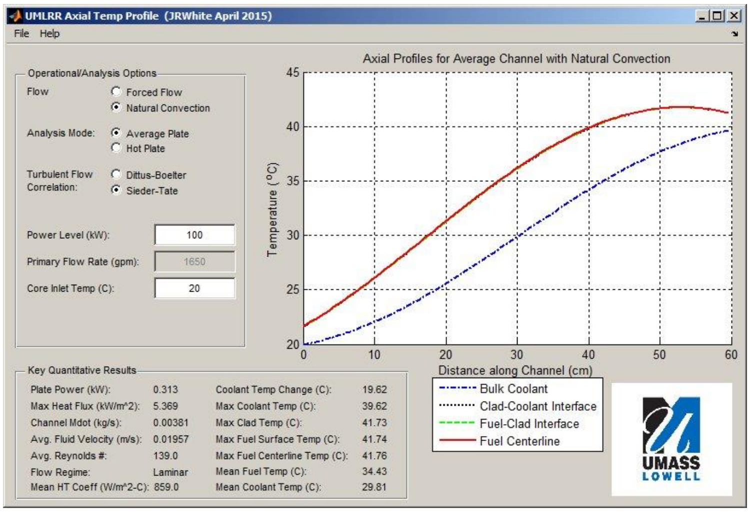
Primary Flow Rate (gpm): 1650

Core Inlet Temp (C): 20

Key Quantitative Results

Plate Power (kW):	3.125	Coolant Temp Change (C):	3.78
Max Heat Flux (kW/m^2):	53.686	Max Coolant Temp (C):	23.78
Channel Mdot (kg/s):	0.19791	Max Clad Temp (C):	31.21
Avg. Fluid Velocity (m/s):	1.01333	Max Fuel Surface Temp (C):	31.32
Avg. Reynolds #:	5996.1	Max Fuel Centerline Temp (C):	31.77
Flow Regime:	Turbulent	Mean Fuel Temp (C):	29.05
Mean HT Coeff (W/m^2-C):	5825.6	Mean Coolant Temp (C):	21.89

sstemp_umlrr GUI (100 kW Natural Convection Flow)


UMLRR Axial Temp Profile (JRWhite April 2015)

Operational/Analysis Options

Flow: ○ Forced Flow ● Natural Convection

Analysis Mode: ● Average Plate ○ Hot Plate

Turbulent Flow Correlation: ○ Dittus-Boelter ● Sieder-Tate

Power Level (kW): 100

Primary Flow Rate (gpm): 1650

Core Inlet Temp (C): 20

Key Quantitative Results

Plate Power (kW):	0.313	Coolant Temp Change (C):	19.62
Max Heat Flux (kW/m^2):	5.369	Max Coolant Temp (C):	39.62
Channel Mdot (kg/s):	0.00381	Max Clad Temp (C):	41.73
Avg. Fluid Velocity (m/s):	0.01957	Max Fuel Surface Temp (C):	41.74
Avg. Reynolds #:	139.0	Max Fuel Centerline Temp (C):	41.76
Flow Regime:	Laminar	Mean Fuel Temp (C):	34.43
Mean HT Coeff (W/m^2-C):	859.0	Mean Coolant Temp (C):	29.81

Solution of the 11-Equation Model

A 1-node kinetics model, thermal model, and I-Xe model, when coupled together, give a complete 11-equation dynamics model for the UMLRR.

Kinetics Model

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \qquad \text{for } i = 1, 2, \cdots 6$$

Coupling Equations

$$\phi = \frac{cf \times P(t)}{\kappa\Sigma_f V_{core}}$$

$$\rho_{T_f} = \alpha_{T_f}\left(T_f(t) - T_{f_{ref}}\right)$$

$$\rho_{T_c} = \alpha_{T_c}\left(T_c(t) - T_{c_{ref}}\right)$$

$$\rho_{Xe}(t) = \frac{X(t)\sigma_{aX}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T}$$

$$\rho = \rho_{RB} + \rho_{T_f} + \rho_{T_c} + \rho_{Xe}$$

Thermal Model

$$C_f \frac{d}{dt}T_f = P - U_{fc}(T_f - T_c)$$

$$C_c \frac{d}{dt}T_c = U_{fc}(T_f - T_c) + U_{cc}(T_{in} - T_c)$$

I – Xe Model

$$\frac{d}{dt}I = \gamma_I \Sigma_f \phi - \lambda_I I$$

$$\frac{d}{dt}X = \lambda_I I + \gamma_X \Sigma_f \phi - \left(\lambda_X + \sigma_{aX}\phi\right)X$$

also need a HT coefficient model

Overall Solution Logic

In the function file for the ODE solver, we perform the following tasks (recall that the state vector, x, is known at time t):

1. Determine the RegBlade position and compute ρ_{RB}.
2. Use the interp1 function to find T_{in} for the current time.
3. Knowing the current values of T_f and T_c, compute ρ_{Tf} and ρ_{Tc}.
4. Knowing the current Xe concentration, compute ρ_{Xe}.
5. Determine total reactivity, ρ, at the current time.
6. Determine U_{fc} and U_{cc} via a call to function htcoeff_umlrr.m
7. Compute the thermal flux, ϕ, for the given power.
8. Finally, compute the derivative of the state vector and pass this back to the ODE solver (ode15s).

$$\underline{x} = \begin{bmatrix} P \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ T_f \\ T_c \\ I \\ Xe \end{bmatrix}$$

Let's look at the actual Matlab m-files…

Feedback Model Validation

(forced flow case from Aug. 20, 2012)

A forced flow validation experiment was performed on Aug. 20, 2012. The initial reactor power was about 5 kW.

The goal of the test was to validate the combined temperature and xenon feedback model for a variable power run.

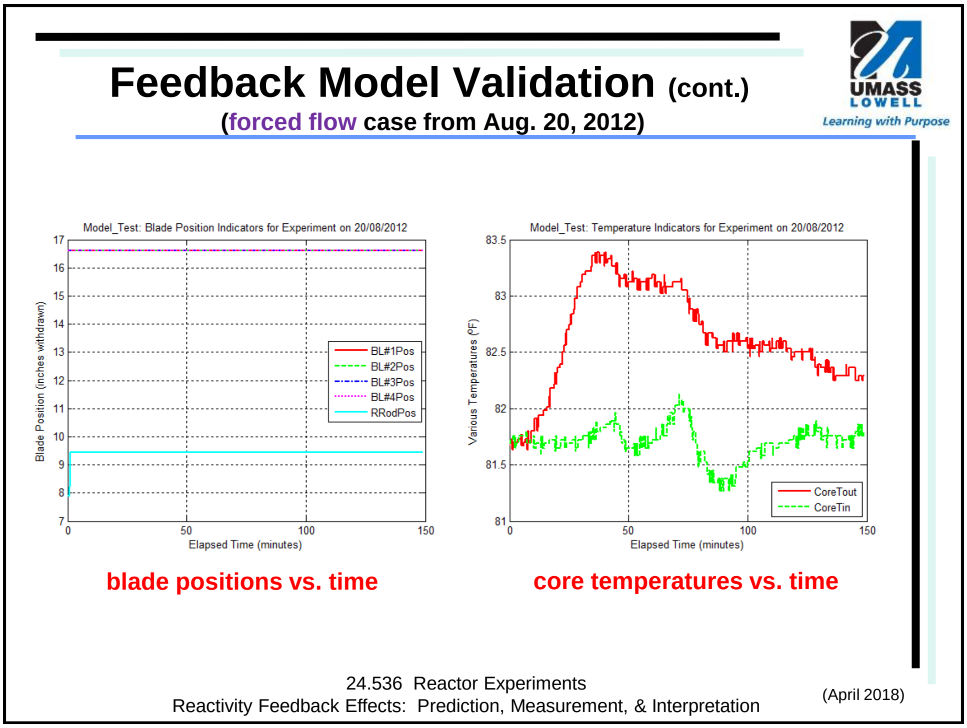
An initial positive reactivity perturbation was made by moving the RegBlade outward from 7.88 inches out to 9.45 inches withdrawn.

The core inlet temperature was kept nearly constant so that subsequent analyses would be more tractable.

However, the core average temperatures increased due to the increasing power caused by the reactivity addition.

Xenon also started to build up as the power increased.

These two effects induced a negative feedback reactivity which eventually stabilized the system and, with xenon buildup, even started to shut itself down.



Feedback Model Validation (cont.)

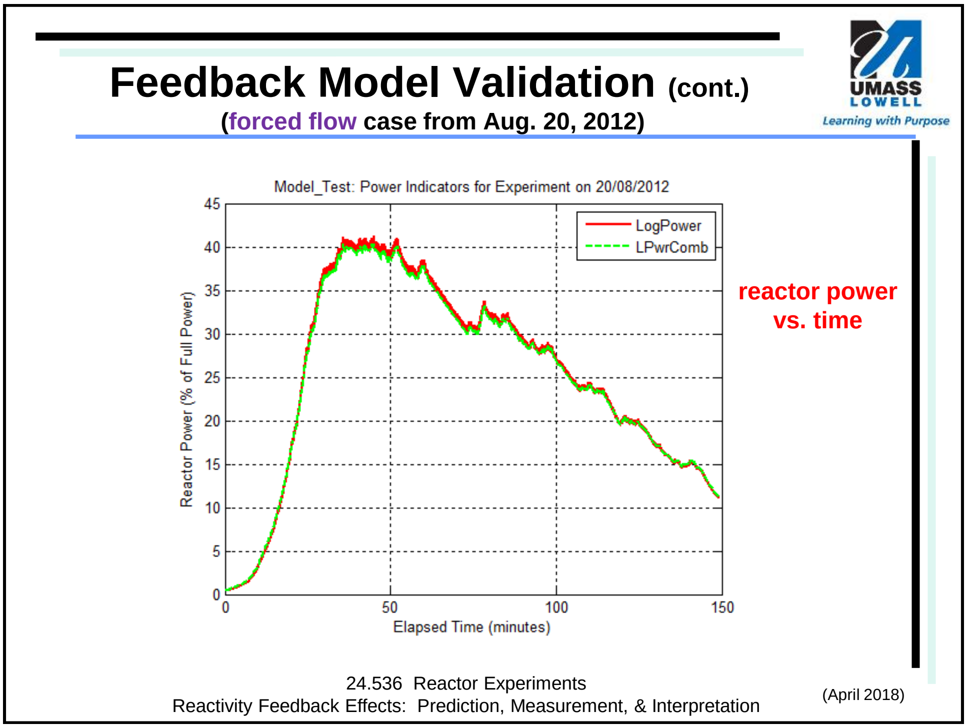
(forced flow case from Aug. 20, 2012)

blade positions vs. time

core temperatures vs. time

24.536 Reactor Experiments
Reactivity Feedback Effects: Prediction, Measurement, & Interpretation

(April 2018)



Feedback Model Validation (cont.)

(forced flow case from Aug. 20, 2012)

reactor power vs. time

24.536 Reactor Experiments
Reactivity Feedback Effects: Prediction, Measurement, & Interpretation

(April 2018)

Feedback Model Validation (cont.)

(forced flow case from Aug. 20, 2012)

Now, the goal of the test is to compare the predicted P(t) profile and feedback reactivity model with the measured P(t) using the combined Linear 1 and 2 channels and the feedback reactivity found via inverse kinetics.

Two cases were studied:

1. **Used the reference fuel temperature coefficient calculated by Michael Pike as part of his MS Thesis:**

   α_{fref} = 1.7e-5 Δk/k per °C

   This gave very poor results!!!

2. **Used the "optimum" fuel temperature coefficient determined in an (undocumented) study in summer 2007:**

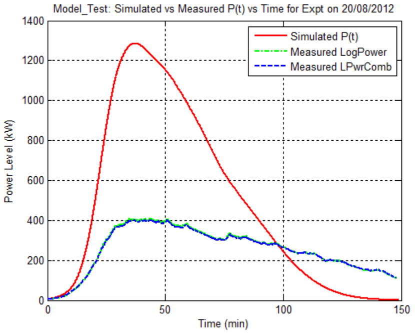
   α_{fopt} = 7.7e-5 Δk/k per °C

   The results here were much more reasonable!!!

These differ by a factor of about 4.5 !!!

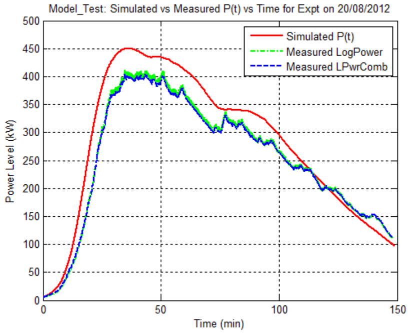
Feedback Model Validation (cont.)

(forced flow case from Aug. 20, 2012)



As apparent, our first try for the feedback reactivity model is horrible!!!

This case used α_{fref} = 1.7e-5 Δk/k-°C

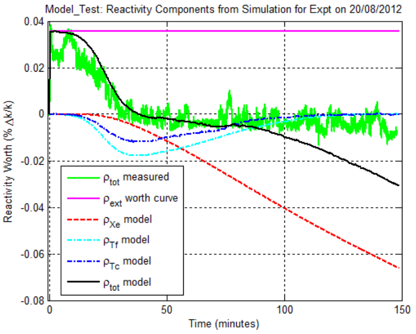


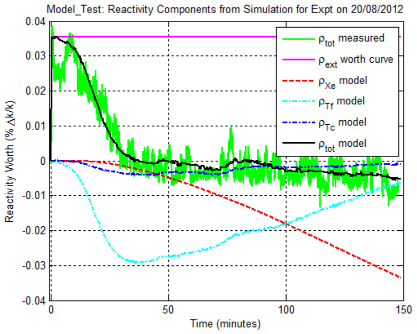
Now this is much better!!!

This case used α_{fopt} = 7.7e-5 Δk/k-°C

Feedback Model Validation (cont.)

(forced flow case from Aug. 20, 2012)





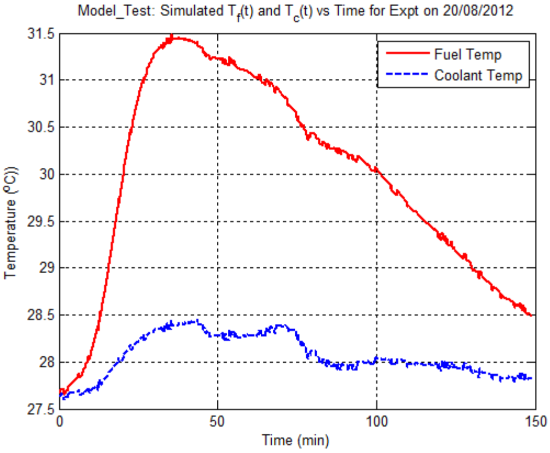
This case used
α_{fref} = 1.7e-5 Δk/k-°C

This case used
α_{fopt} = 7.7e-5 Δk/k-°C

Note: This result suggests that the fuel temperature feedback dominates much of the early part of the transient and that our prediction of the fuel temperature change may be too low???

Feedback Model Validation (cont.)

(forced flow case from Aug. 20, 2012)



This is the current predicted T_f(t) and T_c(t).

However, since we have no measurements of T_f or T_c, we have no information about the accuracy of these results.

Without further information, all we know is that the $\alpha_f \Delta T_f$ product needs to be increased to match the experimental results -- this could be due to poor values of one or both terms!!!

For now, our solution will be to use

α_{fopt} = 7.7e-5 Δk/k-°C

in all subsequent analyses…

Feedback Model Validation

(natural convection flow case from Aug. 20, 2007)

A natural convection flow validation experiment was performed on Aug. 20, 2007. The initial reactor power was about 5 kW.

The goal of the test was to validate the combined temperature and xenon feedback model for a variable power run, with the coolant temperature having a more dominant role.

An initial positive reactivity perturbation was made by moving the RegBlade outward about 2.2 inches.

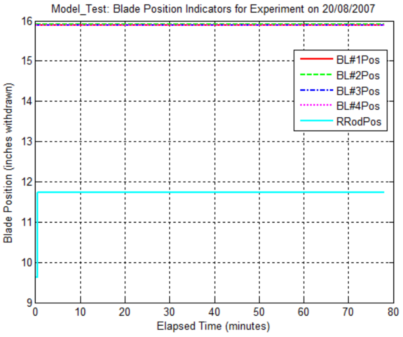
The core average temperatures increased due to the increasing power caused by the reactivity addition.

However, since the power level remained relatively low throughout the run, the xenon effect was negligible.

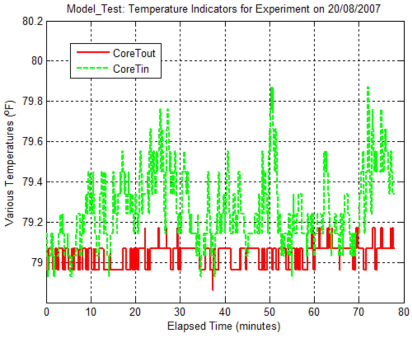
Thus, the negative temperature feedback reactivity eventually stabilized the system at a new steady state power of 26 kW.

Feedback Model Validation (cont.)

(natural convection flow from Aug. 20, 2007)



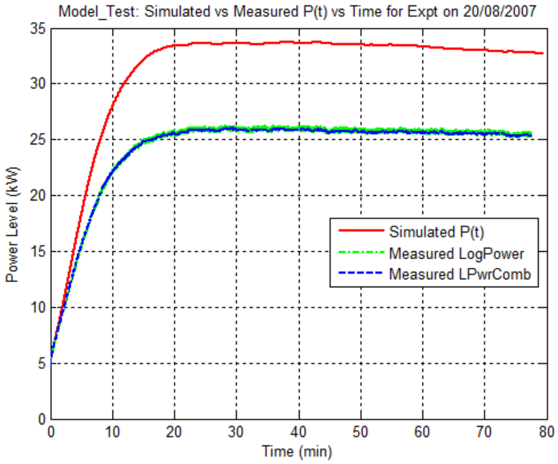
blade positions vs. time



core temperatures vs. time

Feedback Model Validation (cont.)

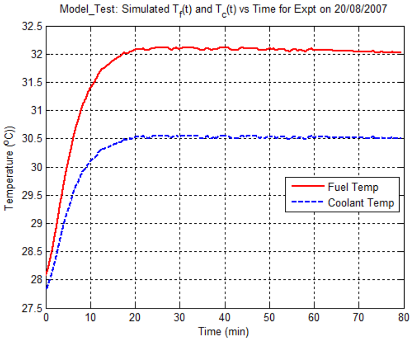
(natural convection case from Aug. 20, 2007)



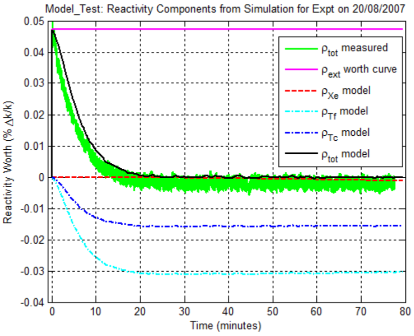
reactor power vs. time

Feedback Model Validation (cont.)

(natural convection case from Aug. 20, 2007)

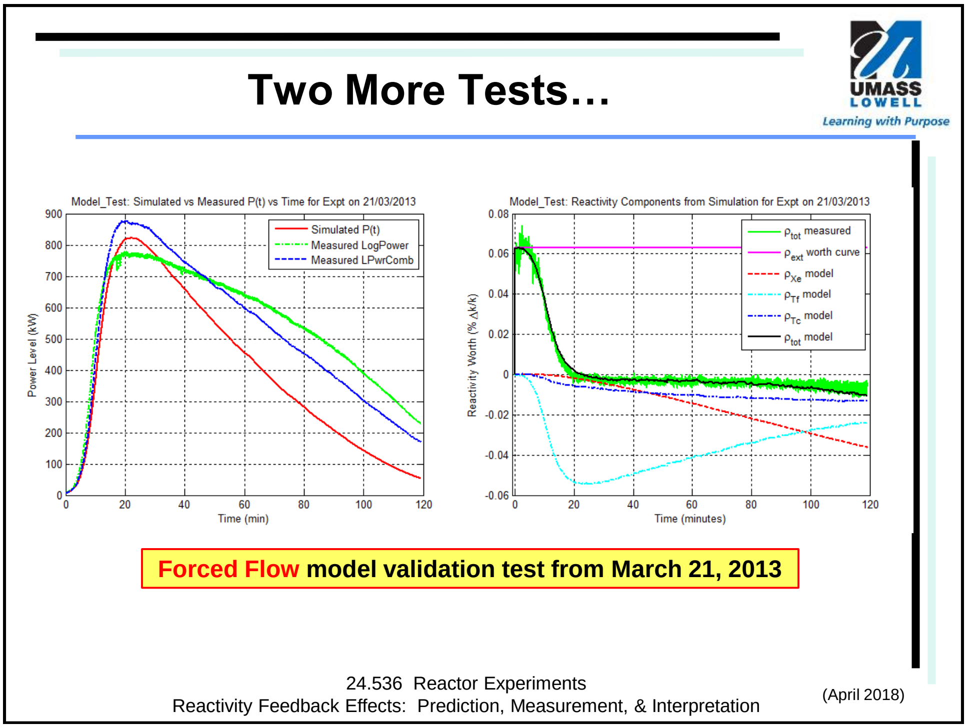


predicted core temperatures



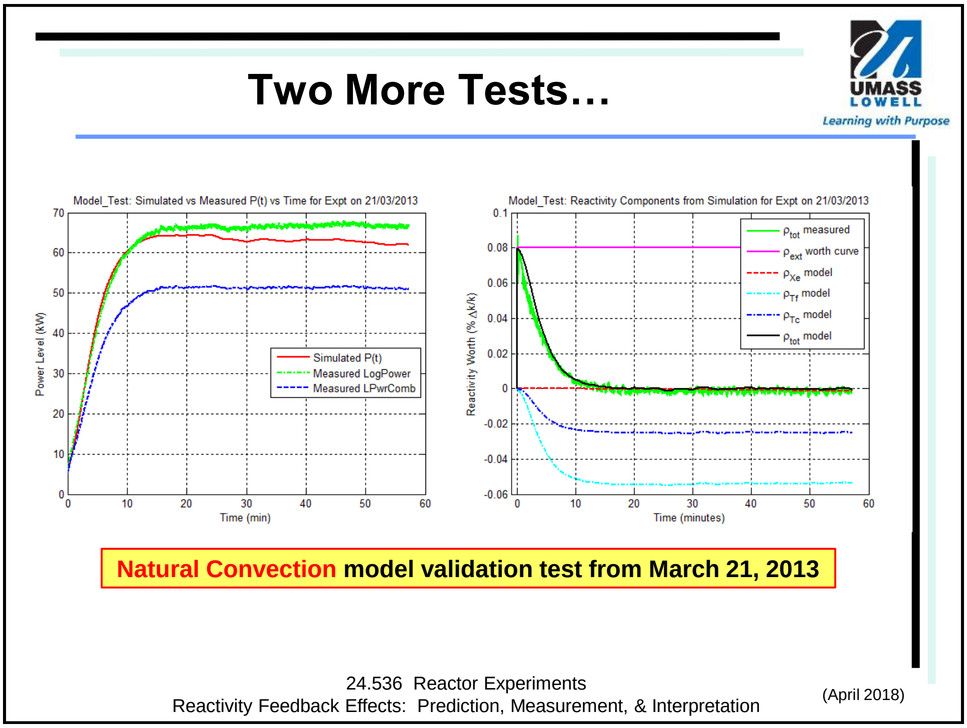
predicted vs. measured feedbacks

Here, both the fuel and coolant temperatures have an important role, and the predicted total feedback reactivity is quite reasonable…



Two More Tests…

Forced Flow model validation test from March 21, 2013



Two More Tests…

Natural Convection model validation test from March 21, 2013

Summary and Take-Aways

Our study of reactivity feedback effects within the UMLRR has accomplished the following:

1. **Established the importance of the inherent negative feedback mechanisms in real systems.**
2. **Showed that the xenon effect and the fuel and coolant temperature feedbacks can all be important, and that their relative contributions can be significantly different under a variety of diverse conditions.**
3. **Developed and validated a "semi-empirical" Feedback Model for the UMLRR that gives "reasonable" results relative to measured data.**

Summary and Take-Aways (cont.)

4. **Identified that our existing thermal model for the fuel temperature may need some improvements (however, where? and how? are the difficult questions yet to be answered).**
5. **Developed and validated an 11-equation dynamic model that can handle both forced and natural convection modes.**

We now have a UMLRR simulator that can predict dynamic behavior under a variety of transient scenarios…

However, there are a number of situations that the model cannot handle -- such as the transition from forced to natural convection flow -- so there is still lots of room for improvement

(as well as resolving some issues with the thermal model for the fuel)

Next Week's Lab

Now we need to show that our simulator works for additional cases and more challenging transient situations…

1. Perform the appropriate simulations and compare to the measured data from the two simple reactor sequences from July 17, 2013.
2. Design two new reactor sequences: one for natural convection and one for force convection flow -- we will run these sequences during the next lab as a test of our new simulation capability...

Rough Lab Procedure/Constraints:

Make a few RegBlade movements while in natural convection mode with the reactor initially at about 5 kW and keep P(t) < 80 kW while in this mode (this will keep the Xe at negligible levels).

Next Week's Lab (cont.)

Rough Lab Procedure/Constraints (cont.):

Ask the reactor staff to transition to forced flow mode at about 5 kW .

In forced flow mode, make some additional RegBlade movements keeping P(t) < 800 kW.

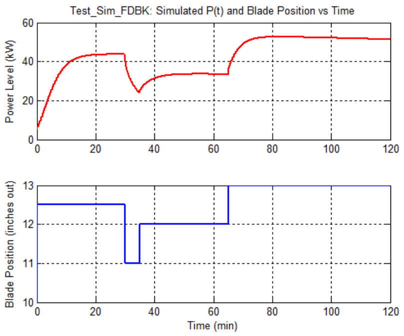
The overall reactor run (for both sequences) should be about 2.5 – 3.0 hours (also be sure to keep the reactor period well above 60 sec for both sequences)…

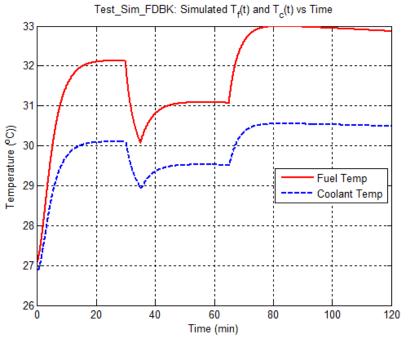
HW #10 asks you to review the theory discussed here and answer a few questions, to do a few additional test comparisons, and to design the proposed sequences to be used in both the natural and forced flow portions of our next lab on Feedback Effects in the UMLRR.

(see details in rexpts_hw10sp18.pdf)

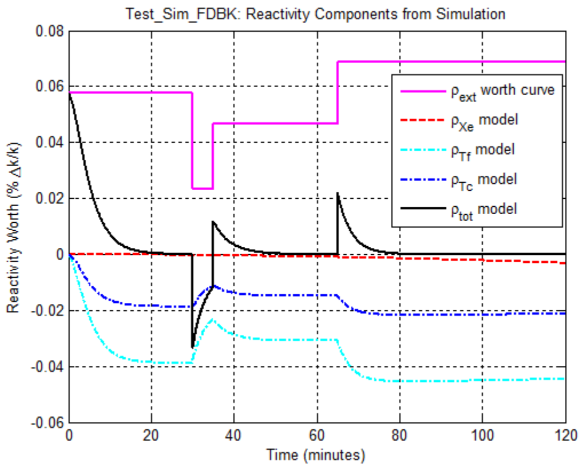
Some sample simulations…

Natural Convection Flow Simulation

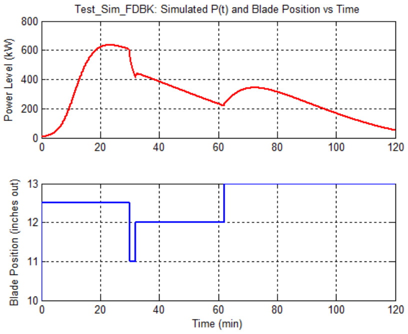


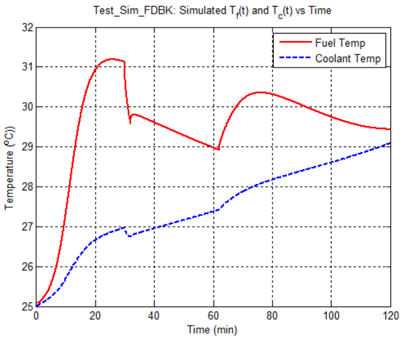


Natural Convection Flow Simulation

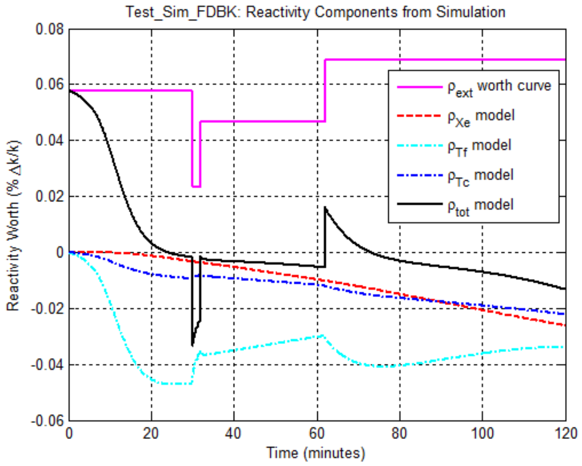


Forced Flow Simulation



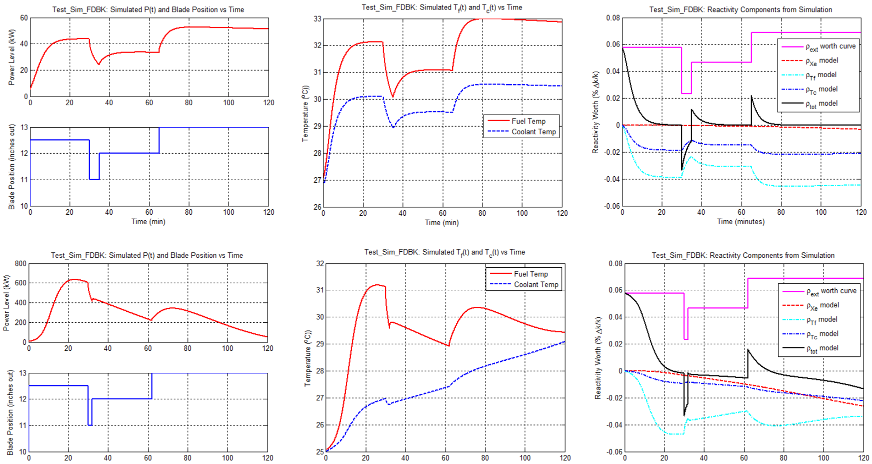


Forced Flow Simulation (cont.)



Comparison of Forced and Free Flow

free flow



forced flow