24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Integral Worth Curves: Theory and Measurement Techniques

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Discussion Outline

Review from previous class: Reactivity Measurements

Brief reviews: Student Presentations

Discussion: All…

Integral Worth Curves

Theoretical Overview & Examples for Three Measurement Techniques

We will take a short break after a little theory…

Homework #8 (see details in rexpts_hw8sp18.pdf)

We have LOTS of Material to cover today -- so let's get started…

Topic Overview

The worth of the control devices within any reactor is one of the most important quantities that must be measured for each system.

For any device that is utilized for routine reactivity control during daily operations, full integral rod worth curves are required.

This lecture and lab combination will address how to develop, interpret, and measure the integral blade worth curves for the UMLRR.

The first step is to establish the basic concepts and terminology, and this will be done using Perturbation Theory Methods and a simple 1-group model of an idealized bare homogeneous reactor.

Although very simplistic, this theoretical treatment allows us to observe a typical worth profile and to establish some insight into what to expect in a real reactor.

Topic Overview (cont.)

For real operating systems, however, the rod or blade worth curves are always established by actual measurement, and the real focus here is to discuss typical techniques for doing this.

We will highlight three experimental methods for generating the desired worth profiles within the UMLRR:

Stable Period Method -- primary tool used by the operations staff for the first 35+ years of operation (1975 to late 2012)

Inverse Count Rate Method -- performed during subcritical operation, but only gives a normalized worth profile (need total worth from some other method)

Inverse Kinetics Method -- relatively new capability within the UMLRR (has now become primary tool for blade worth calibrations)…

Theoretical Development

We will start our development with the operator form of the steady state diffusion equation for a critical reactor,

$$\left(L - \lambda F\right)\phi = 0$$

where $\lambda = 1/k$ and $\rho = (k-1)/k$ relate the eigenvalue, λ, multiplication factor, k, and reactivity, ρ.

Now, for two different states (i.e. a reference and perturbed configuration), we have

$$\Delta\lambda = \lambda' - \lambda = \frac{1}{k'} - \frac{1}{k} = \frac{k - k'}{k'k}$$

and

$$\Delta\rho = \rho' - \rho = \frac{k'-1}{k'} - \frac{k-1}{k} = \frac{k'-k}{k'k}$$

Thus we see that $\Delta\rho = -\Delta\lambda$ -- which says that a reactivity change or worth is simply related to a change in the eigenvalue.

First-Order Perturbation Theory

Now, we multiply the diffusion equation by an arbitrary space and energy dependent weight function, $\theta(\vec{r},E)$, and integrate over all space and energy to give an expression for the eigenvalue λ,

$$\left\langle \theta\left(L - \lambda F\right)\phi \right\rangle = 0 \qquad \text{or} \qquad \lambda = \frac{\langle \theta L \phi \rangle}{\langle \theta F \phi \rangle}$$

If, for example, a perturbation is made in the absorption cross section, Σ_a, in some region in the core, then both the L operator and the flux distribution will change, and the first-order variation in λ associated with $\Delta\Sigma_a$ can be written as

$$\Delta\lambda = \left\langle \frac{\partial\lambda}{\partial\Sigma_a}\Delta\Sigma_a \right\rangle + \left\langle \frac{\partial\lambda}{\partial\phi}\Delta\phi \right\rangle + \text{ higher-order terms } \approx \left\langle \frac{\partial\lambda}{\partial\Sigma_a}\Delta\Sigma_a \right\rangle + \left\langle \frac{\partial\lambda}{\partial\phi}\Delta\phi \right\rangle$$

where we have replaced the change in the L operator by the explicit change in Σ_a (none of the other components of L will change if we only perturb Σ_a).

First-Order Perturbation Theory (cont.)

This result is referred to as the First-Order Perturbation Theory (FOPT) estimate, since we dropped all the higher order terms in $\Delta\Sigma_a$ and $\Delta\phi$, or

$$\Delta\lambda \approx \left\langle \frac{\partial\lambda}{\partial\Sigma_a}\Delta\Sigma_a \right\rangle + \left\langle \frac{\partial\lambda}{\partial\phi}\Delta\phi \right\rangle$$

First-Order Perturbation Theory representation for $\Delta\lambda$

Now, expanding the derivative terms using the definition of λ gives

$$\Delta\lambda \approx \frac{\langle\theta\Delta\Sigma_a\phi\rangle}{\langle\theta F\phi\rangle} + \frac{\langle\theta L\Delta\phi\rangle}{\langle\theta F\phi\rangle} - \frac{\langle\theta L\phi\rangle\langle\theta F\Delta\phi\rangle}{\langle\theta F\phi\rangle^2} = \frac{\langle\theta\Delta\Sigma_a\phi\rangle}{\langle\theta F\phi\rangle} + \frac{\langle\theta(L-\lambda F)\Delta\phi\rangle}{\langle\theta F\phi\rangle}$$

direct effect & *indirect effect*

The integral containing $\Delta\Sigma_a$ involves only a *local integration* and the integral containing the $\Delta\phi$ distribution is a *global integral* -- since $\Delta\Sigma_a$ is non-zero only in the location of the perturbation, yet this change causes the flux to vary throughout the system.

The Weight Function

To evaluate the previous expression for $\Delta\lambda$ for a given $\Delta\Sigma_a$, we need to know the perturbed flux, $\Delta\phi = \phi' - \phi$ -- and this is the real issue here!!!

That is, in practice, we would like to do this without re-solving the neutron balance equation for each variation in the system.

Thus, the goal here is to make a judicious choice for $\theta(\vec{r},E)$ such that the global/indirect term vanishes completely.

The importance of being able to do this cannot be overstated, since it says we can get a good estimate of $\Delta\rho = -\Delta\lambda$ without knowledge of $\Delta\phi$ -- and this is the real power of First-Order Perturbation Theory methods!!!

To perform this "magic", we simply let the weight function be the adjoint flux, or,

$$\theta(\vec{r},E) = \phi^*(\vec{r},E)$$

Adjoint Operators & Functions

("one of the greatest inventions of all time")

To elaborate upon and justify the statement that $\theta = \phi^*$, we introduce the definition of an adjoint operator.

In particular, let's use H* as the *adjoint* of operator H, where H* is defined precisely by the equality

$$\boxed{\langle vHu \rangle = \langle uH^*v \rangle + \text{boundary terms}}$$

This is the definition of the H* operator

where u and v are general functions defined over the same phase space, they satisfy the same type of operator equation, and they have the same kind of boundary conditions.

For our current application involving the neutron diffusion equation, Hu is replaced by $(L - \lambda F)\phi$ and we will write the v function as ϕ^*, where this function is usually referred to as the regular adjoint flux or just simply the adjoint flux.

Adjoint Operators & Functions (cont.)

We now rewrite the basic definition using the specific notation of interest to reactor theory applications, or

$$\langle \phi^*(L-\lambda F)\phi \rangle = \langle \phi(L-\lambda F)^*\phi^* \rangle + BT$$

where BT refers to the "boundary terms".

In most cases, the BTs will vanish with appropriate definition of the boundary conditions for the operator equations -- thus, BT = 0 for our current discussions (see the formal Lecture Notes for further justification of this statement).

Also, with the functions ϕ and ϕ^* satisfying the same type of operator equation and same kind of boundary conditions, we propose, based on the above definition with BT = 0, that the adjoint flux, ϕ^*, satisfy the equation

$$\boxed{(L-\lambda F)^*\phi^* = 0}$$

This is the definition of the adjoint flux, ϕ^*

First-Order Perturbation Theory (cont.)

We now apply the basic definition of an adjoint operator to the numerator of our FOPT expression for $\Delta\lambda$, which gives

$$\langle \phi^*(L-\lambda F)\Delta\phi\rangle = \langle \Delta\phi \underbrace{(L-\lambda F)^*\phi^*}_{0}\rangle + 0 = 0$$

Thus, this term simply goes to zero if the weight function is chosen to be the adjoint flux (i.e. when $\theta = \phi^*$).

This result reduces the FOPT expression for the reactivity change from critical due to a change in material composition or an absorption cross section to

$$\boxed{\Delta\rho = -\Delta\lambda = -\frac{\langle \phi^* \Delta\Sigma_a \phi\rangle}{\langle \phi^* F\phi\rangle}}$$

Final First-Order Perturbation Theory representation for $\Delta\lambda$

and this result was the ultimate goal of this portion of our development!

First-Order Perturbation Theory

(Final Comments)

The goal of this introduction to Perturbation Theory Methods was to define what is meant by the adjoint flux and to justify that $\theta = \phi^*$ is the best choice for the weight function.

However, we purposely left out a lot of detail so that our primary goal was not clouded with too much notation and mathematics.

However, to get a feel for what some of the abstract integral operator notation (such as $<\phi^*H\phi>$) really means, a few explicit detailed examples are given in the formal Lecture Notes.

In addition, the full multigroup, 2-group, and 1-group adjoint diffusion equations are given along with a comparison to the their forward counterparts (in particular, the 1-group equation is self-adjoint!!!).

These additional Lecture Notes simply add some specificity to the operator notation used here -- and these details can be quite useful when actually applying the theory presented here…

Ideal Rod Worth Distributions

Using ***Perturbation Theory Methods***, we have shown that the **worth of a material inserted to an axial depth z** within the reactor is **proportional to the product of the forward and adjoint fluxes** integrated over the perturbed domain.

In particular, assuming **1-group theory** and that **movement of the control rod only perturbs the absorption cross section**, we have

$$\rho_w(z) = \alpha \int_0^z \phi^*(z') \Delta\Sigma_a(z') \phi(z') dz'$$

Recall that $\Delta\rho = -\Delta\lambda = -\frac{\langle \phi^* \Delta\Sigma_a \phi \rangle}{\langle \phi^* F \phi \rangle}$

where α is a proportionality constant and **ϕ^* is the adjoint flux** or **importance function**.

However, since the **1-group diffusion equation is self-adjoint**, the adjoint and forward fluxes are identical -- thus

$$\boxed{\rho_w(z) = \alpha \int_0^z \phi^2(z') \Delta\Sigma_a(z') dz'}$$

1-group theory

Ideal Rod Worth Distributions (cont.)

Now, for a **bare 1-D homogeneous critical reactor of height H**, the **axial flux profile** is given by

$$\phi(z) = A \sin Bz \qquad \text{with} \quad B^2 = \left(\frac{\pi}{H}\right)^2$$

where **z is measured from the top of the reactor**.

If the **rod absorption cross section is constant**, then

$$\rho_w(z) = C \int_0^z \sin^2 \frac{\pi z'}{H} dz' = C \left[\frac{z'}{2} - \frac{H}{4\pi} \sin \frac{2\pi z'}{H} \right]_0^z = C \frac{H}{2} \left(\frac{z}{H} - \frac{1}{2\pi} \sin \frac{2\pi z}{H} \right)$$

where **C is just a new proportionality constant**.

To evaluate this constant, we let $\rho_w(z)|_{z=H} = \rho_w(H) = \rho_{tot}$, which is the **total rod worth**. With this constraint we have

$$\rho_w(H) = C \frac{H}{2}(1 - 0) \qquad \text{or} \qquad C = \frac{2}{H} \rho_w(H)$$

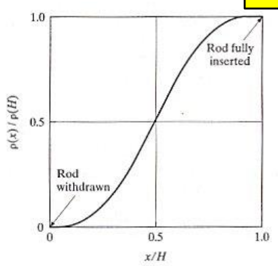
Ideal Rod Worth Distributions (cont.)

Upon substitution, the so-called *ideal integral worth distribution* becomes

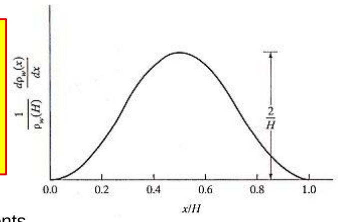
$$\rho_w(z) = \rho_w(H)\left(\frac{z}{H} - \frac{1}{2\pi}\sin\frac{2\pi z}{H}\right)$$

Also of interest is the rate of change of $\rho_w(z)$ per unit distance. This differential worth can easily be obtained by differentiation,

$$\frac{d}{dz}\rho_w(z) = \frac{\rho_w(H)}{H}\left(1 - \cos\frac{2\pi z}{H}\right)$$



When plotted, these distributions give the familiar ideal integral and differential rod worth curves…



Ideal Rod Worth Distributions
(Final Comments)

The rod worth curves for real reactor systems differ somewhat from the ideal curves shown above (note that these were developed using FOPT for a bare homogeneous 1-group system -- a pretty idealized situation indeed!!!).

However, they do give a good qualitative view of reality, with low differential worth near the upper and lower boundaries and a peak differential worth near the core center.

In a real reactor, if the control rods are inserted from the top, then the worth distribution often tends to be slightly bottom peaked (assuming that everything else is axially symmetric).

However, to a first approximation, the ideal distributions should help establish a reasonable set of expectations for the measure-ment of blade or rod worth curves for most real systems -- with the added expectation that some asymmetry may be observed…

Now we will overview three methods for actually **measuring** the blade worth curves…

Three Measurement Techniques

For real operating systems, the rod or blade worth curves are always established by actual measurement, and our focus now shifts to a discussion of typical techniques for doing this.

We will highlight three experimental methods for generating the desired worth profiles:

Stable Period Method -- primary tool used by the UMLRR operations staff for the first 35+ years of operation (1975 to late 2012).

Inverse Count Rate Method -- performed during subcritical operation, but only gives a normalized worth profile (need total worth from some other method).

Inverse Kinetics Method -- recently developed capability at UMass-Lowell for use within the UMLRR (has become the primary tool for blade worth calibrations since 2013)…

Stable Period Method

Procedure:

1. From near full insertion with the reactor at a low-power critical state, move the blade of interest (BOI) out a small Δz.
2. After a short transient time, measure the doubling time and reactor period and use the reactivity equation to convert this to a reactivity change, $\Delta\rho$.
3. This gives one data point, $\Delta\rho/\Delta z$, at the midpoint of the blade position for the given interval.
4. Use the remaining blades to return to a just critical condition at the new reference location for the BOI (this step is time consuming).

Repeat this sequence to get good coverage of the full blade traverse, and then fit the full data set to a mathematical model for the differential worth distribution, such as

$$\frac{d}{dz}\rho_w(z) = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 \cos\left(\frac{2\pi z}{H}\right)$$

Simply integrate this to get the integral worth curve

Stable Period Method -- An Example

Raw Simulated + Computed Data

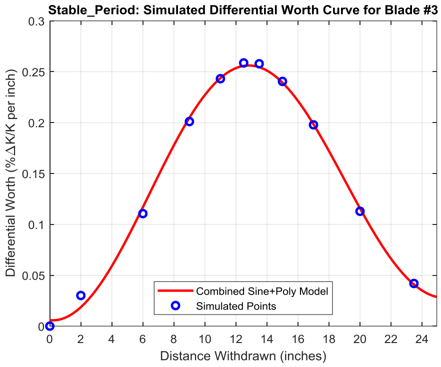
Initial Ht. (in)	Final Ht. (in)	Doubling Time (sec)	Midpoint (in)	Diff. Worth (%Dk/k per in)
0.00	0.00	0.0	0.00	0.0000
0.00	4.00	36.4	2.00	0.0301
4.00	8.00	2.6	6.00	0.1106
8.00	10.00	3.5	9.00	0.2010
10.00	12.00	1.9	11.00	0.2431
12.00	13.00	9.8	12.50	0.2586
13.00	14.00	9.9	13.50	0.2578
14.00	16.00	2.0	15.00	0.2405
16.00	18.00	3.6	17.00	0.1978
18.00	22.00	2.4	20.00	0.1129
22.00	24.95	35.0	23.48	0.0419

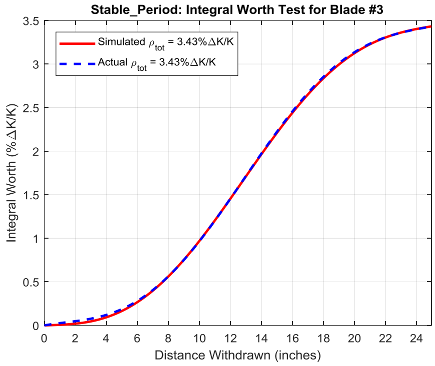
```
Coeffs for Combined Poly-Sinusoid Differential Worth Model:
    1.2172e-01  -1.5154e-03   3.7768e-04  -1.1234e-05  -1.1589e-01

Curve fit coeff of determination (r-squared):    0.9974
Total worth based on curve fit (%Dk/k):          3.4322
Actual total worth of Blade 3 (%Dk/k):           3.4309
```

Output from bw_stable_period.m

Blade 3 was used to simulate the reactivity transients.

Stable Period Method -- An Example





Check out the Matlab code?

Inverse Count Rate Method

This method has its basis in the subcritical multiplication factor techniques that we have discussed previously.

The method can be developed easily with just a few definitions:

- **ρ_{in} = subcriticality level with the BOI fully inserted**
- **ρ_{out} = subcriticality level with the BOI fully withdrawn**
- **$\rho_{tot} = \rho_{out} - \rho_{in}$ = total worth of the control rod or blade**

Now, we can define the integral worth of the blade at any z location as

$$\rho_w(z) = \Delta\rho(z) = \rho(z) - \rho_{in}$$

and multiplying by unity in the form of $\rho_{tot}/(\rho_{out} - \rho_{in})$ gives

$$\rho_w(z) = \left(\rho(z) - \rho_{in}\right)\frac{\rho_{tot}}{\rho_{out} - \rho_{in}} = \rho_{tot}\left(\frac{\rho(z) - \rho_{in}}{\rho_{out} - \rho_{in}}\right)$$

Inverse Count Rate Method (cont.)

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However, with some assumptions about the proportionality constant not changing much from one configuration to another, we have

$$M_r = \frac{C_1}{C_0} \approx \frac{\alpha_1}{\alpha_0}\frac{\rho_0}{\rho_1} \approx \frac{\rho_0}{\rho_1} \quad \text{or} \quad \frac{C(z)}{C_{in}} \approx \frac{\rho_{in}}{\rho(z)} \quad \text{or} \quad \rho(z) \approx \frac{C_{in}}{C_z}\rho_{in}$$

and, upon substitution, this gives

$$\rho_w(z) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z}\rho_{in} - \rho_{in}}{\frac{C_{in}}{C_{out}}\rho_{in} - \rho_{in}}\right) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z} - 1}{\frac{C_{in}}{C_{out}} - 1}\right) = \rho_{tot}\left(\frac{\frac{C_{in}}{C_z} - \frac{C_{in}}{C_{in}}}{\frac{C_{in}}{C_{out}} - \frac{C_{in}}{C_{in}}}\right)$$

or

$$\frac{\rho_w(z)}{\rho_{tot}} = \left(\frac{\frac{1}{C_z} - \frac{1}{C_{in}}}{\frac{1}{C_{out}} - \frac{1}{C_{in}}}\right)$$

As implied by the name Inverse Count Rate Method, the normalized blade worth profile is given in terms of the inverse detector count rates.

Inverse Count Rate Method (cont.)

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Procedure:

1. Record the detector count rate with the blade in its fully inserted position with the reactor at a subcritical state -- C_{in}.
2. Move the blade of interest (BOI) out a small Δz.
3. After a short transient time, measure the steady-state count rate, C_z, for the current blade position z.

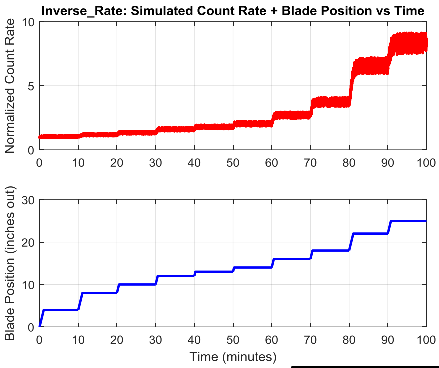
Repeat Steps 2 and 3 to get good coverage of the full blade traverse, and then fit the full data set to a mathematical model for the normalized integral worth distribution, such as

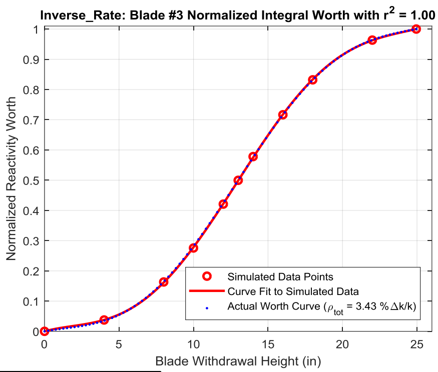
$$\frac{\rho(z)}{\rho_{tot}} = c_1 z^4 + c_2 z^3 + c_3 z^2 + c_4 z + c_5 + c_6 \sin\left(\frac{2\pi z}{H}\right)$$

To get the actual $\rho(z)$, ρ_{tot} must be determined by some other method

Note that this only gives the shape of the integral worth curve

Inverse Count Rate -- An Example





Output from
bw_inverse_rate.m
Blade 3 was used to simulate the reactivity transients.

Check out the Matlab code?

Inverse Point Kinetics

Forward reactor dynamics model

Inverse reactor dynamics problem

See separate presentation on Inverse Kinetics

In the inverse problem, the signal flow is reversed -- that is, given the observed power vs. time behavior, P(t), as the known "input", we want to compute the "output" ρ(t). This perspective is quite different in that we put on our "detective hat" and by observing some measureable system behavior, we try to determine what actually caused the observed response. This is the goal of all inverse problems...

Inverse Kinetics Method

For obtaining blade worth curves, the issue with the "drift" in the $\rho(t)$ prediction due to gamma interference dictates that the deviation from critical should be held within about ± 0.4 %Δk/k and the power swing, especially on the low side, should not be much greater than a factor of 10 below the reference value.

Thus, for applications involving the measurement of the full blade worth curve, the negative reactivity addition associated with the blade of interest (BOI) being inserted into the core must be balanced by the other blades in the system.

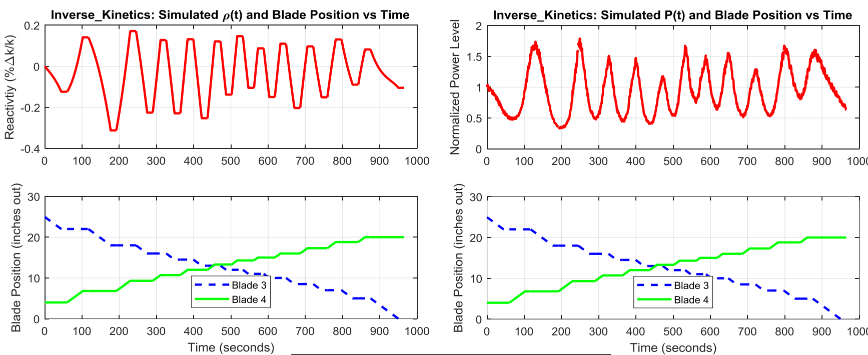
Procedure:

Perform a series of blade movements at low power as follows:

Ramp the BOI in a small amount, wait a few seconds, then remove the other blades as needed to over-compensate the negative insertion and to bring the power back up into the top half of the given range (usually 1 – 15 kW).

Then, as quickly as possible, repeat this sequence as many times as necessary to get the BOI fully inserted.

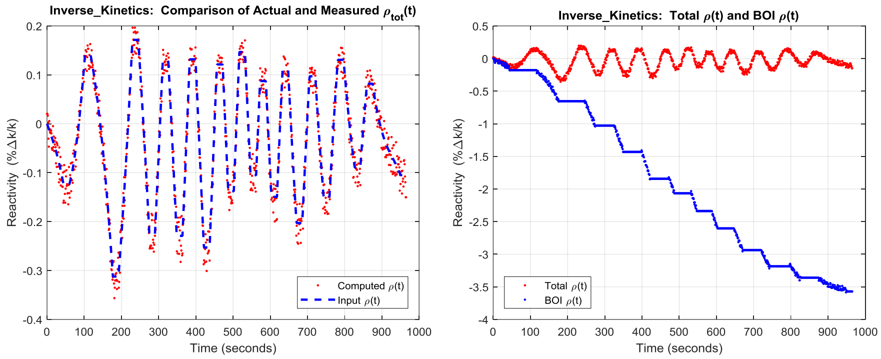
Inverse Kinetics -- An Example



Output from 1st part of bw_inverse_kinetics.m

Note that $\rho(t)$ and P(t) follow the above guidelines.

Inverse Kinetics -- An Example



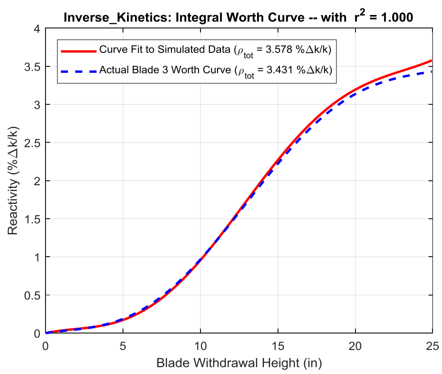
Output from 2^nd^ part of bw_inverse_kinetics.m

The rebank_adjust routine is used to separate $\rho_{BOI}(t)$ from $\rho_{tot}(t)$.

24.536 Reactor Experiments
Integral Worth Curves: Theory and Measurement Techniques

(March 2018)

Inverse Kinetics -- An Example



Output from last part of bw_inverse_kinetics.m

Coding to do the curve fit was taken directly from the umlrr_data GUI

Mathematical Model:

$$\rho(z) = c_1 z^4 + c_2 z^3 + c_3 z^2 + c_4 z + c_5 + c_6 \sin\left(\frac{2\pi z}{H}\right)$$

and the actual worth for the integral blade worth curve is given as:

$$\rho_w(z) = \rho(z) - \min(\rho(z))$$

Check out the Matlab code?

24.536 Reactor Experiments
Integral Worth Curves: Theory and Measurement Techniques

(March 2018)

Summary and Take-Aways

This Lecture first introduces the theoretical concepts needed for the discussion of blade worth curves, and it then summarizes three techniques for actually measuring the blade worth profiles within a real system.

The three experimental methods -- Stable Period Method, Inverse Count Rate Method, and Inverse Kinetics Method -- are each demonstrated via simulation within a sequence of Matlab codes.

These simulations show the basic procedures involved and they validate that each of the methods is theoretically sound and has been implemented correctly.

The key take-aways should be a good understanding of the theoretical basis for the ideal worth curves seen in the literature, and for the various reactivity measurement techniques that can be used to determine these worth profiles in real systems.

Measuring An Integral Worth Curve

In our upcoming lab, the actual lab session will focus on the Inverse Kinetics Method.

The lab will be broken into two phases:

Phase I -- perform a series of movements with the RegBlade to observe the P(t) response and, as part of the post-lab exercises, use inverse kinetics to "measure" the $\rho(t)$ that was inserted.

By comparing to the known RegBlade worth curve, this sequence should help validate the overall Inverse Kinetics methodology…

In addition, we can also simulate the $\rho(t)$ sequence using the point kinetics equations and compare how our simulated P(t) compares to the measured P(t) -- gives check on overall simulation capability…

Measuring An Integral Worth Curve

Phase II -- perform a series of alternating blade insertions with the blade of interest (BOI) and withdrawals of the other blades (as demonstrated in the example) to actually measure the full integral blade worth curve for the BOI.

This simple 2-step procedure will be used in our Blade Worth Curve Measurement Lab

HW #8 asks you to review the theory discussed here and to "design the proposed sequence" to be used in the Phase 1 portion of the upcoming Blade Worth Curve Measurement Lab .
(see details in rexpts_hw8sp18.pdf)