24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Reactivity Feedback Effects – Part I:
Prediction, Measurement, & Interpretation

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Discussion Outline

Review from previous class: Blade Worth Curve Lab

Brief reviews: Student Presentations…

Discussion: All…

Reactivity Feedback Effects

We will take a short break after a little theory…

Prediction, Measurement, and Interpretation

Homework #10 (see details in **rexpts_hw10sp18.pdf**)

We have LOTS of Material to cover today -- so let's get started…

Topic Overview

Every operating reactor must be designed and shown, via measurement, to be inherently stable relative to power, temperature, and reactivity changes in the system.

Both fission product poisoning (primarily due to Xe-135) and temperature feedback effects are important – and both subjects are reviewed here.

The temperature effects, in particular, represent a fast feedback mechanism that can help counter and stabilize reactivity perturbations within the system -- and both fuel and coolant temperature effects are addressed.

The negative Xe reactivity effect is inherent in every thermal system, and its long-term dynamics must be followed carefully -- during both normal operation and in shutdown mode -- to fully account for its dynamic reactivity effect, $\rho_{Xe}(t)$, on the system.

Topic Overview (cont.)

For real systems, we must be able to model, predict, measure, and validate the reactivity effects within the system.

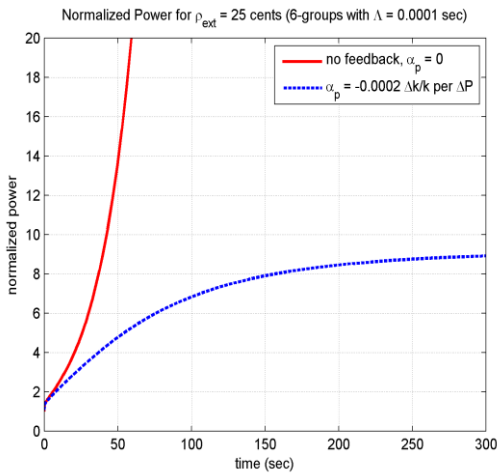
In most cases, it is not easy to measure separate individual effects, so composite effects are evaluated and decomposed in some way to extract the individual effects (as best as possible).

As part of this lecture & lab, we will try to look at "separate-effects tests" to measure specific feedback effects, and use experiments with combined effects as validation runs.

Because of limited temperature data from the UMLRR, we will have to rely on some simple mathematical models to predict and interpret some of the measured temperature coefficient data.

Thus, we will do a little theory, a little modeling, run/review several new/old experiments, and attempt to draw some conclusions concerning the feedback mechanisms within the UMLRR.

Negative Feedback -- An Illustration



No (or positive) feedback leads to an unbounded power transient -- this is NOT acceptable!!!

With inherent negative feedbacks, leveling off at a new steady state power level occurs as the negative feedback compensates for the initial positive reactivity insertion -- this is the only situation that makes sense in a real system.

Temperature Effects

Let's talk about

Temperature Effects

Temperature Effects

In the above illustration, a generic power feedback coefficient was applied to collectively treat a number of feedback effects.

In practice, the individual coefficients for each separate effect are needed since their time constants can be quite different.

For example, in a power excursion, T_f is the first to respond to an increased fission power, then T_c, and finally the temperature of the structural components, and the time delay for the various heat transport mechanisms can be important.

Each reactivity coefficient is defined in a similar fashion. For a temperature effect, we write the temperature coefficient of reactivity as

$$\alpha_T = \frac{\partial \rho}{\partial T}$$

where the temperature might be associated with the fuel (T_f), coolant (T_c), or structural materials.

Temperature Effects (cont.)

Since $\rho = (k - 1)/k$, the reactivity coefficient can be written as

$$\alpha_T = \frac{\partial \rho}{\partial T} = \frac{\partial}{\partial T}\left(1 - \frac{1}{k}\right) = \frac{1}{k^2}\frac{\partial k}{\partial T} \approx \frac{1}{k}\frac{\partial k}{\partial T}$$

where the last approximation ($k^2 \approx k$) is valid for a near-critical or critical reference state.

Now, from the basic definition of a reactivity coefficient, α_T, we see that

$$\rho_f(t) = \Delta\rho(t) \approx \frac{\partial \rho}{\partial T}\Delta T(t) = \alpha_T\left\{T(t) - T_{ref}\right\}$$

Thus, once the reactivity coefficients are known, they can be used to approximate the inherent feedback reactivity within the system.

Note that the units of the temperature coefficient are $\Delta k/k$ per unit temperature -- for example, $\Delta k/k$ per ^oC.

Computation of the Reactivity Coeffs.

The various **temperature coefficients are not easy to quantify!!!**

Often these are **computed using sophisticated computer codes** that attempt to model the reactor in as much detail as possible.

Two discrete temperatures are chosen and the **appropriate cross sections and atom densities are determined for each case**.

The **neutron balance equation is then solved using these data sets to obtain two values of k_{eff}**. Given the T-k combinations,

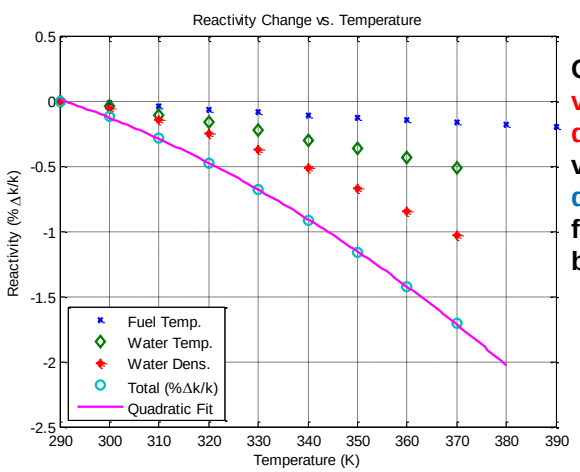
T_1 = reference temp $\rightarrow$ k_1 = reference k_{eff}

T_2 = perturbed temp $\rightarrow$ k_2 = perturbed k_{eff}

The **average temperature coefficient over the temperature range** is

$$\bar{\alpha}_T = \frac{\int_{T_1}^{T_2} \alpha_T(T)\,dT}{\int_{T_1}^{T_2} dT} = \frac{\int_{T_1}^{T_2} \frac{1}{k}\frac{\partial k}{\partial T}\,dT}{T_2 - T_1} = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \frac{dk}{k} = \frac{\ln\left(k_2/k_1\right)}{T_2 - T_1}$$

Computation of the Reactivity Coeffs.



One can also **plot Δk/k vs. T for a set of discrete T-k pairs** and, via a **curve fit** or **a finite difference estimate**, form $\bar{\alpha}_T$ vs. T from the basic definition.

These data are for the UMLRR M-2-5 core

(from Michael Pike's MS Thesis)

$\alpha_{ITC}(T)$ is the slope of the total ρ(T) vs. T curve

Reactivity Coefficients for the UMLRR

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Reactivity Coefficients at 25 ºC for several UMLRR Models (Δk/k/ºC)

Component	M-1-3 (BOL)	**M-2-5 (BOL)**	M-2-5 WPI Fuel	ANL Data	**M-2-5 Measured**
Water Temp Only	-4.4E-05	-4.2E-05	-1.3E-05	-4.8E-05	---
Water Density Only	-5.3E-05	-5.8E-05	-5.3E-05	-4.6E-05	---
total $T_{coolant}$	-9.7E-05	-1.0E-04	-6.6E-05	-9.4E-05	---
T_{fuel}	-1.7E-05	-1.7E-05	-1.7E-05	-1.5E-05	---
$T_{coolant}$ + T_{fuel}	**-1.1e-4**	**-1.2e-4**	**-8.3e-5**	**-1.1e-4**	**-5.9e-5**
void (Δk/k/%void)	-2.3E-03	-2.6E-03	-2.3E-03	-2.4E-03	---

$$\alpha_V = \frac{V}{k}\frac{dk}{dV} = \frac{V}{k}\frac{dk}{dT}\frac{dT}{dV} = V\alpha_{T_{den}}\frac{dT}{dV}$$

factor of two difference

These data are also from Michael Pike's MS Thesis

Measuring α_{ITC} for the M-2-5 Core

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A pool heat-up run followed by a pool cooldown experiment were conducted in Jan. 2013.

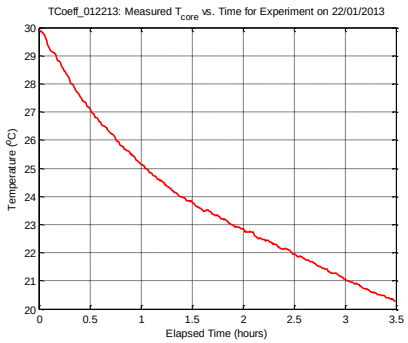
Heat-up operations occurred on a Friday, and the reactor was shutdown over a long weekend to allow the Xe-135 to completely decay.

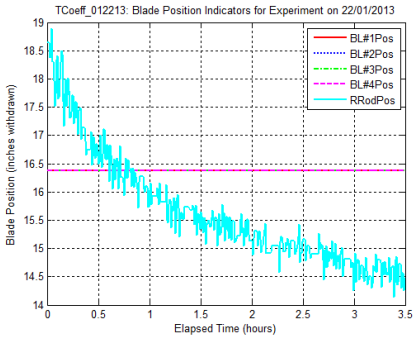
On Tuesday morning, the reactor was brought to low power critical with the RegBlade in auto mode, and the secondary cooling system was turned on to cool down the system.

The decreasing temperatures induced a positive reactivity and the RegBlade moved further into the core to compensate.

The RegBlade position, converted into reactivity via the worth curves, gives the inherent temperature feedback -- and, from this information and a simple curve fit, we can estimate the isothermal (fuel + coolant) temperature coefficient, α_{ITC} …

Measuring α_{ITC} for the M-2-5 Core

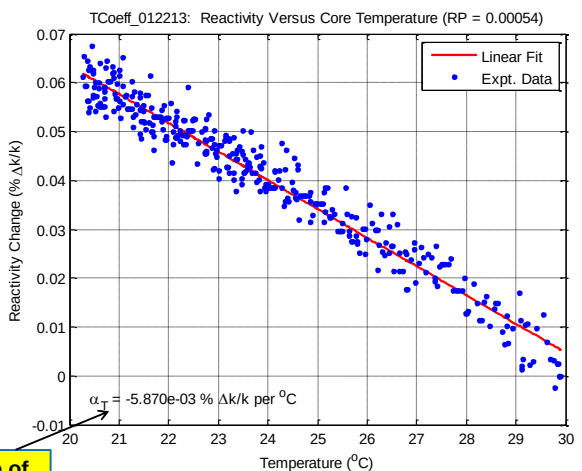




Note: In this analysis, we assume that $\Delta T = \Delta T_f = \Delta T_c$, thus

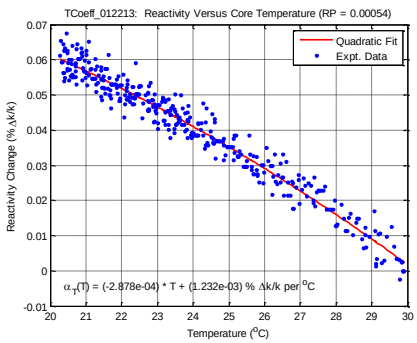
$$\Delta\rho = \alpha_{T_f}\Delta T_f + \alpha_{T_c}\Delta T_c = \left(\alpha_{T_f} + \alpha_{T_c}\right)\Delta T = \alpha_{ITC}\,\Delta T$$

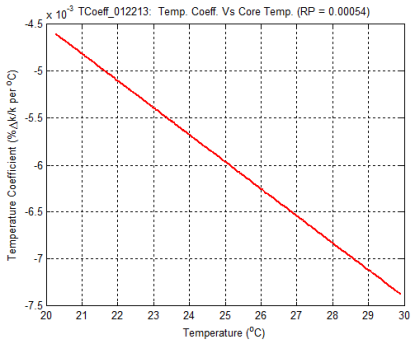
Measuring α_{ITC} for the M-2-5 Core



Best estimate of average α_{ITC}

Measuring α_{ITC} for the M-2-5 Core





Clearly α_{ITC} is a function of temperature

Xenon Effects

Let's talk about

Xenon Effects

Special Saturating FPs

Certain fission product nuclides play an especially important role in thermal systems because of their extremely large thermal absorption cross sections and relatively large equilibrium yields.

For example, for Xe-135, $\sigma_a(E_o) = 2.65\times10^6$ barns ($\gamma_{eq} \approx 0.0663$)

for Sm-149, $\sigma_a(E_o) = 41{,}000$ barns ($\gamma_{eq} \approx 0.0107$)

for Sm-151, $\sigma_a(Eo) = 15{,}200$ barns ($\gamma_{eq} \approx 0.0042$)

for a typical fission product, $\sigma_a(E_o) \approx$ 20-40 barns ($\gamma_{eq} \approx 1.92$)

The time constants associated with the dynamics of the important FP chains are on the order of hours to days (not a safety concern).

Here, we will highlight the dynamics of the I-Xe chain because of its large equilibrium yield and its extremely large thermal absorption cross section.

Dynamics of the I-Xe Chain

The detailed I-Xe chain can be visualized as follows:

$$\mathrm{Te-135} \xrightarrow[11\ \mathrm{sec}]{\beta^-} \mathrm{I-135} \xrightarrow[6.7\ \mathrm{hr}]{\beta^-} \mathrm{Xe-135} \begin{cases} \xrightarrow[9.2\ \mathrm{hr}]{\beta^-} \mathrm{Cs-135} \\ \xrightarrow[2.65\times10^6\ \mathrm{b}]{n,\gamma} \mathrm{Xe-136} \end{cases}$$

$\uparrow 0.0350$ fission $\quad \uparrow 0.0289$ fission $\quad \uparrow 0.00237$ fission

But, because Te-135 decays so rapidly and, since the daughter products from Xe-135 decay and absorption are not of interest, we can simplify the above scheme, as follows:

$$\mathrm{I-135} \xrightarrow[6.7\ \mathrm{hr}]{\beta^-} \mathrm{Xe-135} \begin{cases} \xrightarrow[9.2\ \mathrm{hr}]{\beta^-} \\ \xrightarrow[2.65\times10^6\ \mathrm{b}]{n,\gamma} \end{cases}$$

$\uparrow 0.06390$ fission $\quad \uparrow 0.00237$ fission

Dynamics of the I-Xe Chain (cont.)

Based on the simplified scheme, the basic isotope balance equations that define the I-Xe dynamics of interest in thermal systems can be written as (accumulation rate = production rate – loss rate):

I-135 balance equation: $$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I$$

Xe-135 balance equation: $$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \left(\lambda_X + \sigma_{aX}\phi\right) X$$

In these expressions, the thermal neutron flux is directly related to the operating power level.

Thus, a change in power can be treated as a driving force that moves the I-135 and Xe-135 densities from their equilibrium or steady-state values to some new state.

Dynamics of the I-Xe Chain (cont.)

There are several subtle approximations built into these eqns:

1. The balance eqns have been integrated over space, leading to the spatial independence associated with the given equations.

2. For the energy dependence, the $\Sigma_f \phi$ and $\sigma_{aX}\phi$ terms are interpreted as integrals over energy.

3. However, the Xe-135 cross section is extremely large at thermal energies relative to that at high energy. Thus, ignoring the fast component of the Xe-135 reaction rate is certainly justifiable.

4. But, for 2-group theory, we can write

$$\Sigma_f \phi \Rightarrow \Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2 = \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi_2 \Rightarrow \left(\Sigma_{f1}\frac{\phi_1}{\phi_2} + \Sigma_{f2}\right)\phi$$

Thus, we will let Σ_f represent the effective fission cross section.

Equilibrium Xenon Reactivity

To determine the reactivity effect for the equilibrium condition (i.e. production rate = loss rate) , we set the derivatives, dI/dt and dX/dt, to zero, which gives

$$I_\infty = \frac{\gamma_I \Sigma_f \phi_\infty}{\lambda_I} \qquad \text{and} \qquad X_\infty = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty}$$

Working, in particular, with the expression for X_∞, we can write the macroscopic Xe-135 absorption cross section at equilibrium as

$$\Sigma_{a\infty} = X_\infty \sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\lambda_X + \sigma_{aX}\phi_\infty}\sigma_{aX} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_\infty}{\phi_X + \phi_\infty}$$

where ϕ_∞ is the average thermal flux at equilibrium conditions and ϕ_X is given by

$$\phi_X = \lambda_X / \sigma_{aX}$$

Equilibrium Xenon Reactivity (cont.)

Finally, note that the reactivity due to a homogenous poison in a thermal system can be approximated as

$$\rho(t) = \frac{\Sigma_{aP}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T} = \frac{X(t)\sigma_{aX}/\Sigma_{f2}}{\nu p \varepsilon P_F P_T}$$

time-dependent xenon reactivity

Thus, putting the expression for X_∞ into this equation gives the reactivity effect of equilibrium xenon, or

$$\rho_\infty = \frac{(\gamma_I + \gamma_X)}{\nu p \varepsilon P_F P_T}\frac{\Sigma_f}{\Sigma_{f2}}\frac{\phi_\infty}{(\phi_X + \phi_\infty)}$$

equilibrium xenon reactivity

This is the expression of real interest in our pursuit of the dynamic xenon reactivity, $\rho_{Xe}(t)$

(see the formal Lecture Notes for a derivation of this expression)

Solution for the General Case

Given some initial condition and the time-dependent thermal flux (or power) as input, we need to solve the iodine and xenon balance equations simultaneously to give X(t), which can then be used to give $\rho_{Xe}(t)$ for the particular case of interest.

For the general case, where a general P(t) or $\phi(t)$ is used, the easiest way to solve these equations is via numerical integration with a standard ODE solver (such as Matlab's ode45 or ode15s routines, for example).

However, for the case where P(t) or $\phi(t)$ is constant over some interval, the balance equations represent a set of sequential, linear, constant coefficient ODEs, that can be solved analytically with relative ease.

Solution for the General Case (cont.)

The result for the constant thermal flux case, after suitable manipulation, is given by the following equations:

$$I(t) = I_o e^{-\lambda_I t} + \frac{\gamma_I \Sigma_f \phi}{\lambda_I}\left(1 - e^{-\lambda_I t}\right)$$

Analytical solution for piecewise constant flux or power

$$X(t) = X_0 e^{-wt} + \frac{(\gamma_I + \gamma_X)\Sigma_f \phi}{w}\left(1 - e^{-wt}\right) + \frac{\gamma_I \Sigma_f \phi - I_0 \lambda_I}{w - \lambda_I}\left(e^{-wt} - e^{-\lambda_I t}\right)$$

where $w = \sigma_{aX}\phi + \lambda_X$ and I_o and X_o represent the initial I-135 and Xe-135 concentrations.

These equations are only valid over a time period of constant flux (or power) given by the value of the thermal flux.

However, the thermal flux (or power) can often be approximated as a simple piecewise constant function.

Application to the UMLRR

To **quantify the Xe-135 reactivity effect** for a given application, we require specific information for the components of the **six factor formula** for the particular system of interest.

For the **UMLRR**, the **needed parameters** associated with the **six-factor formula** are summarized as follows (from 2-group cross section data generated in summer 2010):

$\nu = 2.43$ $\quad p = 0.879$ $\quad \varepsilon = 1.067$

$P_F = 0.665$ $\quad P_T = 0.969$

$\Sigma_{f1} = 1.21\times10^{-3}\ cm^{-1}$ $\quad \Sigma_{f2} = 5.04\times10^{-2}\ cm^{-1}$ $\quad \phi_1/\phi_2 = 2.75$

and $\Sigma_f = \Sigma_{f1}\dfrac{\phi_1}{\phi_2} + \Sigma_{f2}$

Application to the UMLRR (cont.)

Finally, we write the **thermal flux in terms of the relative power, rp**, and then vary this quantity for the various cases of interest, as follows:

$$P = \kappa\left(\Sigma_{f1}\phi_1 + \Sigma_{f2}\phi_2\right)V_{core} = \kappa\Sigma_f\phi V_{core}$$

Solving for $\phi = \phi_2$ gives

$$\phi = \frac{P}{\kappa\Sigma_f V_{core}} = \frac{rp\times P_{full}}{\kappa\Sigma_f V_{core}}$$

In the code, we add a **correction factor, cf**, to account for all the model approximations:

$$\phi = \frac{cf\times rp\times P_{full}}{\kappa\Sigma_f V_{core}}$$

where $P_{full} = 1\ MW = 10^6\ W$ and the **relative power, rp, is now the driving force for simulating I-Xe dynamics within the UMLRR**.

For evaluating ϕ for the subsequent simulations, we used a **20 element core** where the fuel assembly cross section is 7.7724 cm × 7.7724 cm and the active fuel height is 59.69 cm (**core volume = $7.212\times10^4\ cm^3$**).

Measuring the Correction Factor, cf

A xenon-effects experiment was conducted in Jan. 2013.

The goal was to validate our analytical xenon reactivity model for the UMLRR -- that is, to find the correction factor, cf.

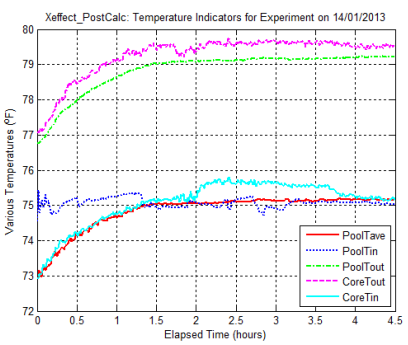
The reactor was run at steady state at near full power with approximately constant temperature for about 4.5 hours.

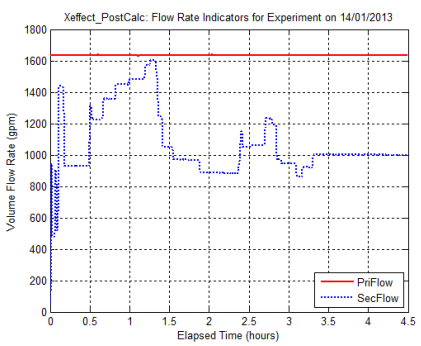
The RegBlade was in auto mode, and it continually moved out of the core while maintaining the reactor at the critical state.

The positive reactivity associated with the RegBlade moving out was needed to compensate for the negative reactivity associated with the buildup of xenon in the system.

The RegBlade position, converted into reactivity, represents the inherent xenon feedback effect -- and, with this information, we can obtain the desired value of cf that gives good model prediction vs. measurement for this experiment.

Measuring the Correction Factor, cf





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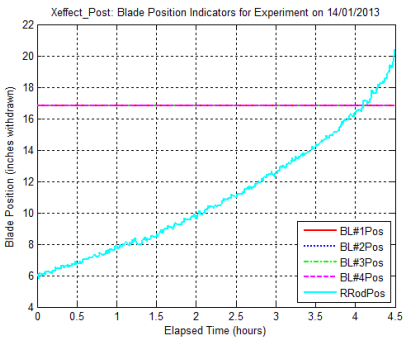
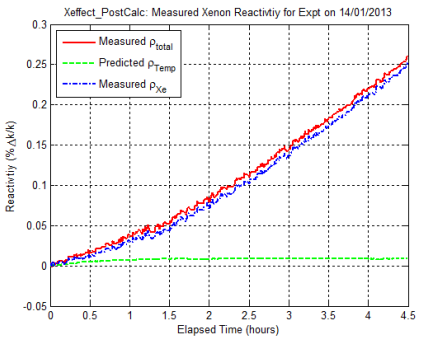
Reactivity Feedback Effects: Prediction, Measurement, & Interpretation
(March 2018)

Measuring the Correction Factor, cf



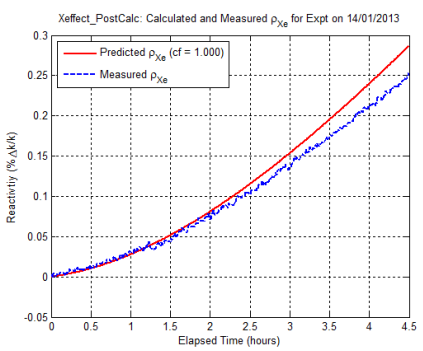


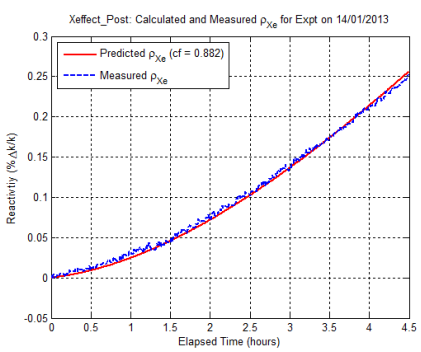
$$\rho_{Temp}(t) = \alpha_{ITC}(t)\,\Delta T_{core}(t)$$ **temperature effect**

α_{ITC} = -5.870e-3 %Δk/k/°C

$$\rho_{Xe}(t) = \rho_{total}(t) - \rho_{Temp}(t)$$ **xenon effect**

Measuring the Correction Factor, cf





The correction factor, cf = 0.88, is the same value as determined from a similar experiment performed back in 2005.

Thus, the xenon model developed above, with a correction factor of 0.88, should give a reasonable simulation of the xenon reactivity, ρ_{Xe}, for the UMLRR.

Temperature + Xenon Effects

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Now let's include both

Temperature and Xenon Effects

Feedbacks Validation Experiment

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A validation experiment was performed on Aug. 16, 2012.

The goal of the test was to validate the combined temperature and xenon feedback models developed previously.

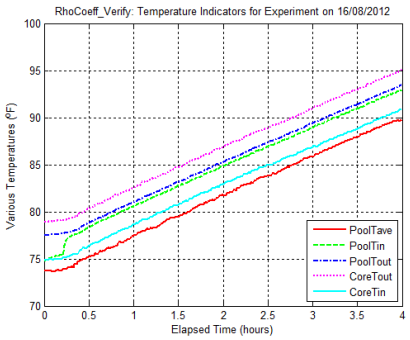
The reactor was run at near full power for 4 hours with no cooling (after a weekend to assure that no xenon was present initially).

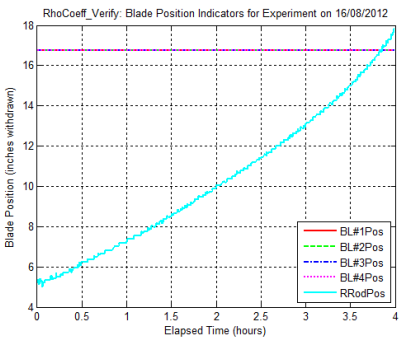
The secondary pump was off to allow the pool and core temperatures to increase linearly during the experiment.

While in auto mode, the RegBlade continually moved out of the core during the run to compensate for the negative reactivity associated with the buildup of xenon and the increase in the average core temperature.

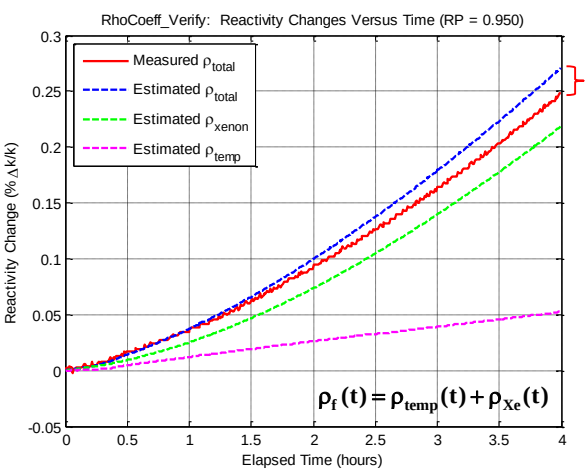
With the RegBlade position giving the total feedback effect -- one can easily compare the accuracy of our temperature and xenon feedback models.

Feedbacks Validation Expt. (cont.)





Feedbacks Validation Expt. (cont.)



As apparent, the feedback reactivity model slightly over predicts the total feedback…

used
α_{ITC} = -5.870e-3 %Δk/k/°C
cf = 0.882

Reactivity Effects with P(t) ≠ constant

The above cases were constant power runs, where $\Delta T_f = \Delta T_c$ is a pretty good assumption. When this is true, only the isothermal temperature coefficient is needed -- and ρ_{temp} is given by

$$\Delta\rho_{temp} = \alpha_{T_f}\Delta T_f + \alpha_{T_c}\Delta T_c = \left(\alpha_{T_f} + \alpha_{T_c}\right)\Delta T = \alpha_{ITC}\Delta T$$

However, for variable power cases, $\Delta T_f \neq \Delta T_c$, so we will need a mathematical model to estimate these quantities -- since there are no direct measurements for T_f and T_c within a fuel assembly within the UMLRR.

With estimates of ΔT_f and ΔT_c, and estimates of α_{Tf} and α_{Tc}, then an estimate of ρ_{temp} is given by

$$\Delta\rho_{temp} = \alpha_{T_f}\Delta T_f + \alpha_{T_c}\Delta T_c$$

Clearly there are too many "estimates" here, but this is all we have to work with…

Fuel & Coolant Temperatures

So now, let's discuss a mathematical model to estimate the

fuel & coolant temperatures within the UMLRR

to be continued -- see Part II…

If we have not already done so, let's take a
15 minute break…

When we come back, we will overview a simple **steady state thermal model** for predicting the **fuel and coolant temperatures** within the UMLRR…