24.331 Fundamentals of NSE

Lesson 11: Neutron Interactions with Matter III

The Fission Reaction

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See Section 6.6 in your text by Shultis & Faw (with some additional info)

Lesson 11 Objectives

Describe the basic fission process within the context of the liquid drop model of the nucleus.

Describe the typical distribution of fission products versus mass number A.

Discuss the relative magnitude and importance of decay heat considerations in the design of nuclear systems.

Discuss the number and spectrum of neutrons emitted in the fission process, including both prompt and delayed neutrons.

Explain the importance of the small delayed neutron component relative to the control of nuclear systems.

Write formal expressions for the fission source using both the continuous and multigroup formulations.

Write a formal expression for the power level in a nuclear system.

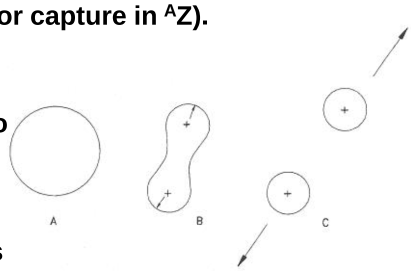
The Fission Reaction

Considering U235 fission, the fission process can be written as

$$U235 + n \rightarrow (U236)^{*} \rightarrow 2FPs + \nu n's + 200\text{ MeV}$$

The excited compound nucleus has an energy equal to the kinetic energy (KE) of the incident neutron plus the binding energy of the last neutron (BE_n) in nuclide ^{A+1}Z (for capture in ^{A}Z).

As the excitation energy of the compound nucleus increases, the nucleus can elongate -- possibly to the point where the electrostatic repulsive force exceeds the short range attractive nuclear forces between nucleons in the two lobes of the elongated droplet -- thus causing the compound nucleus to actually break into two intermediate mass fission fragments.



The Fission Reaction (cont.)

For fission to happen, a critical energy (E_{crit}) associated with the compound nucleus must be reached.

If $KE + BE_n > E_{crit}$ for isotope ^{A+1}Z, then fission can occur.

If $BE_n > E_{crit}$, then fission can occur with any incident neutron energy, and such a nuclide is denoted as a fissile isotope.

It is common practice to say that U235 fissions even though it is U236* that actually fissions.

For U236, E_{crit} = 5.3 MeV and BE_n = 6.4 MeV.

Since $BE_n > E_{crit}$, U235 (not U236) is said to be a fissile isotope.

TABLE 3.3 CRITICAL ENERGIES FOR FISSION, IN MeV

Fissioning Nucleus ^{A}Z	Critical Energy	Binding Energy of Last Neutron in ^{A}Z
^{232}Th	5.9	*
^{233}Th	6.5	5.1
^{233}U	5.5	*
^{234}U	4.6	6.6
^{235}U	5.75	*
^{236}U	5.3	6.4
^{238}U	5.85	*
^{239}U	5.5	4.9
^{239}Pu	5.5	*
^{240}Pu	4.0	6.4

The Fission Reaction (cont.)

As another example, notice that, for U239, Table 3.3 from Lamarsh lists E_{crit} = 5.5 MeV and BE_n = 4.9 MeV.

Thus, U238 is not fissile and it requires a threshold kinetic energy of about 0.6 MeV for fission ($KE_{thres} = E_{crit} - BE_n$).

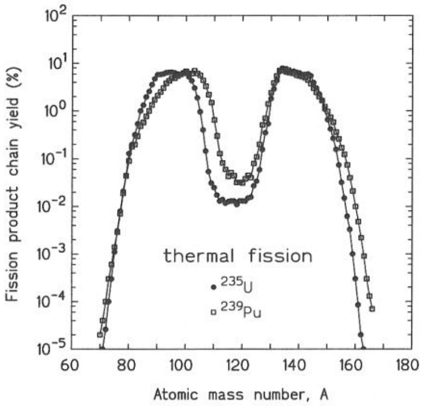
U238 is said to be a fissionable isotope.

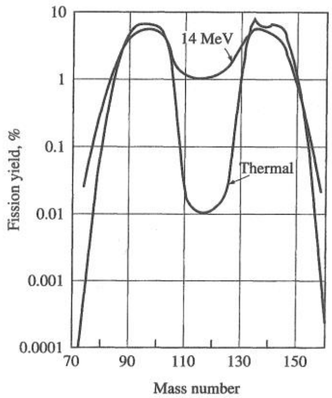
TABLE 3.3 CRITICAL ENERGIES FOR FISSION, IN MeV

Fissioning Nucleus ^AZ	Critical Energy	Binding Energy of Last Neutron in ^AZ
^{232}Th	5.9	*
^{233}Th	6.5	5.1
^{233}U	5.5	*
^{234}U	4.6	6.6
^{235}U	5.75	*
^{236}U	5.3	6.4
^{238}U	5.85	*
^{239}U	5.5	4.9
^{239}Pu	5.5	*
^{240}Pu	4.0	6.4

Fission Products

Fission is almost always asymmetric. The masses of the two fission fragments are usually substantially different, and a plot of the FP yields vs. A has a characteristic double hump shape.





Fission Products (cont.)

The fission product yield is typically written as γ_{ij} which is defined as

γ_{ij} = # atoms of isotope i produced per fission in nuclide j

where $\sum_i \gamma_{ij} = 2$ for any j (i.e. there are two FPs per fission event)

Note that fission products are not only important from a neutron balance point of view.

In addition, most fission product nuclides are radioactive and most of these are neutron rich and decay via β^- decay.

This radioactivity and its associated decay heat cause many of the problems associated with the use of nuclear energy.

In general, the radioactive fission products must be contained, cooled after shutdown, and stored as waste after removal from the reactor.

Decay Heat Considerations

Relative to the decay heat concern, consider the table below that gives the approximate average distribution of recoverable energies per fission.

The dominant component is the KE of the FP fragments.

This energy contribution, as well as the energy associated with the prompt fission & capture gammas and the neutrons emitted from the fission reaction, go away very quickly after the reactor is shutdown (becomes subcritical).

Distribution of Recoverable Energy from the Fission Process.

Energy Component	Approx. MeV per fission
kinetic energy of FPs	168
kinetic energy of neutrons	5
prompt fission gammas	7
prompt capture gammas	5
FP decay -- beta particles	8
FP decay -- gamma rays	7
total	200

Decay Heat Considerations

In contrast, the **7- 8 % associated with the FP decay** (i.e. **that due to the FP beta and gamma decay**), **does not vanish immediately**.

In fact, **this decay heat represents a significant contribution for a long time after shutdown!!!**

see plot on next slide…

For example, for a reactor with P = 3000 MW$_{th}$, the power level shortly after shutdown is about 0.075 × 3000 = 225 MW$_{th}$ -- which is still a very significant power source.

Even after 2.75 hours ≈ 10^4 seconds, the decay heat is still almost 1% of the initial reactor power before shutdown -- which corresponds to about 30 MW$_{th}$ for the current example!!!

Thus, cooling is required for a long time after shutdown!

This was the real issue in the Fukushima accident!!!

Decay Heat Considerations (cont.)

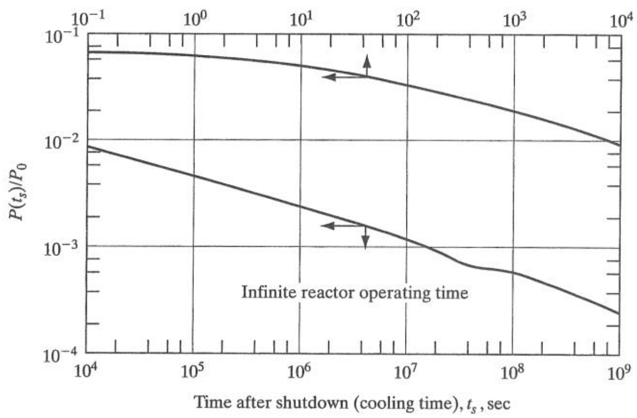


Figure 8.3 The ratio $P(t_s)/P_0$ of the fission product decay power to the reactor operating power as a function of time t_s after shutdown.

Fission Neutrons

The neutrons emitted from fission are important since they represent the mechanism for keeping the chain reaction going.

The average number of neutrons emitted per fission is given the symbol ν_T (usually just ν).

The contributions to ν_T are made up from prompt neutrons and delayed neutrons.

Prompt neutrons are emitted directly in the fission event (essentially instantaneously).

The delayed component occurs from the neutrons that are released in the decay of certain fission products.

These delayed neutron precursors are produced from fission and the neutrons they emit can be thought of as resulting from fission with a characteristic delay before release.

Fission Neutrons (cont.)

The delayed neutrons are usually grouped into six separate groups with effective decay constants λ_i and yields β_i, where

$$\beta_i = \frac{\nu_i}{\nu_T} = \frac{\text{delayed neutrons from precursor group i per fission}}{\text{total neutrons emitted per fission}}$$

Note also that

$$\nu_d = \sum_i \nu_i = \text{total delayed neutrons per fission}$$

where

$$\beta = \sum_i \beta_i = \frac{\nu_d}{\nu_T} = \text{total delayed neutron fraction}$$

Note also that β is quite small -- only about 0.00685 for U235 and about 0.0023 for Pu239.

Fission Neutrons (cont.)

Finally, we note that, without the small fraction of delayed neutrons that exist, we would not be able to control the chain reaction (see example in the next few slides).

However, we will postpone a detailed treatment of this subject until we study reactor kinetics (next semester in the Reactor Theory course).

Thus, we will re-visit this subject in more detail later in your NSE studies…

Example: Control of Nuclear Systems

Consider a U235 fueled and water moderated critical reactor where we have the following typical parameters:

$\beta = 0.0068$ = total delayed neutron fraction

$l_p \approx \Lambda_p = 10^{-4}$ s = prompt neutron generation time

$l_d = 12.7$ s = average delayed neutron lifetime

Also recall that the multiplication factor, k, is given by

$$k = \frac{\text{neutrons produced in one generation}}{\text{neutrons produced in preceeding generation}}$$

Thus, if n_o is the neutron population at time zero, then the total population after N generations is simply

$$n_N = k^N n_o$$

Example: Control of Nuclear Systems

Now, let's assume that we only have prompt neutrons in the system.

If $n_o = 1$ and $k = 1.001$ (just slightly above critical), then in 1 second there will be about 10000 neutron generations, where

$$N = \frac{\Delta t}{\Lambda_p} = \frac{1\,\text{sec}}{10^{-4}\,\text{sec/gen}} = 10{,}000 \text{ generations}$$

and, after this many generations, the neutron population will be

$$n_{10{,}000} = (1.001)^{10{,}000}(1) \approx 21{,}900$$

Thus, the power level would increase by a factor of more than 21,000 in just 1 second.

Clearly, this is not a controllable situation!!!

Example: Control of Nuclear Systems

Now, assume that k is still 1.001, but that only $(1-\beta) = 0.9932$ of the neutrons produced are prompt -- the remaining 0.68% are delayed neutrons.

In this case, if we consider only the prompt neutrons, the system will be subcritical -- that is, $k_{prompt} = (0.9932)(1.001) \approx 0.9942$ -- and, in 1 second, the new neutron population produced from these prompt neutrons would be

$$n_{10{,}000} = (0.9942)^{10{,}000}(1) \approx 5.5\times10^{-26} \approx 0$$

Thus, in 1 second, all the original prompt neutrons are gone -- that is, the system is subcritical on prompt neutrons alone.

Example: Control of Nuclear Systems

However, if we also consider the delayed neutron contribution, with an overall effective neutron lifetime given by

$$l_{eff} = (1-\beta)\,\Lambda_p + \beta\, l_d = (0.9932)(10^{-4}) + (0.0068)(12.7) \approx 0.086\ s$$

then, in 1 second, there will only be about 12 neutron generations, where

$$N = \frac{\Delta t}{l_{eff}} = \frac{1\,\text{sec}}{0.086\,\text{sec/gen}} = 11.6 \text{ generations}$$

and, after this many generations, the neutron population will be

$$n_{12} = (1.001)^{12}(1) \approx 1.012$$

Thus, in 1 second, there is only about a 1% increase in neutron level and overall reactor power -- and we see that, with delayed neutrons, we have a much more manageable control scenario!!!

Example: Control of Nuclear Systems

Clearly this example is not overly rigorous since, in practice, we need to treat the time dependence of the prompt and various delayed neutron groups separately.

However, this "back of the envelope calculation" gives us a rough picture of the effects of delayed neutrons…

In particular, it clearly illustrates that, without the small fraction of delayed neutrons present, we would not be able to control a nuclear system!!!

Delayed neutrons are essential to the control of nuclear systems…

Total Fission Source

For our current work we will consider only the steady state fission source and write this as

$$\frac{\text{neutrons emitted}}{\text{cm}^3\text{-sec}} = \int (1-\beta)\nu_T \Sigma_f(E)\phi(E)dE + \int \beta \nu_T \Sigma_f(E)\phi(E)dE$$

$$\text{prompt} \qquad + \qquad \text{delayed}$$

or

$$S_{fis} = \int \nu_T \Sigma_f \phi \, dE = \int \nu \Sigma_f \phi \, dE$$

The first expression separates the total source into prompt and delayed components.

However, a key point that is not addressed here is the time at which the neutrons are emitted -- thus, this statement is only valid for the steady state (time-independent) case, where the different time dependence of the prompt and delayed neutrons is not important.

Total Fission Source

Also, we should note that the symbol η is used to refer to the average number of neutrons emitted per absorption in the fuel.

With this definition, we can also write the total steady state fission source as

$$S_{fis} = \int \nu \Sigma_f \phi \, dE = \int \eta \Sigma_a^F \phi \, dE$$

where

$$\eta(E) = \frac{\nu \Sigma_f(E)}{\Sigma_a^F(E)}$$

F $\rightarrow$ fuel material

The Fission Spectrum

Not only is it important to determine the number of neutrons emitted per fission, but we also need to know the energy of these neutrons.

Fission neutrons are emitted with a continuous energy spectrum which is Maxwellian in nature but shifted in energy such that the peak of the curve is slightly less than 1 MeV and the average energy is about 2 MeV.

This distribution of fission neutron energies is known as the fission spectrum (typically called the prompt fission spectrum), where

$\chi(E)dE$ = probability that a fission neutron will be born within energy interval dE around E

The Fission Spectrum

Since this must be a properly normalized distribution function (i.e. the probability of finding the neutron with energies between 0 and ∞ is unity), then

$$\int_0^\infty \chi(E)dE = 1$$

In a multigroup formulation, χ_g is defined as

$$\chi_g = \int_{\Delta E_g} \chi(E)dE \quad \text{and} \quad \sum_g \chi_g = 1$$

For prompt neutrons, a modified Maxwellian distribution known as the Watt fission spectrum is often used to describe $\chi(E)$.

The Fission Spectrum (cont.)

The general form of the Watt fission spectrum is given by

$$\chi(E) = a e^{-bE} \sinh\sqrt{cE}$$

where the parameters a, b, and c are found by fitting this equation to appropriate experimental data for each fissionable nuclide of interest.

For example, for U235 fission, Lamarsh gives

$$\chi(E) = 0.453 e^{-1.036E} \sinh\sqrt{2.29E}$$

where E is in MeV.

In Shultis and Faw, they write things a little differently as

$$\chi(E) = a e^{-E/b} \sinh\sqrt{cE}$$

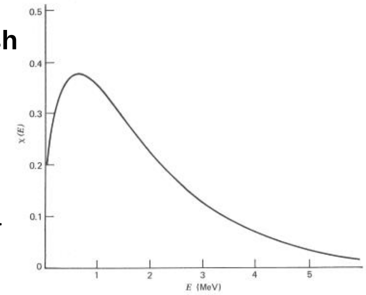


FIGURE 2-21. Fission spectrum for thermal neutron induced fission in ^{235}U.

see eqn. 6.43 and Table 6.4 in Shultis & Faw

The Fission Spectrum (cont.)

Delayed neutrons also a have a spectrum of energies, $\chi_d(E)$, and the profile tends to be peaked at a lower energy [relative to $\chi_p(E)$].

However, since β is so small, the difference in delayed versus prompt neutron spectra is negligible for steady state analyses -- so $\chi(E) \approx \chi_p(E)$.

For transient problems, the differences between $\chi_p(E)$ and $\chi_d(E)$ are very important and this distinction needs special treatment (at a later time)!

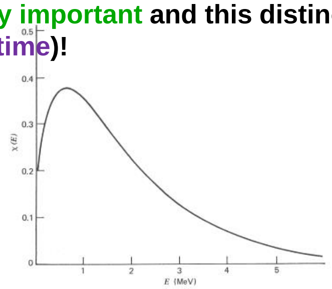


FIGURE 2-21. Fission spectrum for thermal neutron induced fission in ^{235}U.

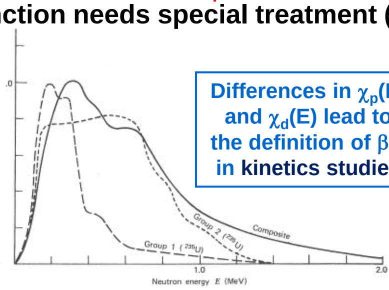


FIGURE 2-23. Composite delayed neutron spectrum.

Differences in $\chi_p(E)$ and $\chi_d(E)$ lead to the definition of β_{eff} in kinetics studies

Energy Dependent Fission Source

Now, if we combine the discussion of the neutron energy distribution with the treatment of the number of neutrons emitted per fission, we can formally define the energy dependent fission source as

$$\begin{matrix}\text{\# of neutrons emitted per} \\ \text{cm}^3\text{ - sec in energy interval dE}\end{matrix} = \begin{pmatrix}\text{fraction of fission neutrons} \\ \text{emitted in energy interval dE}\end{pmatrix} \times \begin{pmatrix}\text{total neutron production} \\ \text{rate from all fission}\end{pmatrix}$$

or

$$\begin{matrix}\text{number of neutrons emitted per} \\ \text{cm}^3 - \text{sec in energy interval dE}\end{matrix} = \chi(E)\left[\int_{\text{all }E'} \nu\Sigma_f(E')\phi(E')dE'\right]dE$$

and, in a discrete multigroup energy formulation, this becomes

$$\begin{matrix}\text{number of neutrons emitted per} \\ \text{cm}^3\text{-sec in energy group g}\end{matrix} = \chi_g \sum_{g'} \nu\Sigma_{fg'}\phi_{g'}$$

Energy Dependent Fission Source

This latter representation for the fission source will be one of the primary source terms within the full multigroup balance equation…

$$S_g^{fis} = \chi_g \sum_{g'} \nu\Sigma_{fg'}\phi_{g'}$$

Fission source used in the steady state neutron balance equation

Fission Power

The last and probably most important product from fission is the energy released, since the practical application of this available energy is the primary reason for studying nuclear reactor physics and reactor engineering.

Up to now we have been using a nominal value of 200 MeV per fission event.

However, this value is not only nuclide dependent, it is also made up of several components.

The dominant source of energy is in the form of the kinetic energy of the fission fragments.

These fragments deposit their energy locally (essentially at the spot of fission).

Fission Power

Fission product decay heat and the emission of γ-rays following radiative capture (i.e. the n, γ reaction) are also important contributions to the total.

In detailed computations, many of the individual components can be treated explicitly, giving a better representation of energy deposition.

However, most preliminary work simply assumes 200 MeV per fission deposited at the point of fission.

We will make this assumption in most of our work!!!

We have already noted that the flux magnitude in the reactor is determined by the imposed power level (recall that power and criticality are decoupled at least to first order).

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Fission Power (cont.)

If we denote κ as the recoverable energy per fission, then the reactor power can be expressed in terms of the fission rate as

$$P = \kappa \int \Sigma_f(\vec{r},E)\phi(\vec{r},E)d\vec{r}dE$$

Several references use E_R instead of κ

where the integration is over all space and energy.

A consistency check on the units associated with this expression gives

$$\text{watts} = \left(\frac{\text{joules}}{\text{fission}}\right)\left(\frac{\text{fissions/sec}}{\text{cm}^3 - \text{energy}}\right)\left(\text{cm}^3\right)\left(\text{energy}\right) = \frac{\text{joules}}{\text{sec}} = \text{watts}$$

The power equation is typically used as a normalization of the flux distribution in a critical reactor.

The nominal value of κ is $3.204\text{x}10^{-11}$ joules/fission, which corresponds to 200 MeV per fission event.

Lesson 11 Summary

In this Lesson we have briefly discussed the following subjects:

The basic fission process within the context of the liquid drop model of the nucleus.

The typical distribution of fission products versus mass number.

The relative magnitude and importance of the decay heat source term in the design of nuclear systems.

The number and spectrum of neutrons emitted in the fission process, including both prompt and delayed neutrons.

The importance of the small delayed neutron component relative to the control of nuclear systems.

A set of formal expressions for the fission source using both the continuous and multigroup formulations.

A formal expression for the power level in a nuclear system.