24.536 Reactor Experiments
407.403 Advanced Nuclear Lab

Reactor Kinetics & Dynamics and UMLRR Startup Demo

Prof. John R. White
Nuclear Engineering Program
UMass-Lowell, Lowell MA

Discussion Outline

- **Review from previous class and HW#1**
- **General Operational Control Concepts**
- **Perform a Reactor Startup Demonstration**
- **Review of Reactor Kinetics and Dynamics**
  - **Space-Time Kinetics → Point Kinetics**
  - **The Generation Time Formulation of Point Kinetics**
  - **Solution to the Kinetics Eqns. (*the kinetics_gui* code)**
- **Homework #2 (see details in rexpts_hw2sp18.pdf)**

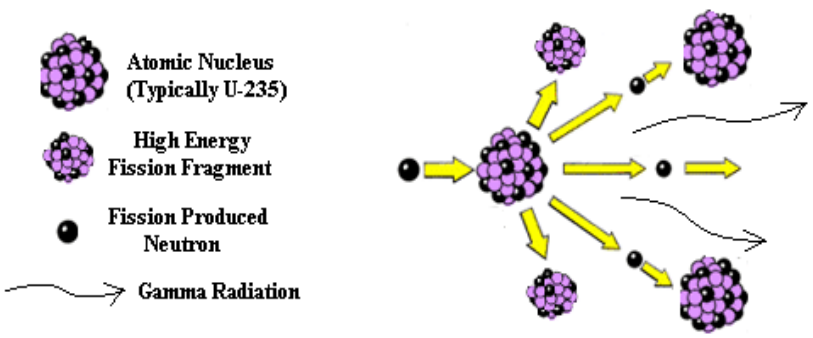
Review: Previous Class and HW#1

Any Questions:

- Introduction/Course Policy
- The UMLRR Facility Overview
- Data Acquisition Tools
  - UMLRR_Online Demo (remote real-time capability)
  - *umlrr_data* GUI Demo (offline data analysis tool)
  - Matlab Analysis of Reduced Data File
- Homework #1 (see details in rexpts_hw1sp18.pdf)

UMLRR Operations…

Reactor operations is all about controlling the neutron population in the system…



U235 + n → fission products + 2 or 3 n's + 200 MeV

Neutron Life Cycle (thermal systems)

Fission neutrons are born at high energy, they slow down via elastic and inelastic neutron scattering, and then, as thermal neutrons, they cause additional fissions to continue the cycle…

Fission is the primary neutron source in nuclear reactors.

Scattering only changes the neutron energy level.

Absorption and leakage are the ultimate loss mechanisms.

When the neutron production and loss rates are in balance, then the neutron population remains constant and the system operates at constant power (power is related to the fission rate).

Rate of Change = Production Rate – Loss Rate

The Multiplication Factor

The multiplication factor, k, is a term used to describe the neutron balance in a nuclear system.

$$k = \frac{\text{production}}{\text{loss}} = \frac{\text{production}}{\text{absorption} + \text{leakage}}$$

Critical	→	production = absorption + leakage	**k = 1**
Supercritical	→	production > absorption + leakage	**k > 1**
Subcritical	→	production < absorption + leakage	**k < 1**

Power Level Control (from critical)

Reactor power can be controlled by regulating the absorption rate via movement of the control rods.

Power Increase: From steady-state, remove control to decrease the absorption term. Production becomes greater than loss (k > 1) and the neutron population begins to increase.

Power Decrease: From a stable condition, insert control to increase the relative number of parasitic absorptions. The loss component becomes greater than neutron production (k < 1) and the neutron population begins to decrease.

When the new target power level is reached, control is moved towards its previous position until k becomes unity -- the reactor is now critical again at the new steady state power level.

A reactor can be critical at any power level…

Reactivity

In an operating reactor, the value of k is always very close to unity (i.e. near critical) .

Thus, we define the term reactivity, ρ, as a measure of the deviation from critical, or

$$\rho = \frac{k-1}{k}$$

For example, we often talk about the insertion of positive or negative reactivity when the control rods are moved:

-- If the rods are inserted, this adds more absorption, k becomes less than unity, and we say that negative reactivity has been added to the system.

-- If the control rods are moved outward a little, then positive reactivity has been added since the absorption term decreases, and k becomes slightly greater that unity.

Typical Reactor Startup

Do Live UMLRR Startup Demo…

Typical Reactor Startup

Within the context of the general control considerations discussed above, let's take a quick look at a typical startup sequence in the UMLRR (specific data are from an April 2008 Energy Balance Experiment plotted using the *umlrr_data* GUI.)

This should be similar to the observations from today's UMLRR Startup Demo -- you will analyze this demo as part of HW#2…

Basic Sequence (external source inserted and blades full in at start):

1. **Pull RegBlade out to near mid position (manual mode).**
2. **Systematically pull remaining blades out one at a time keeping the blades banked nearly evenly. This is done in stages, with much smaller blade movements as criticality is approached (usual procedure is to move about ½ the Δz expected for criticality – which is known from previous operation).**

Typical Reactor Startup (cont.)

3. The startup counter is used to monitor operations at subcritical and Linear P1 and P2 are used during power operation (these are auto-ranging detectors).

4. When the count rate on the SUC starts to increase rapidly (2000-4000 cps), the reactor is critical (or slightly supercritical).

5. At 500 W the system is stabilized for a short time by putting the RegBlade in auto mode.

6. At this point, the external source and startup counter are usually removed from the core.

7. After removal of the source and SUC, the RegBlade is put back into manual mode, and the reactor is taken to the desired power level.

Typical Reactor Startup (cont.)

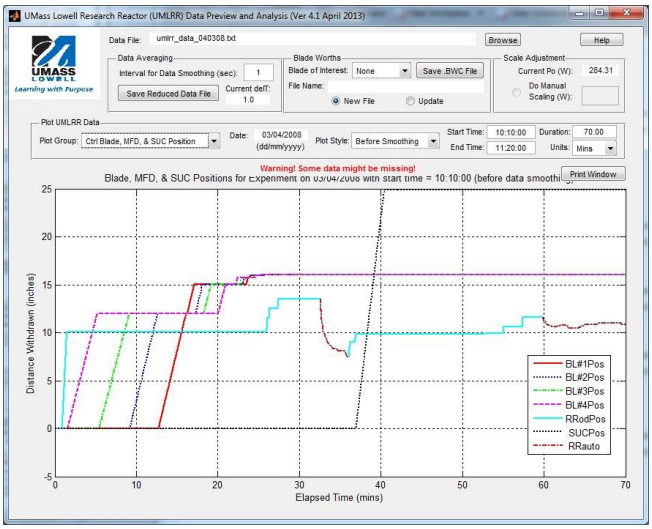
8. The RegBlade is put into auto mode for routine operation to automatically counter any transient reactivity effects (xenon buildup and/or temperature changes).

9. For extended operation, regular re-banking is needed to keep the RegBlade near the middle of its 25-26 inch movable range.

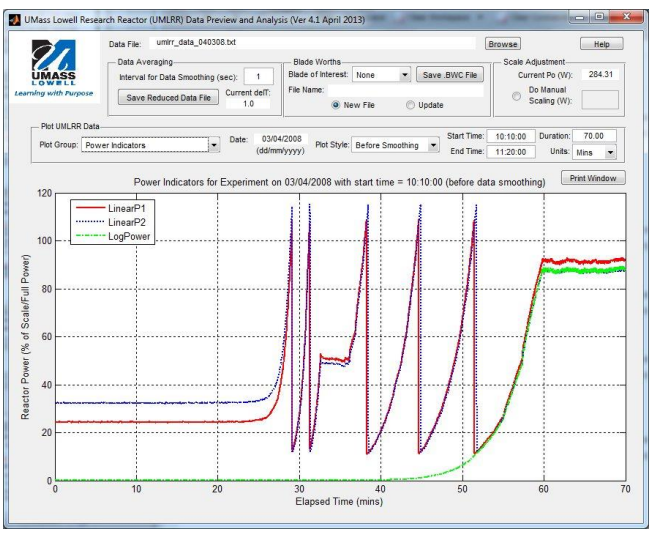
10. For shutdown, the blades, SUC, and source are reinserted…

See the sequence of plots for blade position, power, SUC rate, and core temperatures from a typical startup in April 2008…

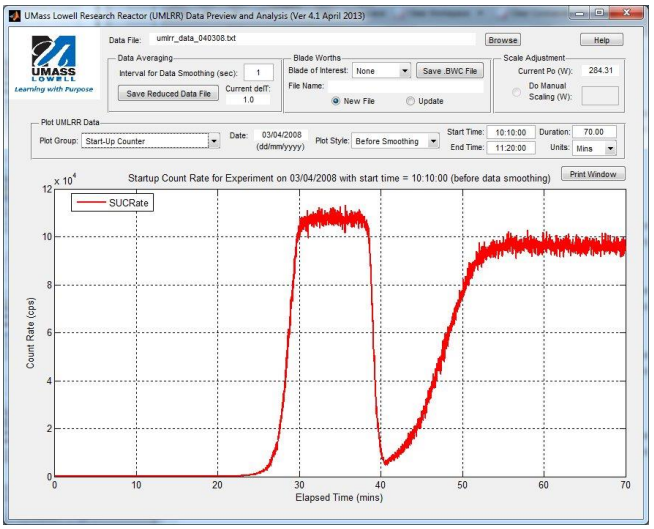
Reactor Startup: Blade Position



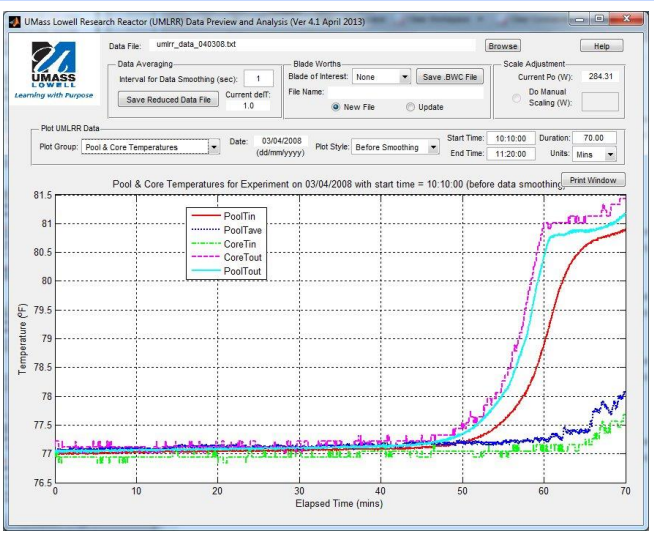
Reactor Startup: Power



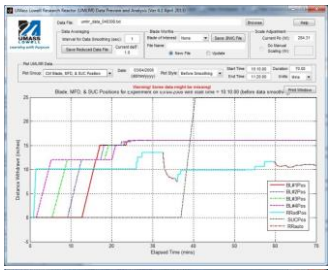
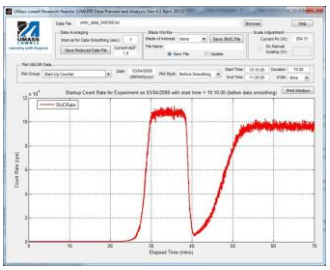
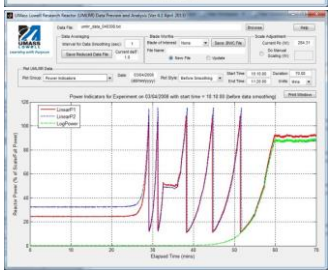
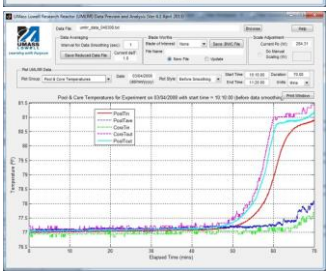
Reactor Startup: SUC Rate



Reactor Startup: Core Temps



Typical Reactor Startup Results

Reactor Kinetics

Now let's discuss some formal Reactor Kinetics…

Time-Dependent Diffusion Equation

The remainder of this lesson will elaborate on the subject of Reactor Kinetics.

The starting point here is the 1-group time-dependent diffusion equation.

In words, this equation states that the

rate of change of neutron density = production rate of neutrons per unit volume − loss rate of neutrons per unit volume

Note that, since both prompt and delayed neutrons are produced, we must take into account the timing associated with these separate components of the fission source.

To keep track of the amount of delayed neutron precursors, precursor balance equations are also needed (the precursors are usually grouped into six separate groups with six effective decay constants, λ_i, and yields, β_i).

1-Speed Space-Time Kinetics

Using standard notation, we can write the complete 1-speed space-time kinetics equations as follows:

Neutron Balance

$$\frac{1}{v}\frac{\partial}{\partial t}\phi = \left[(1-\beta)\nu\Sigma_f\phi + \sum_i \lambda_i C_i + Q\right] - \left[-\vec{\nabla}\cdot D\vec{\nabla}\phi + \Sigma_a\phi\right]$$

Precursor Balance

$$\frac{\partial C_i}{\partial t} = \beta_i \nu\Sigma_f\phi - \lambda_i C_i \quad \text{for } i = 1, 2, \cdots 6$$

These equations represent a set of seven coupled PDEs, where the cross sections, fluxes, and source are all functions of both space and time.

Computer codes are available to solve the space-time kinetics problem -- but this subject is outside the scope of this course…

In general, these equations are rather difficult to solve!!!

1-Speed Point Kinetics Model

There are many applications in reactor operations when the spatial flux shape does not change significantly with time.

For these cases, the general space-time description can be reduced to a point model (spatially integrated model) that includes time as the only independent variable.

This procedure reduces the system to seven ordinary differential equations (ODEs) – which are significantly easier to solve.

There is a formal procedure for doing this reduction, during which, the "effective" kinetics parameters are defined precisely.

The most general procedure usually starts with the multigroup neutron balance equation, but the 1-speed approximation allows a more straightforward development that gives identical point kinetics equations -- with slightly less rigor in the definition of some parameters.

1-Speed Point Kinetics Model (cont.)

Since the resultant differences in definition do not affect our present discussion and application of the final equations, we will proceed here with the 1-speed formulation (since the notation is much easier to follow).

Starting with the 1-speed space-time model, we assume that the flux can be separated into a slowly varying (or time-independent) spatial distribution and a more rapidly varying amplitude function,

$$\phi(\vec{r},t) = \psi(\vec{r},t)T(t) \approx \psi_o(\vec{r})T(t)$$

where the spatial distribution with the 'o' subscript represents the initial steady state flux shape and T(t) represents the time-dependent amplitude of the neutron flux (or power level).

1-Speed Point Kinetics Model (cont.)

Now, we substitute this approximation into the neutron and precursor balance equations and integrate the resultant equations over the spatial domain of interest to give

$$\frac{1}{v}\langle\psi_o\rangle\frac{dT}{dt}=\left[(1-\beta)\langle\nu\Sigma_f\psi_o\rangle T+\sum_i\lambda_i\langle C_i\rangle+\langle Q\rangle\right]-\left[\left\langle-\vec{\nabla}\cdot D\vec{\nabla}\psi_o\right\rangle+\langle\Sigma_a\psi_o\rangle\right]T$$

$$\frac{d}{dt}\langle C_i\rangle=\beta_i\langle\nu\Sigma_f\psi_o\rangle T-\lambda_i\langle C_i\rangle \qquad \text{for } i=1,2,\cdots 6$$

These represent a set of seven coupled first-order ordinary differential equations (ODEs) -- that is, the Point Kinetics Model.

1-Speed Point Kinetics Model (cont.)

The Point Kinetics model given on the previous slide is usually not used in this form for practical application.

In particular, since the cross sections can be time dependent and under operator control (i.e. movement of a control rod affects Σ_a, etc.), almost every term in these equations can be modified to initiate a transient case.

However, from an operational perspective, the effect of a change in cross section (or material composition) manifests itself as a change in the multiplication factor, k, or in the reactivity, ρ.

Changing the above equations to incorporate k or ρ directly leads to the traditional Lifetime Formulation (uses k) and Generation Time Formulation (uses ρ) of point kinetics.

In these formulations, the multiplication factor, k(t), or reactivity, $\rho(t)$, becomes the driving force for initiating most transient analyses.

The Generation Time Formulation

In this lesson we will focus on the Generation Time Formulation, but the formal Lecture Notes develop both schemes in detail.

One first defines the prompt neutron generation time by arguing that, at steady state, the neutron production rate from fission in a critical system is the total neutron population divided by the neutron generation time.

In equation form, this can be written as

$$\text{production rate} = \frac{\text{neutron population}}{\text{generation time}} \quad \text{or} \quad \text{generation time} = \frac{\text{neutron population}}{\text{production rate}}$$

Defining Λ as the prompt neutron generation time, the one-speed approximation gives

$$\Lambda = \frac{1}{v}\langle \psi_o \rangle \Big/ \langle \nu\Sigma_f \psi_o \rangle$$

The Generation Time Formulation

Note: In the context of the 1-group diffusion equation, k and ρ have the following formal definitions (these are used in subsequent manipulations):

$$k = \frac{\text{neutron production rate from fission}}{\text{loss rate}} = \frac{\text{production}}{\text{loss}}$$

$$k = \frac{\langle \nu\Sigma_f \psi_o \rangle}{\langle -\vec{\nabla} \cdot D\vec{\nabla}\psi_o \rangle + \langle \Sigma_a \psi_o \rangle}$$

and

$$\rho = \frac{k-1}{k} = \frac{\text{production - loss}}{\text{production}}$$

$$\rho = \frac{\langle \nu\Sigma_f \psi_o \rangle - \left[\langle -\vec{\nabla} \cdot D\vec{\nabla}\psi_o \rangle + \langle \Sigma_a \psi_o \rangle \right]}{\langle \nu\Sigma_f \psi_o \rangle}$$

Generation Time Formulation (cont.)

Now we divide every term in the point kinetics equation for the neutron level by the neutron production rate from fission to give

$$\frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}\frac{dT}{dt} = (1-\beta)\frac{\langle\nu\Sigma_f\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle - \frac{\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o\rangle + \langle\Sigma_a\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}T$$

and use the definitions of ρ and Λ to simplify to

$$\Lambda\frac{dT}{dt} = \left[\frac{\langle\nu\Sigma_f\psi_o\rangle - \left[\langle -\vec{\nabla}\cdot D\vec{\nabla}\psi_o\rangle + \langle\Sigma_a\psi_o\rangle\right]}{\langle\nu\Sigma_f\psi_o\rangle} - \beta\right]T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle$$

or

$$\Lambda\frac{dT}{dt} = (\rho-\beta)T + \sum_i \lambda_i \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle C_i\rangle + \frac{1}{\langle\nu\Sigma_f\psi_o\rangle}\langle Q\rangle$$

Generation Time Formulation (cont.)

Now, we define the normalized precursor and external source amplitudes as

$$c_i(t) = \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle C_i(t)\rangle \qquad q(t) = \frac{1}{\frac{1}{v}\langle\psi_o\rangle}\langle Q(t)\rangle$$

When these expressions are substituted into the above equation and we use the definition of the generation time, the final neutron balance equation results

$$\Lambda\frac{dT}{dt} = (\rho-\beta)T + \sum_i \lambda_i \frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}c_i + \frac{\frac{1}{v}\langle\psi_o\rangle}{\langle\nu\Sigma_f\psi_o\rangle}q$$

or neutron amplitude

$$\frac{dT}{dt} = \left(\frac{\rho-\beta}{\Lambda}\right)T + \sum_i \lambda_i c_i + q$$

Generation Time Formulation (cont.)

Finally, to complete the generation time formulation, we divide the precursor equation by the total neutron population and again use the definition of Λ and the normalized precursor amplitude to give

$$\frac{d}{dt}\frac{\langle C_i\rangle}{\frac{1}{v}\langle\psi_o\rangle} = \beta_i \frac{\langle \nu\Sigma_f\psi_o\rangle}{\frac{1}{v}\langle\psi_o\rangle} T - \lambda_i \frac{\langle C_i\rangle}{\frac{1}{v}\langle\psi_o\rangle} \qquad \text{for } i = 1, 2, \cdots 6$$

or

precursor amplitudes

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} T - \lambda_i c_i \qquad \text{for } i = 1, 2, \cdots 6$$

The last two highlighted equations represent the Generation Time Formulation of Point Kinetics.

Normalization Considerations

The solution of the kinetics equations usually leads to relative results -- that is, one computes $T(t)/T_o = n(t)/n_o = P(t)/P_o$ etc., where these represent the time-dependent relative flux amplitude, neutron level, power level, etc.

However, when reactivity feedbacks are important, knowledge of the absolute neutron level or power level becomes essential.

As detailed in the Lecture Notes, one can formally derive a set of point kinetics equations that directly include the actual reactor power level, P(t), in watts and the neutron source level, <Q(t)>, in neutrons/sec. The resultant equations are:

$$\frac{d}{dt}P(t) = \frac{(\rho-\beta)}{\Lambda}P(t) + \sum_i \lambda_i c_i(t) + \frac{\kappa}{\nu}\frac{1}{\Lambda}\langle Q(t)\rangle$$

$$\frac{d}{dt}c_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i c_i(t) \qquad \text{for } i = 1, 2, \cdots 6$$

These Point Kinetics equations will be highlighted in the remainder of these Lecture Notes...

Solution of the Point Kinetics Eqn.

In general, analytical solution of the point kinetics equations is not easy -- recall that we have a coupled set of seven ODEs!

In most cases, these equations are evaluated for a given $\rho(t)$ using numerical methods (such as Matlab's ode15s solver -- where a stiff equation solver is needed because of the large difference in time constants that results).

For a few specific cases, an analytical solution is possible -- and the resultant solutions give considerable insight into the general behavior of the time dependent neutron balance in real systems.

One common situation that can be solved analytically involves a step change in reactivity in a critical reactor operating at low power ("low power" means that feedback effects are negligible).

The solution of this case allows us to introduce some common terminology, and to gain a good understanding of the expected behavior in several common situations.

Step Change in Reactivity

Starting with the generation time formulation of point kinetics with no external source, we have

$$\frac{dP}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right)P + \sum_i \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda}P - \lambda_i c_i \quad \text{for } i = 1, 2, \cdots 6$$

In most applications of these equations, the kinetics parameters (Λ, β_i, and λ_i) are assumed to be constant, the reactivity is the driving force for the transient, and P(t) and $c_i(t)$ are the dependent variables that vary with time due to some changing $\rho(t)$.

However, for a step change in reactivity, $\rho(t) = \rho$ = constant, and the above equations become a system of seven linear constant coefficient ODEs -- and this falls into a class of problems that we know how to handle analytically.

Step Change in Reactivity (cont.)

The standard approach for solving linear time-invariant systems is to assume a solution of the form of a simple exponential.

Following this technique, we assume that

$$P(t) = A_o e^{\omega t} \qquad \text{and} \qquad c_i(t) = A_i e^{\omega t}$$

Now, we substitute these assumed solutions into the precursor balance equations, to obtain

$$A_i \omega e^{\omega t} = \frac{\beta_i}{\Lambda} A_o e^{\omega t} - \lambda_i A_i e^{\omega t} \quad \Rightarrow \quad A_i (\omega + \lambda_i) = \frac{\beta_i}{\Lambda} A_o$$

or

$$A_i = \frac{\beta_i / \Lambda}{\omega + \lambda_i} A_o$$

Step Change in Reactivity (cont.)

Now, putting the assumed solutions, along with the above result, into the P(t) equation gives

$$A_o \omega e^{\omega t} = \left(\frac{\rho - \beta}{\Lambda} \right) A_o e^{\omega t} + \sum_i \lambda_i A_i e^{\omega t} = \left(\frac{\rho - \beta}{\Lambda} \right) A_o e^{\omega t} + \sum_i \lambda_i \frac{\beta_i / \Lambda}{\omega + \lambda_i} A_o e^{\omega t}$$

Cancelling the common $A_o e^{\omega t}$ factor in each term and multiplication by Λ gives

$$\Lambda \omega = (\rho - \beta) + \sum_i \lambda_i \frac{\beta_i}{\omega + \lambda_i}$$

and solving for ρ gives

$$\rho = \Lambda \omega + \beta - \sum_i \frac{\beta_i \lambda_i}{\omega + \lambda_i}$$

Step Change in Reactivity (cont.)

To put this expression into standard form, note that $\beta = \sum \beta_i$.

Now, using this equality, we have

$$\rho = \Lambda\omega + \sum_i \left(\beta_i - \frac{\beta_i \lambda_i}{\omega + \lambda_i} \right) = \Lambda\omega + \sum_i \left(\frac{\beta_i \omega + \beta_i \lambda_i - \beta_i \lambda_i}{\omega + \lambda_i} \right)$$

or

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$

reactivity equation

This equation is the standard form of the so-called reactivity equation (or inhour equation) obtained from the generation time formulation of point kinetics.

Step Change in Reactivity (cont.)

From a pure mathematical viewpoint, the reactivity equation is simply the characteristic equation associated with the original seven linear constant coefficient ODEs -- and, for a given value of reactivity, the roots of this equation give the values of ω that satisfy the original form of the assumed solution.

Also, since we have seven coupled first-order ODEs, we should expect seven roots (i.e. seven values of ω) that will satisfy the so-called reactivity equation.

Assuming that each ω_j for j = 1, 2, ··· 7 is distinct, the linear super-position principle allows us to write the general solution as a linear combination of the linearly independent individual solutions.

Step Change in Reactivity (cont.)

Thus, we can write a general solution for the time-dependent power level P(t) as

$$P(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \cdots + A_7 e^{\omega_7 t}$$

The A_j coefficients are determined from the ICs.

where the ω_j's are the roots of the reactivity equation and the A_j's are the seven arbitrary coefficients needed for the general solution of a 7th order initial value problem (IVP).

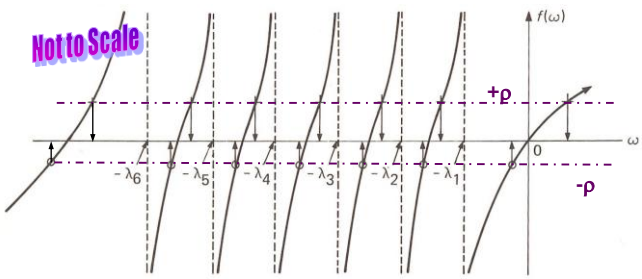
Although the above development establishes a solid mathematical foundation, it really has not shed a lot of insight into the actual behavior of P(t).

To do this, we must first get a better understanding of the reactivity equation and the values of its roots.

Interpretation of the Reactivity Eqn.

We can think of the right hand side (RHS) of the reactivity equation as some function of ω, say f(ω), and simply plot f(ω) vs. ω for a wide range of ω.

Then, if we superimpose the LHS [i.e. ρ(ω) = constant] on the plot, the intersections of the two curves give the desired roots, ω_j.



reactivity equation

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i}$$

positive ρ: one positive root and six negative roots

negative ρ: seven negative roots

Interpretation of the Reactivity Eqn.

If we order the roots ω_j from most positive to most negative, then, after a relatively short transient time, the last six terms for P(t) decay away (because $\omega_2, \omega_3, \cdots, \omega_7 < 0$ for both positive and negative reactivity), leaving only the term containing ω_1, or

$$P(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \quad \cdots \quad + A_7 e^{\omega_7 t} \approx P_1 e^{\omega_1 t} = P_1 e^{\pm t/\tau}$$

where $\tau = 1/|\omega_1|$ is called the reactor period and P_1 is the power level (or flux amplitude) just after the short transient period.

If ρ is positive, $\omega_1 > 0$, and the reactor period is positive -- so P(t) grows indefinitely as $e^{+t/\tau}$ (remember that we assumed no feedbacks up to this point).

Interpretation of the Reactivity Eqn.

For negative reactivity, just the opposite occurs.

That is, $\omega_1 < 0$ and the reactor period is negative -- so the power level decreases towards zero with the form $e^{-t/\tau}$.

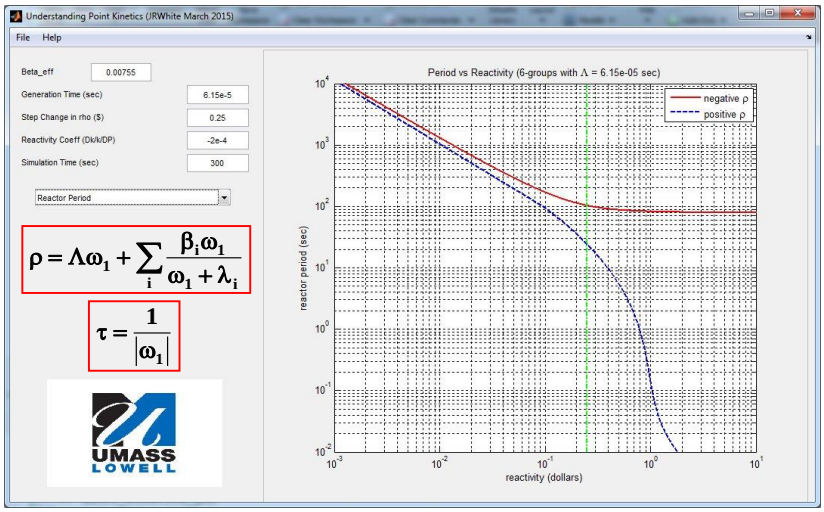
Thus, after a short transient time, the dominate behavior of P(t) is simply associated with the most positive root of the reactivity equation and it is represented as a simple growing or decaying exponential,

$$P(t) = \left(\frac{P_1}{P_o}\right) P_o e^{\omega_1 t} \quad \text{or} \quad \frac{P(t)}{P_o} = \frac{P_1}{P_o} e^{\pm t/\tau}$$

P_1/P_o is the prompt jump/drop and τ is the reactor period

where all the above descriptions assume no feedbacks

Reactivity Equation: τ vs. ρ



Inherent Safety

Low power operations means no feedback effects (i.e. no changes in fuel and coolant temperatures, no Xe buildup, etc.)

However, for high power operation, changes in the temperature of the fuel and coolant, for example, can result in Doppler broadening of the resonances and material density changes, which changes the various production and loss rates, which changes k…

These are referred to as inherent feedbacks within the system,

$$\Delta T \rightarrow \Delta k \rightarrow \Delta\rho \rightarrow \Delta P \rightarrow \Delta T \text{ etc.}$$

We define the temperature coefficient of reactivity, α_T, as the rate of change of reactivity with temperature, or

$$\alpha_T = \frac{d\rho}{dT} \quad \text{and} \quad \Delta\rho \approx \frac{d\rho}{dT}\Delta T \approx \alpha_T \Delta T$$

Inherent Safety

In the Point Kinetics model, the inherent feedbacks are incorporated through a "feedback reactivity, ρ_f"

$$\rho(t) = \rho_{ext}(t) + \rho_f(t)$$

where $$\Delta\rho_f(t) = \alpha_{T_f}\Delta T_f(t) + \alpha_{T_c}(t)\Delta T_c(t) + \Delta\rho_{Xe}(t)$$

Now, consider the sign of α_T:

positive α_T, $T\uparrow \rightarrow \rho\uparrow \rightarrow P\uparrow \rightarrow T\uparrow \rightarrow \rho\uparrow \rightarrow P\uparrow$ etc. unbounded

negative α_T, $T\uparrow \rightarrow \rho\downarrow \rightarrow P\downarrow \rightarrow T\downarrow \rightarrow \rho\uparrow \rightarrow P\uparrow \rightarrow T\uparrow$ etc. stable

All reactors MUST be designed to have NEGATIVE α_T to ensure inherent safety.

Typical Solution Profiles

To wrap up our formal discussion of the reactivity equation and the solution of the Generation Time Formulation of Point Kinetics for a step change in reactivity, it makes sense to show the typical P(t) behavior for a specific change in reactivity.

This was accomplished in a simple Matlab code, with and without feedbacks, for the case of both positive and negative reactivity ($\rho/\beta = \pm 0.25$).

To illustrate the stabilizing effect associated with negative feedback, we define a generic power feedback coefficient as

$$\alpha_p = \frac{\partial\rho}{\partial P} = \frac{1}{k^2}\frac{\partial k}{\partial P} \approx \frac{1}{k}\frac{\partial k}{\partial P}$$

Typical Solution Profiles (cont.)

With this definition, the actual reactivity that enters into the point kinetics equation is a combination of the externally applied reactivity, ρ_{ext} and the feedback reactivity, ρ_f, or

$$\rho(t) = \rho_{ext} + \rho_f(t) = \rho_{ext} + \alpha_p\left(P(t) - P_o\right)$$

Note that, with negative α_p, then an increase in power reduces ρ, which decreases P, which increases ρ, etc. until a new steady state condition is realized.

Since criticality is achieved when $\rho = 0$, the new steady state power level associated with the negative feedback case will be reached when the feedback reactivity exactly cancels the applied external reactivity,

$$\alpha_p\left(P_{new} - P_o\right) = -\rho_{ext} \qquad \text{or} \qquad P_{new} = P_o - \frac{\rho_{ext}}{\alpha_p}$$

Typical Solution Profiles (cont.)

With this brief background, we can now actually simulate and compare the dynamics of a system with and without feedback.

For the feedback-free case, we set $\alpha_p = 0$, and for the simulation with inherent feedbacks, we set the power feedback coefficient to its appropriate value for the system of interest (for this case, $\alpha_p = -2\times10^{-4}$ $\Delta k/k$ per unit ΔP).

The results of the Matlab simulation for the two cases with and without feedback are shown in the next few slides:

Positive ρ: For ρ_{ext} = +25 cents, we see the expected unbounded exponential increase in the flux or power level for the case of a positive reactivity insertion with no feedback.

However, for the case where the inherent negative feedback is treated, the power level rises less rapidly and it eventually levels off at a new steady state power.

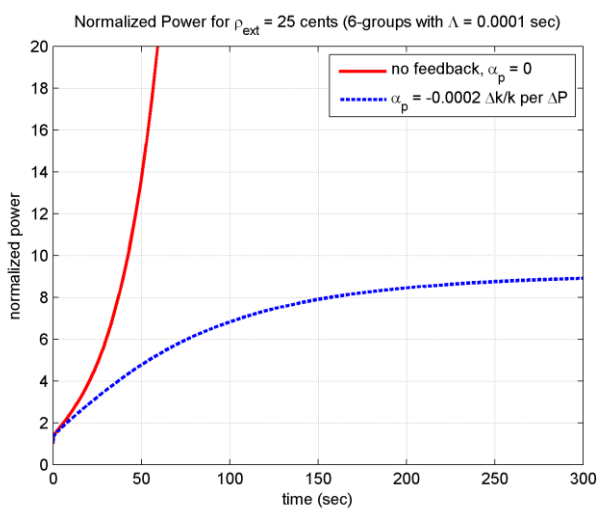
Typical Solution Profiles (cont.)

Negative ρ: For ρ_{ext} = -25 cents, both simulations lead to a decreasing power level.

In the case with inherent feedbacks, the exponential decrease is reduced slightly, but not enough to keep the reactor from complete shutdown.

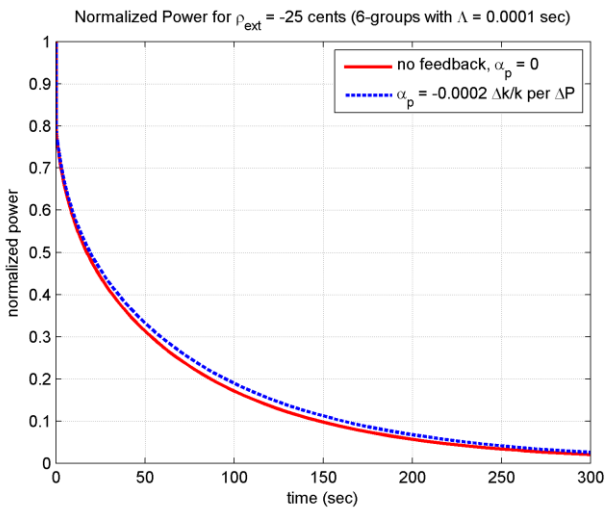
This is true because the positive reactivity due to the power feedback is not sufficient to overcome the original negative external reactivity added to the system.

Typical Solution Profiles (cont.)



24.536 Reactor Experiments
Reactor Kinetics & Dynamics and UMLRR Startup Demo
(Jan. 2018)

Typical Solution Profiles (cont.)



Numerical vs. Analytical Solution

The main point of the above discussion of the analytical solution method was so we could get a good understanding of the expected behavior and to introduce some important terminology associated with reactor kinetics.

However, actually computing accurate values for all seven roots of the reactivity equations, and then setting up the appropriate equations and solving for the seven coefficients for each transient situation of interest is not really easy to implement.

Also, remember that the analytical solution method can only be applied for ρ = constant -- it does not work for the general case of ρ = ρ(t).

Numerical vs. Analytical Solution

However, even for the simple situation where ρ = constant, the numerical solution of the seven coupled ODEs using an available ODE solver is a much easier path to follow (and this was the technique chosen here to do the actual simulations).

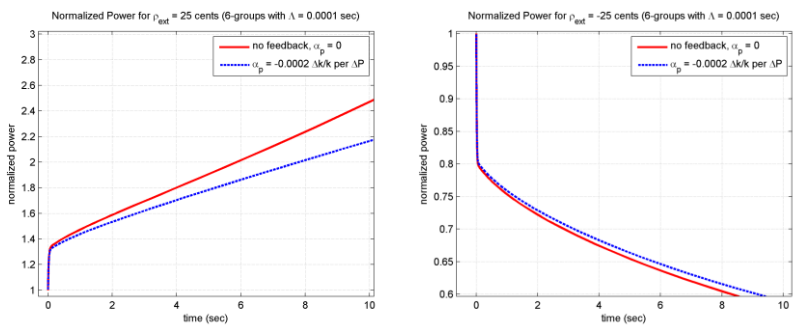
Thus, the numerical approach was selected for two important reasons:

- The numerical solution is much easier to obtain.
- The numerical solution allows the treatment of feedback effects.

The Prompt Jump/Drop

If we expand the first few seconds of the transient profiles shown previously, we see a nearly instantaneous rise or fall in the normalized power immediately after the step change in reactivity is made.

The rapid change seen here is due to the most negative root, ω_7, of the reactivity equation. Because ω_7 has such a large negative value, this term goes to zero nearly instantaneous after initiation of the transient.



The Prompt Jump/Drop (cont.)

Since the prompt jump/drop is an inherent feature of each transient, it would be convenient if we could get a quick and easy-to-use estimate of the magnitude associated with this phenomenon.

In particular, since we have already argued that the $A_1 e^{\omega_1 t}$ term dominates the transient response after a short period (for the no feedback case), if we could determine the normalized power, P_1, just after the prompt jump/drop, we would have a simple way to estimate the complete power profile versus time, or

$$P(t) = \left(\frac{P_1}{P_o}\right) P_o e^{\omega_1 t} \qquad \text{or} \qquad \frac{P(t)}{P_o} = \frac{P_1}{P_o} e^{\pm t/\tau}$$

where P_1/P_o is the desired magnitude of the prompt jump ($P_1/P_o > 1$) or prompt drop ($P_1/P_o < 1$).

The Prompt Jump/Drop (cont.)

The development of an approximation for the prompt jump/drop is given in detail in the formal Lecture Notes, with the result

$$\frac{P_1}{P_o} = \frac{\beta}{\beta - \rho}$$

where P_1 is the power level just after the prompt jump/drop.

By way of example, for the simulations shown previously, $\beta = 0.0065$ and $\rho = \pm 0.25\beta$.

Now, using the prompt jump/drop approximation, we have

$$\frac{P_1}{P_o} = \frac{\beta}{\beta - \rho} = \frac{1}{1 - 0.25} = 1.33 \quad (\text{for } \rho = +25 \text{ cents})$$

and

$$\frac{P_1}{P_o} = \frac{\beta}{\beta - \rho} = \frac{1}{1 + 0.25} = 0.80 \quad (\text{for } \rho = -25 \text{ cents})$$

These values agree very nicely with the prompt jump and drop transients seen in the previous figures!!!

Small Reactivity Values

One last approximation that often simplifies hand calculations, concerns the treatment of small reactivity insertions.

For $\rho \approx 0$ (either positive or negative), the magnitude of the most positive root of the reactivity equation is small compared to the magnitude of all the λ_i values (i.e. $|\omega| << |\lambda_i|$).

With this observation, the reactivity equation becomes

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{\omega + \lambda_i} \approx \Lambda\omega + \omega \sum_i \frac{\beta_i}{\lambda_i} = \omega\left(\Lambda + \sum_i \frac{\beta_i}{\lambda_i}\right)$$

and, since the reactor period, τ, is just the inverse of the most positive root, ω_1, we have

$$\tau = \frac{1}{\rho}\left(\Lambda + \sum_i \frac{\beta_i}{\lambda_i}\right) \qquad \text{(for small } \rho\text{)}$$

Small Reactivity Values

Also, in all practical cases, the generation time, Λ, is small compared to the 2nd term inside the brackets.

Thus, we can estimate the reactor period as

$$\tau = \frac{1}{\rho}\sum_i \frac{\beta_i}{\lambda_i} = \frac{t_d \beta}{\rho} \qquad \text{(for small } \rho\text{)}$$

t_d is the mean lifetime of the delayed neutrons

Capabilities of the *kinetics_gui* Code

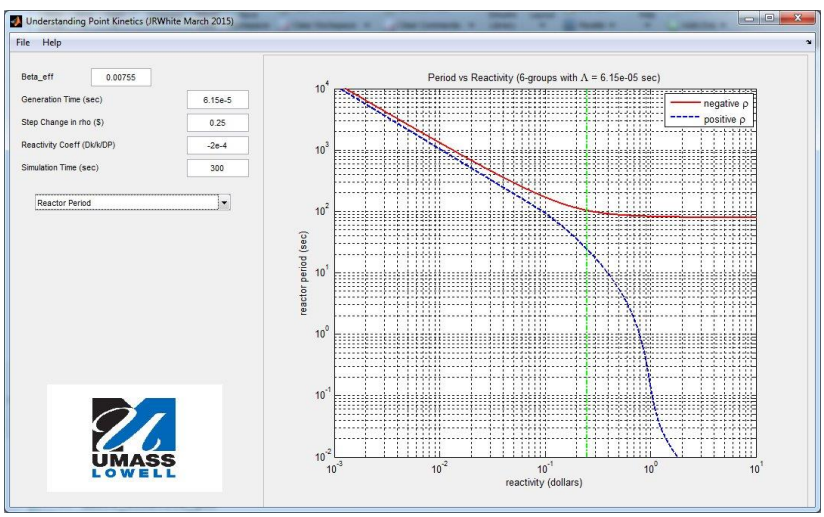
Finally, we note that, as shown previously, one can simply evaluate the reactivity equation and generate a plot of reactor period vs. reactivity -- this is a very useful operations/design tool.

The capability to do this, as well as plot the reactivity equation and generate the solution profiles for a given ρ has been incorporated into the *kinetics_gui* code.

The code is very simple to use and it gives a tremendous amount of insight into the workings of point kinetics -- you should give it a test drive!!!

The Matlab-based *kinetics_gui* code is available for use from the course Dropbox folder.

The *kinetics_gui* Interface



Summary and Take-Aways

How power maneuvers are performed within typical reactor systems.

Good overview of the specific startup procedure for the UMLRR.

How to convert the 1-group space-time kinetics formulation into the 1-speed point kinetics model.

An understanding of the primary advantage associated with the Generation Time Formulation relative to the standard time-dependent diffusion equation.

How to solve the Generation Time Formulation of point kinetics for a step change in reactivity.

Summary and Take-Aways (cont.)

A good understanding of the reactivity equation -- including a discussion concerning the sign and magnitude of the roots and the time dependent behavior of the power following a step change in ρ.

How the reactor period and prompt jump/drop are used to estimate the behavior of the power following a step change in ρ:

$$\frac{P(t)}{P_o} = \frac{P_1}{P_o} e^{\pm t/\tau} \quad \text{and} \quad \frac{P_1}{P_o} = \frac{\beta}{\beta - \rho}$$

The concept of reactivity feedback and the observed power profiles associated with a step change in reactivity with and without negative feedback.