# 24.536 Reactor Experiments
# 407.403 Advanced Nuclear Lab

## Subcritical Multiplication & Approach to Critical Experiment

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## Discussion Outline

**Review from previous class and HW#3**

- **Reactor Operations Demo**
- **Point Kinetics Simulations in Matlab**

**Subcritical Multiplication**

**Approach to Critical Experiment**

**Take a short break around midpoint of class time…**

**Blade Worth Curves** (demo the bw_display GUI)

**Homework #4** (see details in rexpts_hw4sp18.pdf)

## Review/Discussion…

**Any Questions??? Now is the time…**

**Analytical Solution of Kinetics Eqns.** (step change in $\rho$)

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$ and $$\frac{P(t)}{P_o} = \frac{P_1}{P_o} e^{\pm t/\tau}$$ and $$\frac{P_1}{P_o} = \frac{\beta}{\beta - \rho}$$

**Reactor Operations Demo**

Positive/negative $\rho$ changes, manual/auto mode operation, pump-on/pump-off transients, inherent stability via negative feedbacks, etc.

**Point Kinetics Simulations in Matlab**

Do we need to do more examples (maybe from the HW)?



## Subcritical Multiplying Systems

In steady state, a subcritical system can be described symbolically by

$$(L - F)\phi = Q$$

where $L\phi$ includes the leakage, absorption, and both the outscatter and inscatter terms, $F\phi$ represents the fission source, and Q is the fixed source.

Without an external source, Q, there would be no steady state flux in a subcritical arrangement.

The most common situation of a subcritical system in a reactor arises during the reactor startup and shutdown periods.

## Subcritical Multiplying Systems (cont.)

Clearly, a reactor core has a substantial fission potential, but it may be arranged in a subcritical configuration (either by having some assemblies missing or by having large amounts of control inserted).

In most fuel there is an inherent neutron source due to the spontaneous fission and $\alpha$-decay (which leads to $(\alpha,n)$ reactions) that are associated with the higher actinides.

The neutrons emitted from these reactions, or from an externally applied fixed source, undergo subcritical multiplication (they cause fission in the fuel material) and give rise to a steady state neutron distribution throughout the system.



## Subcritical Multiplying Systems (cont.)

Our goal here is to develop a relationship between the fixed source, Q or S, and the total neutron source, N, within a subcritical system, where

N = fission source + fixed source

and to apply this relationship for monitoring reactivity changes during subcritical operation -- and, in particular, for predicting when criticality will occur as additional reactivity is added to the system.

This "relationship" is usually simply written as

$$N = MQ \quad \text{or} \quad N = MS$$

where M is the subcritical multiplication factor and Q or S are used interchangeably to represent the fixed source term.

## Subcritical Multiplying Systems (cont.)

Non-Multiplying System: $M \rightarrow 1$ and $k \rightarrow 0$ which indicates that the total neutron source is due only to the original source neutrons (i.e. no fission, which means no fuel)

Critical System: $M \rightarrow \infty$ and $k \rightarrow 1$ which says that the total neutron source is dominated by the fission neutron source (i.e. fission source >> Q)



## Subcritical Multiplication (Method #1)

Assume that a steady state source of neutrons producing q neutrons per generation is available.

At time t = 0, with no neutron population present, we turn on the source.

Thus, initially, q neutrons are introduced into the multiplying system which can be characterized by the multiplication factor, k.

The total neutron population after one generation, $n_1$, after two generations, $n_2$, etc. can be written as the sum of the fission neutrons produced in the current generation and the original source neutrons added in that generation.

## Subcritical Multiplication (Method #1)

Writing this sequence, in detail, gives

$$n_o = q$$

$$n_1 = kn_o + q = (k+1)q$$

$$n_2 = kn_1 + q = (k^2 + k)q + q = (k^2 + k + 1)q$$

$$n_3 = kn_2 + q = (k^3 + k^2 + k)q + q = (k^3 + k^2 + k + 1)q$$

At steady state (after many generations), we have

$$n_\infty = (1 + k + k^2 + k^3 + \cdots)q = \left(\sum_{p=0}^{\infty} k^p\right)q$$



## Subcritical Multiplication (Method #1)

However, this infinite series, for k < 1, is the binomial series, which reduces to

$$n_\infty = \frac{1}{1-k} q$$

Above we let q be the number of source neutrons added per generation.

If the resultant expression is divided by the neutron generation time, $\Lambda$, (with units of seconds per generation), then the interpretation becomes more straightforward in terms of a standard neutron source with units of neutrons per second,

$$\frac{n_\infty}{\Lambda} = \frac{1}{1-k}\frac{q}{\Lambda} \quad \text{or} \quad N = \frac{1}{1-k} S = MS$$

where $M = \dfrac{1}{1-k}$ subcritical multiplication factor

## Subcritical Multiplication (Method #2)

Now, to gain a little more insight, let's also consider a more mathematical approach for developing an expression for the subcritical multiplication factor, M.

Starting with the Generation Time Formulation of point kinetics, we have

$$\frac{dT}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right)T + \sum_i \lambda_i c_i + q$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda}T - \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6$$

with $c_i(t) = \dfrac{1}{\frac{1}{v}\langle \psi_o \rangle}\langle C_i(t) \rangle$ and $q(t) = \dfrac{1}{\frac{1}{v}\langle \psi_o \rangle}\langle Q(t) \rangle$



## Subcritical Multiplication (Method #2)

Now, at steady state, the precursor equation becomes

$$0 = \frac{\beta_i}{\Lambda}T(0) - \lambda_i c_i(0) \qquad \text{or} \qquad \sum_i \lambda_i c_i(0) = \frac{\beta}{\Lambda}T(0)$$

and putting this into the steady state neutron balance equation yields

$$0 = \left(\frac{\rho - \beta}{\Lambda}\right)T(0) + \frac{\beta}{\Lambda}T(0) + q(0)$$

and, canceling the terms containing $\beta/\Lambda$, gives

$$0 = \frac{\rho}{\Lambda}T(0) + q(0)$$

or

$$T(0) = -\frac{1}{\rho}\Lambda q(0) = -\frac{k}{k-1}\Lambda q(0) = \frac{k}{1-k}\Lambda q(0)$$

## Subcritical Multiplication (Method #2)

Now, using the definition of the normalized input source and the definition of the prompt generation time, $\Lambda$, we have

$$T(0) = \frac{k}{1-k}\frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}\frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle Q(0)\rangle = \frac{k}{1-k}\frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q(0)\rangle$$

and, multiplying both sides by $\langle\nu\Sigma_f\psi_o\rangle$ gives

$$S_{fis} = \langle\nu\Sigma_f\psi_o\rangle T(0) = \frac{k}{1-k}\langle Q(0)\rangle = \frac{k}{1-k}S_{ext}$$

As a last step, we let the total neutron source, N, be the sum of the fission source, $S_{fis}$, and the input source, $S_{ext}$, or

$$N = S_{fis} + S_{ext} = \frac{k}{1-k}S_{ext} + S_{ext} = \left(\frac{k}{1-k}+1\right)S_{ext} = \frac{1}{1-k}S_{ext} = M\,S_{ext}$$



## Subcritical Multiplication (Method #2)

Thus, as before,

$$M = \frac{1}{1-k} \quad \text{and} \quad N = M\,S_{ext}$$

This says that, at steady state subcritical conditions, the total neutron source is simply M, the subcritical multiplication factor, times the input external source strength, $S_{ext}$.

Non-Multiplying System: $M \to 1$ as $k \to 0$ that is, the total neutron source is due only to the original source neutrons (i.e. no fuel $\to$ $S_{fis} = 0$)

Critical System: $M \to \infty$ as $k \to 1$ that is, the total neutron source is dominated by the fission neutron source (i.e. $S_{fis} >> S_{ext}$)

## Measurement Considerations

In practice, measuring the parameters within the expressions for subcritical multiplication is not particularly easy:

The value of k for the system is not usually known

The input source strength, S, is not readily available

The total neutron source strength, N, is not directly measurable

Instead, what is available from the detector is a count rate, C, in counts per second that is proportional to the total neutron source level, or

$$C_i = \alpha_i N_i = \alpha_i \left( \frac{1}{1-k_i} \right) S_i = \alpha_i M_i S_i$$

where $\alpha_i$ is the proportionality constant and the subscript i refers to the $i^{th}$ configuration (i.e. the count rate in a particular configuration is a function of the given configuration).



## Measurement Considerations (cont.)

However, if the source strength remains constant (independent of i) for a series of measurements, then a ratio of measurements in two specific configurations removes the dependence on S,

$$\frac{C_i}{C_o} = \frac{\alpha_i M_i S}{\alpha_o M_o S} = \frac{\alpha_i M_i}{\alpha_o M_o} = \frac{\alpha_i (1-k_o)}{\alpha_o (1-k_i)}$$

where $C_o$ is the initial count rate in the base system.

Since the true value of the absolute subcritical multiplication factor, M, is often not available (need k for some base state), a common definition for the relative subcritical multiplication factor in the $i^{th}$ configuration, $M_{ri}$, is

$$M_{ri} = \frac{\alpha_i M_i}{\alpha_o M_o} = \frac{C_i}{C_o}$$

which can be determined from the recorded detector count rates.

## Measurement Considerations (cont.)

Finally, taking the inverse of $M_{ri}$ gives

$$\frac{1}{M_{ri}} = \frac{C_o}{C_i} = \frac{\alpha_o(1-k_i)}{\alpha_i(1-k_o)} = \beta_i(1-k_i)$$

where $\beta_i$ is just another (unknown) proportionality constant as implied in the above equation.

The important feature here is that the inverse relative subcritical multiplication factor, $1/M_r$, is approximately a linear function of the neutron multiplication factor, k.

In particular, a plot of $1/M_r$ using two known values of k can be easily extrapolated to the $1/M_r = 0$ point -- which gives a rough prediction of where the system will be critical, since $M_r \rightarrow \infty$ as $k \rightarrow 1$.



## Measurement Considerations (cont.)

It is important to note that extrapolating linearly to the $1/M_r = 0$ point implies that $\beta_i$ (and $\alpha_i$) is insensitive to the configuration change from the $(i\text{-}1)^{th}$ to the $i^{th}$ arrangement.

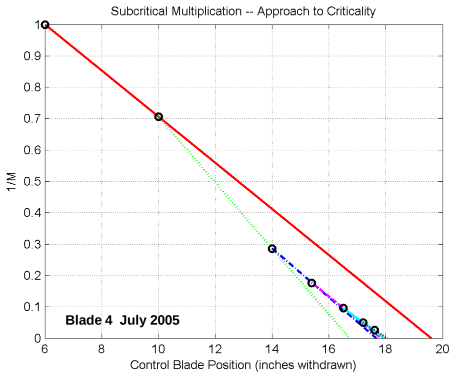
Although this is often a relatively poor assumption in the early stages of an approach to critical sequence, it usually becomes a reasonably good approximation as one slowly approaches the critical configuration.

However, it should also be emphasized that a plot of $1/M_r$ vs. k is not very useful since the value of k is not directly measurable.

Instead, the real independent variable, say η, is some operator-controlled measurable parameter that is used to bring the reactor to critical (such as the number of fuel assemblies loaded in the core, the position of a control rod or group of control elements, or the amount of soluble boron in the system…).

## Measurement Considerations (cont.)

A change in the control parameter, η, varies k and, instead of a $1/M_r$ vs. k plot, one uses a $1/M_r$ vs. η plot, where criticality is still reached when $1/M_r \rightarrow 0$.




Thus, linear extrapolation of the $1/M_r$ vs. η curve to zero still gives an estimate of the value of η needed for criticality.

In addition, as criticality is approached via progressively smaller changes in the control parameter, η, the relationship between k and η also becomes approximately linear, so the approximate linear behavior indicated above can still be expected.



## Measurement Considerations (cont.)

Finally, we note that many of the above expressions are often written in terms of reactivity, ρ, where

$$\rho = \frac{k-1}{k} \qquad \text{or} \qquad k = \frac{1}{1-\rho}$$

In particular, substituting these expressions gives

$$M_{ri} = \frac{C_i}{C_o} = \frac{\alpha_i\left(1-\frac{1}{1-\rho_o}\right)}{\alpha_o\left(1-\frac{1}{1-\rho_i}\right)} = \frac{\alpha_i}{\alpha_o}\frac{\rho_o}{\rho_i}\frac{1-\rho_i}{1-\rho_o} \approx \frac{\alpha_i}{\alpha_o}\frac{\rho_o}{\rho_i}$$

where the last approximation assumes that ρ << 1.

# Measurement Considerations (cont.)

**Furthermore, if the proportionality constant between the total neutron source, $N_i$, and detector count rate, $C_i$, is relatively insensitive to the configuration change, then this simplifies further to**

**Important**

$$M_{ri} = \frac{C_i}{C_o} \approx \frac{\rho_o}{\rho_i}$$

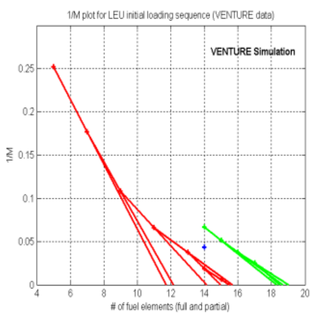
**If the count rate is doubled, then the relative multiplication factor has doubled, and the reactivity level has been reduced by about one-half (and vice versa)**

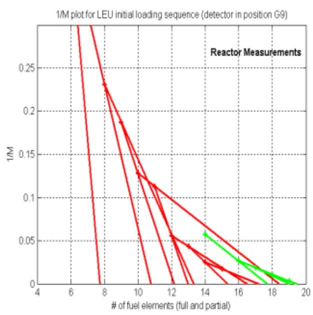


# Application: Approach to Critical

**As an illustration, the Lecture Notes briefly review the initial critical loading analysis that was performed for the conversion of the UMLRR from HEU to LEU fuel.**

**The actual critical loading took place in August 2000 using an approach that was based roughly on a previous computational study of the expected core loading sequence.**



## Approach to Critical Experiment

**Performing an approach to critical experiment by moving fuel elements as illustrated on the previous slide is quite time consuming, somewhat tedious for the reactor staff, and potentially problematic if done too frequently.**

**Thus, an alternate application, that uses that same concepts yet is more straightforward, is certainly preferable for illustrating the basic concepts of subcritical multiplication.**

**In particular, in our Approach to Critical experiment, we will apply this same methodology to estimate the critical height of a control blade within the UMLRR.**

**The basic idea here starts with the blade of interest near full insertion, with the other three control blades and the regulating blade at some fixed position -- where the overall blade configuration, of course, must give a subcritical configuration.**



## Approach to Critical Experiment (cont.)

**Then, the blade of interest is pulled out by some amount and some time is given for the count rate on the startup counter to stabilize.**

**From an estimate of the count rate for the current configuration, one can determine the $M_r$ and make a rough estimate, using a 1/M plot, of the critical height of the blade.**

**With a new estimated critical height, a new blade position is requested (usually about 1/2 of the predicted change needed for criticality), and the process is repeated.**

**After several steps, one should have a very good estimate of the real critical location (for the given configuration of the other blades, the fuel configuration, and the overall system state)…**

**This is the procedure we will use in next week's Approach to Critical Experiment**

## Summary and Take-Aways

A better understanding of **subcritical systems**.

The concept of **subcritical multiplication** and how the expressions

$$M = \frac{1}{1-k} \quad \text{and} \quad N = M\,S_{ext}$$

can be developed from **fundamental principles** and/or from **manipulation of the Point Kinetics equations**.

An understanding for the various **measurement considerations** needed to apply the theory to practical situations -- including the definition and use of the **relative subcritical multiplication factor**:

$$M_{ri} = \frac{\alpha_i M_i}{\alpha_o M_o} = \frac{C_i}{C_o}$$



## Summary and Take-Aways (cont.)

The fact that $1/M_r$ is **approximately a linear function of the neutron multiplication factor, k**.

$$\frac{1}{M_{ri}} = \frac{C_o}{C_i} = \frac{\alpha_o(1-k_i)}{\alpha_i(1-k_o)} = \beta_i(1-k_i)$$

$k_i = f(\eta_i)$ where $\eta$ is the desired control variable

That a plot of $1/M_r$ vs. $\eta$ using two known values can be easily extrapolated to the $1/M_r = 0$ point -- which gives a **prediction of where the system will be critical**, since as $k \to 1$, $M_r \to \infty$, and $1/M_r \to 0$ .

**HW #4 addresses the above items and provides the proper background for our upcoming Approach to Critical Lab**
**(see details in rexpts_hw4sp18.pdf)**

# bw_display GUI

One of our subsequent labs will focus on the **back-ground theory** and **actual measurement** of the **Blade Worth Curves for the UMLRR**.

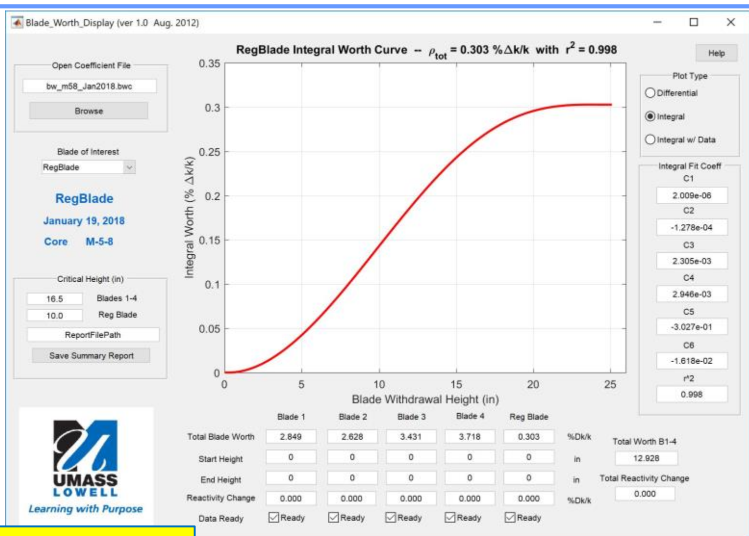
However, in the meantime, **it is important to be able to extract and apply information from the existing blade worth curves**.

Thus, we will finish this class with a **brief demo** of the **bw_display GUI** -- the Matlab code that is used locally to **obtain qualitative and quantitative data** about the worth of the control devices within the UMLRR.

(see the main screen for the bw_display GUI on the next slide)



# bw_display GUI




**Show Live Demo**