24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Reactivity Measurement Techniques

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Discussion Outline

Review from previous class: Approach to Critical Lab

Brief review: Student Presentations

Not this semester…

Discussion: All…

Reactivity Measurement Techniques

Overview & Examples of Four Different Methods

Take a short break -- maybe after Method #1???

Homework #6 (see details in rexpts_hw6sp18.pdf)

Topic Overview

Reactor systems are usually configured within one of the following states:

subcritical -- system either has control inserted or the core is not fully loaded with sufficient fuel for criticality

critical -- operating at steady state (mostly near full power, but sometimes at cold zero power or hot zero power during startup testing)

near critical -- either slightly subcritical or slightly super critical during startup, shutdown, or during power maneuvers

In all cases, it is essential to know the reactivity level of the given configuration and the reactivity worth of the control devices.

In this lecture, we will address different ways to measure both reactivity level and reactivity worth.

Four Measurement Techniques

Initial System is Critical

Asymptotic Period Technique -- limited to small positive and negative reactivity changes from critical

Rod Drop Method -- used to measure large negative reactivity insertions from critical

Initial System is Subcritical

Source Jerk Method -- can measure the absolute subcriticality level, ρ_o

Subcritical Multiplication Factor Approach -- useful for determining changes in reactivity during subcritical operation (must know reference ρ_o)

Each method is developed individually, along with a simple Matlab simulation that illustrates the basic application of the method (using simulated data).

The Starting Point for Each Method

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Generation Time Formulation of Point Kinetics

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \qquad \text{for } i = 1, 2, \ldots 6$$

where P(t) is the power level in watts and <Q(t)> represents the total external source strength in neutrons/sec.

The relationship between the normalized neutron amplitude, T(t), and reactor power, P(t), is given by

$$T(t) = \frac{\nu}{\kappa}P(t)$$

Matlab's ode15s routine will also be used to solve these eqns. to allow us to quantitatively illustrate each method.

These expressions are the basis for much of the theoretical development for the various reactivity measurement techniques discussed here…

Asymptotic or Stable Period Method

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The solution to the point kinetics equations for a step change in reactivity in a critical system with no external source is given by the so-called reactivity equation.

For the Generation Time Formulation, the reactivity equation can be written as

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$

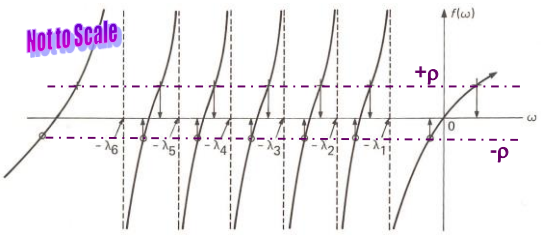
This is simply the characteristic equation associated with the original seven coupled linear constant coefficient ODEs -- and, for a given ρ, there will be seven distinct roots, and the root locations dictate the dynamics of the system.

For example, we can write a general solution for the time-dependent power level, P(t), as

$$P(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \cdots + A_7 e^{\omega_7 t}$$

Stable Period Method (cont.)

The ω_j are the roots of the reactivity equation as illustrated here.



positive ρ: one positive root and six negative roots

negative ρ: seven negative roots

If we order the roots from most positive to most negative then, after a relatively short transient time, the last six terms decay away (because $\omega_2, \omega_3, \cdots, \omega_7 < 0$), leaving only

$$P(t) = P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau} \quad \text{(after some transient time)}$$

where $\tau = 1/|\omega_1|$ is called the stable reactor period and P_1 is the power level after the short transient time.

Stable Period Method (cont.)

Measurement Philosophy: If a small reactivity change is made in a low-power critical system then, after a short transient time, the asymptotic power level will behave approximately as

$$P(t) = P_1 e^{\pm t/\tau}$$

and, from observation of the measured P(t), one should be able to measure the reactor period, τ -- which, via the reactivity equation, gives an indirect measurement of ρ, the reactivity change that initiated the transient in the first place.

In particular, taking the natural logarithm of the P(t) equation gives a straight line,

$$\ln \frac{P(t)}{P_1} = \pm \frac{1}{\tau} t$$

with a slope $m = 1/\tau$, where m will be positive for $P(t)/P_1 > 1$ and negative for $P(t)/P_1 < 1$.

Stable Period Method (cont.)

Once the asymptotic period is known, the reactivity equation evaluated with $\omega = \omega_1 = 1/\tau$ gives

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i} = \frac{\Lambda}{\tau} + \frac{1}{\tau}\sum_i \frac{\beta_i}{\frac{1}{\tau} + \lambda_i} \approx \frac{1}{\tau}\sum_i \frac{\beta_i}{\frac{1 + \lambda_i \tau}{\tau}}$$

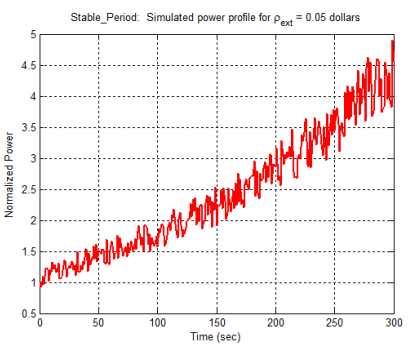
or

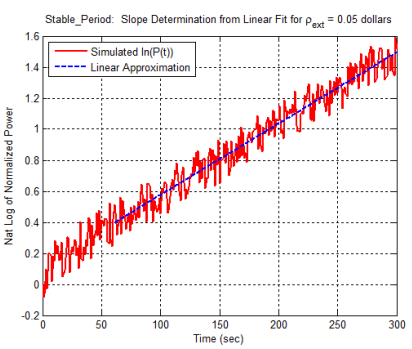
$$\rho = \sum_i \frac{\beta_i}{1 + \lambda_i \tau}$$

Implementation:

- **Take the natural logarithm of the P(t) data**
- **Do a linear fit to determine the slope, m**
- **Compute the reactor period as $\tau = 1/m$**
- **Use the reactivity equation to find ρ**

Stable Period Method – An Example

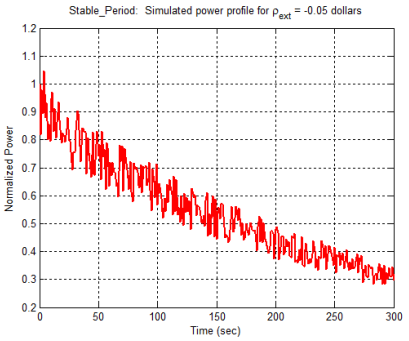


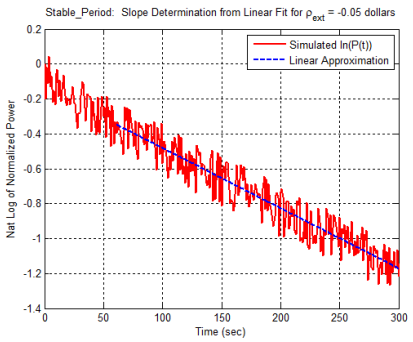


Small Positive Step Change in Reactivity
ρ = +0.05 dollars

Measured stable period (sec): 217.76
Measured rho (dollars): 0.0505
Error in Predicted rho (%): 1.05

Stable Period Method – An Example





Small Negative Step Change in Reactivity
ρ = -0.05 dollars

Measured stable period (sec): -290.34
Measured rho (dollars): -0.0507
Error in Predicted rho (%): 1.39

Let's check out the Matlab code …

Doubling Time Estimate

Another common approach for obtaining τ is to measure the doubling time, t_d.

That is, t_d is the time it takes for the reactor power to change by a factor of two after a short transient time following the initial reactivity change.

Mathematically, this can be written as

$$\frac{P(t_d)}{P_1} = 2 = e^{t_d/\tau}$$ **and this gives** $$\tau = \frac{t_d}{\ln 2}$$

Thus, the desired asymptotic period can be easily obtained with a simple measurement of the doubling time in the system following a reactivity change.

Doubling Time Estimate (cont.)

Note also that, for a negative reactivity, the same expression is obtained if we associate t_d with the time it takes the flux amplitude or power to decrease by a factor of two.

Finally we note that, although the doubling time method is easy to use, the noisy detector signals that are characteristic of most real systems can lead to some uncertainty in selecting t_d and in the measurement of τ.

We can use the doubling time approach during the actual lab to estimate the reactor period

If the detector signal is indeed noisy, then determining the period from a linear fit to the ln $P(t)/P_1$ profile will probably give the best result.

Then we will perform a formal linear fit to the raw data in the post-lab exercises to get a better value of τ

Limitations

For large reactivity changes, the stable period method breaks down for both positive and negative ρ -- for quite different reasons.

For large positive ρ, the reactor period becomes too small such that P(t) increases too rapidly, quickly causing an unsafe reactor condition that clearly must be avoided in all cases.

In addition, even for moderate $+\rho$, the reactor power often approaches a level where we can no longer assume that the feedbacks are negligible (which is implicit in the constant ρ assumption).

Thus, positive reactivity changes beyond about 0.10 dollars simply cannot be measured with the stable period method (actually there is no simple dynamic method that can be used to measure large positive changes in reactivity).

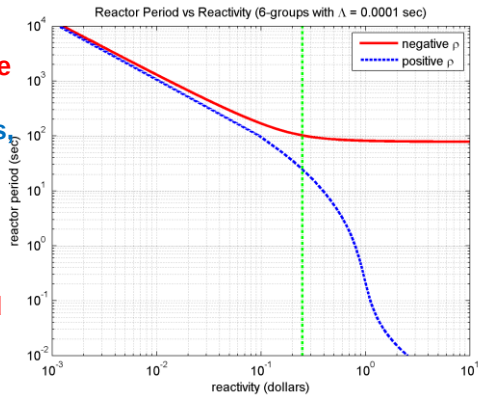
Limitations (cont.)

For large negative ρ, the situation is quite different.

Here the reactor power is decreasing, so safety is not the concern.

But, τ vs. ρ approaches a constant for large negative ρ values -- thus, beyond about -0.10 to -0.20 dollars, it is not possible to relate a unique combination of τ and ρ via the reactivity equation.

Thus, an alternate method is clearly needed for measuring large negative ρ -- as discussed next...



Let's take a short break…

When we come back, we will develop the theory behind the Rod Drop and Source Jerk Methods…

Rod Drop and Source Jerk Methods

As noted, the stable period method only works for small reactivity changes in critical systems.

Thus, we definitely need to expand on our capability to measure ρ for a wider variety of situations.

Two techniques that do this for us include the Rod Drop and Source Jerk methods:

- Rod Drop Method -- used to measure large negative reactivity insertions in critical systems
- Source Jerk Method -- used to determine the absolute reactivity level of subcritical systems

Both situations are of interest in practical applications!!!

Although these methods address different situations, they share a common theory, so will address both methods within a single development…

Rod Drop & Source Jerk Methods (cont.)

As implied by the methods' names, the intent here is to either introduce a large negative step change in reactivity into a critical system (for the Rod Drop case), or to make a source perturbation to a subcritical system by instantaneously removing the external source (for the Source Jerk method).

Both these instantaneous changes occur at t = 0 and they lead to the following situations:

for $t > 0^+$, ρ = constant and $\int_{0^+}^{\infty} \langle Q(t) \rangle dt = 0$

Both these statements are true for both the Rod Drop and Source Jerk scenarios:

-- for the critical case, there is no source present

-- for the subcritical case, the source is zero for $t > 0^+$

so the 2nd statement is true in both cases

Rod Drop & Source Jerk Methods (cont.)

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-- a "step insertion" is associated with the Rod Drop case

-- the assumption is made that the instantaneous removal of the source only affects the source strength, not the reactivity level of the system (this is often only a rough approximation)

so the 1st statement is approximately true in both cases

In our development, we take these two statements as valid assumptions for both the Rod Drop and Source Jerk methods.

To develop the theory for both methods, we start with the generation time formulation and integrate from 0^+ to ∞,

these already include the above approximations

$$\int_{0^+}^{\infty} dP = \frac{(\rho-\beta)}{\Lambda}\int_{0^+}^{\infty} P(t)dt + \sum_i \lambda_i \int_{0^+}^{\infty} c_i(t)dt$$

$$\int_{0^+}^{\infty} dc_i = \frac{\beta_i}{\Lambda}\int_{0^+}^{\infty} P(t)dt - \lambda_i \int_{0^+}^{\infty} c_i(t)dt \qquad \text{for } i = 1, 2, \cdots 6$$

Rod Drop & Source Jerk Methods (cont.)

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We now solve the precursor equation for the $\lambda_i \int_{0^+}^{\infty} c_i(t)dt$ term, and substitute this result into the integral neutron balance equation, to give

$$\int_{0^+}^{\infty} dP = \frac{\rho}{\Lambda}\int_{0^+}^{\infty} P(t)dt - \frac{\beta}{\Lambda}\int_{0^+}^{\infty} P(t)dt + \frac{\beta}{\Lambda}\int_{0^+}^{\infty} P(t)dt - \sum_i \int_{0^+}^{\infty} dc_i$$

and, with the middle terms cancelling, we have

$$\rho = \frac{\Lambda\left[\int_{0^+}^{\infty} dP + \sum_i \int_{0^+}^{\infty} dc_i\right]}{\int_{0^+}^{\infty} P(t)dt}$$

The exact differentials can be integrated to give:

$$\int_{0^+}^{\infty} dP = P_{\infty} - P_{0^+} \approx -P_{0^+}$$

$$\int_{0^+}^{\infty} dc_i = c_{i\infty} - c_{i0^+} \approx -c_{i0^+}$$

$\longrightarrow$ assumes power and precursors become negligible as $t \to \infty$

$$\rho = \frac{-\Lambda\left[P_{0^+} + \sum_i c_{i0^+}\right]}{\int_{0^+}^{\infty} P(t)dt}$$

Rod Drop Method

For this case, the system is initially at critical with no source, so the initial conditions are

$$P(0^-) = P_{0^-} \qquad \text{and} \qquad c_{i0^-} = \frac{1}{\Lambda}\frac{\beta_i}{\lambda_i}P_{0^-} \quad \text{for } i = 1, 2, \ldots 6$$

where the 0^- notation implies before the instantaneous perturbation is made.

Also, ρ in the highlighted equation represents the worth of the rod that is instantly inserted into the critical system at t = 0.

Now, the relationship between quantities evaluated at $t = 0^-$ and at $t = 0^+$ is associated with the usual prompt jump/drop effect due to the fast response of the prompt neutrons.

Thus, because of the very rapid change due to the prompt neutrons, we have that $P_{0+} \neq P_{0-}$ but, due to the much slower response of the delayed neutrons, we can argue that $c_{i0+} = c_{i0-}$.

Rod Drop Method (cont.)

Making the indicated substitutions into the reactivity equation for this situation gives

$$\left|\frac{\rho}{\beta}\right| = \frac{\frac{\Lambda}{\beta}\left[P_{0^+} + \sum_i \frac{1}{\Lambda}\frac{\beta_i}{\lambda_i}P_{0^-}\right]}{\int_{0^+}^{\infty} P(t)dt} = \frac{\frac{\Lambda}{\beta}P_{0^+} + \left(\frac{1}{\beta}\sum_i \frac{\beta_i}{\lambda_i}\right)P_{0^-}}{\int_{0^+}^{\infty} P(t)dt} = \frac{\frac{\Lambda}{\beta}P_{0^+} + t_d P_{0^-}}{\int_{0^+}^{\infty} P(t)dt}$$

where, t_d is the mean lifetime of the delayed precursors

$$t_d = \frac{1}{\beta}\sum_i \frac{\beta_i}{\lambda_i}$$

As a last step, since $t_d >> \Lambda/\beta$, the above expression reduces to:

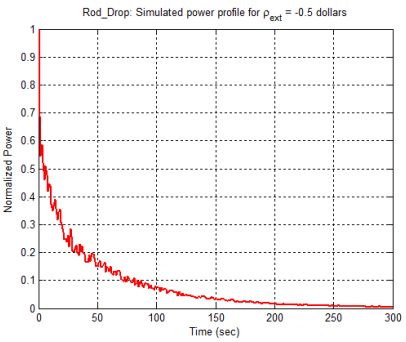
Implementation:

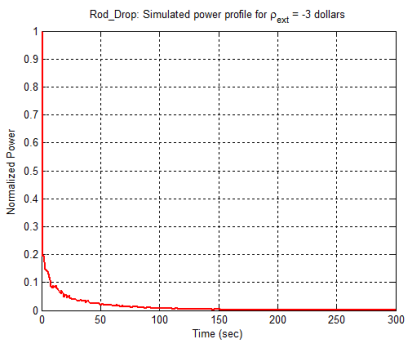
- Measure P(t)
- Numerically integrate P(t)
- Evaluate ρ/β

$$\left|\frac{\rho}{\beta}\right| = \frac{t_d P_{0^-}}{\int_{0^+}^{\infty} P(t)dt}$$

This is the final expression for the Rod Drop Method

Rod Drop Method – An Example





Actual reactivity used ($): **-0.500**
Measured reactivity worth ($): **-0.517**
Error in measured result (%): **3.48**

Actual reactivity used ($): **-3.000**
Measured reactivity worth ($): **-3.151**
Error in measured result (%): **5.02**

Check out the Matlab code?

Source Jerk Method

For this case, the system is initially subcritical with $\rho_{0-} = \rho_o$ being the degree of subcriticality.

The initial conditions for this system can be determined by setting dP/dt = 0 and $dc_i/dt = 0$, to give

$$P(0^-) = P_{0^-} = -\frac{1}{\rho_o}\frac{\kappa}{\nu}\langle Q(0^-)\rangle$$

and

$$c_{i0^-} = \frac{1}{\Lambda}\frac{\beta_i}{\lambda_i}P_{0^-} \quad \text{for } i = 1, 2, \ldots 6$$

Here it is important to note that these initial conditions are essentially identical with the Rod Drop case if we simply think of P_{0-} as the initial power level and don't dwell on how the steady state neutron level is developed (i.e. critical vs. subcritical).

Source Jerk Method

With identical initial conditions, we have a similar development, resultant formula, and implementation scheme as given above to measure the desired system subcriticality, ρ_o, or

$$\left|\frac{\rho_o}{\beta}\right| = \frac{t_d P_{0^-}}{\int_{0^+}^{\infty} P(t)dt} = \frac{t_d T_{0^-}}{\int_{0^+}^{\infty} T(t)dt}$$

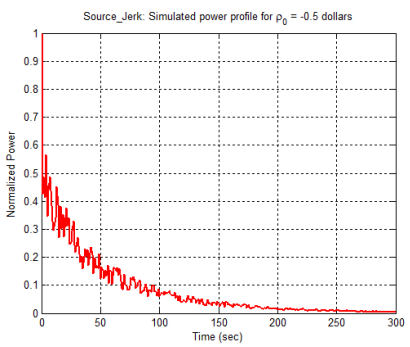
This is the final expression for the Source Jerk Method

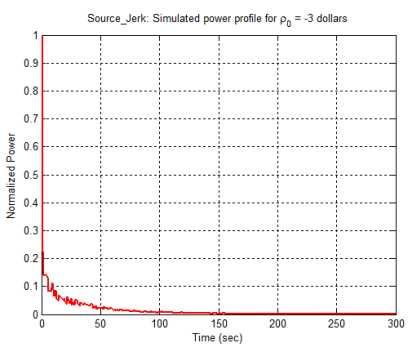
Notes:

The Source Jerk Method is used at subcritical conditions where the power level is low.

In the UMLRR, this requires use of the startup counter (SUC), where the ratio of the detector count rates is used to evaluate the above expression, since $C(t)/C_o = T(t)/T_o = P(t)/P_o$.

Source Jerk Method – An Example





Actual subcriticality ($):	-0.500
Measured subcriticality ($):	-0.522
Error in measured result (%):	4.47

Actual subcriticality ($):	-3.000
Measured subcriticality ($):	-3.246
Error in measured result (%):	8.20

Check out the Matlab code?

Subcritical Multiplication Factor Method

Thus far we can measure reactivity changes from critical and measure the absolute subcriticality level of a subcritical system, so the only situation that is missing is to account for reactivity changes while the system is still subcritical.

This situation was addressed to some extent in our previous Approach to Critical lab, where we introduced the concept of subcritical multiplication and the relative subcritical multiplication factor, M_r.

Recall that M_r is simply the ratio of detector count rates for two different subcritical configurations,

$$M_r = \frac{C_1}{C_0}$$

In addition, we also defined the absolute subcritical multiplication factor for the i^{th} state as $M_i = \frac{1}{1-k_i}$

Subcritical Multiplication Factor Method

With just basic definitions, we can write M_r in a number of useful forms,

$$M_r = \frac{C_1}{C_0} = \frac{\alpha_1 M_1 S}{\alpha_0 M_0 S} = \frac{\alpha_1 M_1}{\alpha_0 M_0} = \frac{\alpha_1 (1-k_o)}{\alpha_0 (1-k_1)} = \frac{\alpha_1 \left(1-\frac{1}{1-\rho_0}\right)}{\alpha_0 \left(1-\frac{1}{1-\rho_1}\right)} = \frac{\alpha_1}{\alpha_0}\frac{\rho_0}{\rho_1}\frac{1-\rho_1}{1-\rho_0} \approx \frac{\alpha_1}{\alpha_0}\frac{\rho_0}{\rho_1} \approx \frac{\rho_0}{\rho_1}$$

where ρ_0 and ρ_1 represent the two subcriticality levels for the two different states.

The change in reactivity from state 0 to state 1 is given by

$$\Delta\rho = \rho_1 - \rho_0 = \rho_0\left(\frac{\rho_1}{\rho_0} - 1\right) = \rho_0\left(\frac{1}{M_r} - 1\right) = \rho_0\left(\frac{1-M_r}{M_r}\right) = \rho_0\left(\frac{1-C_1/C_0}{C_1/C_0}\right)$$

Thus, if we know the subcriticality level for the reference state, ρ_0, then $\Delta\rho$ can be determined by comparing the steady state detector count rates before and after the configuration change.

Subcritical Multiplication Factor Method

A similar relationship for $\Delta\rho$ written in terms of the neutron amplitude at two different states can also be formally developed by starting with the point kinetics equations.

To see this, set the time derivatives to zero to establish a steady state relationship between the neutron and source amplitudes for each state,

$$P_0 = -\frac{1}{\rho_0}\frac{\kappa}{\nu}\langle Q_0\rangle \quad \text{or} \quad \rho_0 = -\frac{1}{P_0}\frac{\kappa}{\nu}\langle Q_0\rangle$$

$$P_1 = -\frac{1}{\rho_1}\frac{\kappa}{\nu}\langle Q_1\rangle \quad \text{or} \quad \rho_1 = -\frac{1}{P_1}\frac{\kappa}{\nu}\langle Q_1\rangle$$

If the source is not modified (i.e. $\langle Q\rangle = \langle Q_0\rangle = \langle Q_1\rangle$), then the reactivity change can be written as

$$\Delta\rho = \rho_1 - \rho_0 = \rho_0\left(\frac{\rho_1}{\rho_0} - 1\right) = \rho_0\left(\frac{P_0}{P_1} - 1\right)$$

Subcritical Multiplication Factor Method

To put the desired relationship in final form, we multiply and divide by P_1/P_0, to give

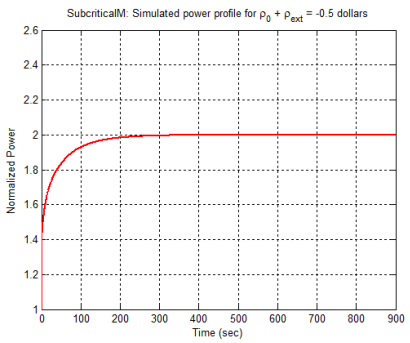
$$\Delta\rho = \rho_0\left(\frac{P_0}{P_1} - 1\right)\frac{P_1/P_0}{P_1/P_0} = \rho_0\left(\frac{1 - P_1/P_0}{P_1/P_0}\right) = \rho_0\left(\frac{1 - T_1/T_0}{T_1/T_0}\right)$$

which is clearly the same as on the previous slide which uses the ratio of count rates, C_1/C_0, since in all this development we have been saying that

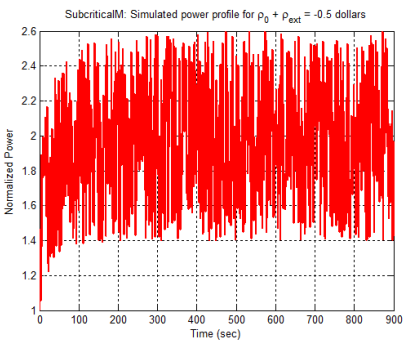
$$M_r = \frac{C_1}{C_0} = \frac{P_1}{P_0} = \frac{T_1}{T_0}$$

Thus, if we know any of these relative quantities (i.e. ratio of count rates, ratio of power levels, or ratio of flux amplitudes) and the reference subcriticality level, $\rho_0 = \rho_o$, then obtaining an estimate of $\Delta\rho$ is a rather trivial task (in theory).

Subcritical Multiplication Factor Example

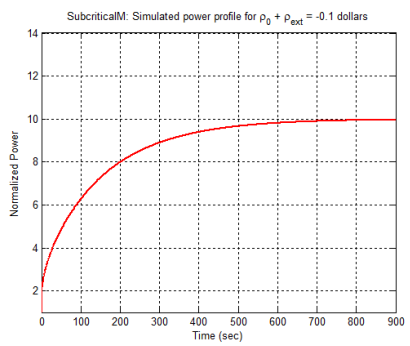


Subcriticality level($): **-1.000**
Actual delrho used ($): **0.500**
Measured delrho ($): **0.500**
Error in measured delrho (%): **-0.002**

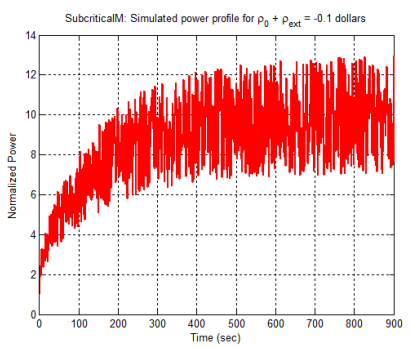


Subcriticality level($): **-1.000**
Actual delrho used ($): **0.500**
Measured delrho ($): **0.492**
Error in measured delrho (%): **-1.673**

Subcritical Multiplication Factor Example



Subcriticality level($): **-1.000**
Actual delrho used ($): **0.900**
Measured delrho ($): **0.899**
Error in measured delrho (%): **-0.098**



Subcriticality level($): **-1.000**
Actual delrho used ($): **0.900**
Measured delrho ($): **0.897**
Error in measured delrho (%): **-0.292**

Check out the Matlab code?

Summary and Take-Aways

This set of Lecture Notes summarizes four experimental techniques for measuring reactivity levels or reactivity changes within actual reactor systems:

- Asymptotic Period Technique and Rod Drop Method (starting from a critical system)
- Source Jerk Method and Subcritical Multiplication Factor Approach (starting from a subcritical system)

The four methods combine to give good coverage of most of the situations that can occur within real systems -- for both critical and subcritical configurations.

Summary and Take-Aways

In addition to the basic theory, actual use of the methods was also illustrated via simulation within a series of short Matlab codes so that one can better visualize how to apply these techniques in practical applications.

The key take-aways should be a good understanding of the various reactivity measurement techniques addressed here, and the confidence and knowledge of how to use these methods within subsequent experiments within the UMLRR...

HW #6 addresses the above items and provides the proper background for our upcoming Reactivity Measurements Lab
(see details in rexpts_hw6sp18.pdf)

Reactivity Measurements Lab

In our Reactivity Measurement Lab we will actually test out the above four reactivity measurement techniques within the UMLRR with the following reactor operations sequence:

Phase I -- use the Asymptotic Period Technique to measure the reactivity change associated with small positive and negative movements of the regulating blade (RegBlade) from its initial reference position.

Phase II -- disengage the electromagnet for one of the large control blades and drop the blade nearly instantaneously into the core, and then use the Rod Drop Technique to estimate the amount of reactivity that was inserted into the system.

Reactivity Measurements Lab (cont.)

Phase III -- Re-engage the blade drive shaft and electromagnet and withdraw the blade to within a few inches of its original position. This represents a large positive reactivity insertion while still remaining subcritical and, by observing the ratio of count rates for these two subcritical states, we can use the Subcritical Multiplication Factor Approach to determine the magnitude of the reactivity associated with the blade withdrawal.

Phase IV -- after the subcritical system has stabilized at the end of Phase III, the source will be removed from the system as quickly as possible. This should cause the flux level to quickly drop towards zero (or its background level), and we will use the Source Jerk Method to evaluate the subcriticality level of this particular configuration.

This is the basic 4-step procedure we will use in our up-coming Reactivity Measurements Lab