(a) Convert the Maxwellian distribution in energy

$$f(E) = C_1 E^{1/2} e^{-E/kT}$$

to be a function of neutron speed, where $E = \frac{1}{2}mv^2$. that is, what is the corresponding $f(v)$ distribution?

Hint: to do this formally set $f(E)dE = f(v)dv$ and solve for $f(v)$

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with $E = \frac{1}{2}mv^2$ $\qquad dE = mv\,dv$

then $f(E)\,dE = f(v)\,dv$

$$= C_1\left(\frac{m}{2}\right)^{1/2} v\, e^{-mv^2/2kT}\, mv\,dv$$

now combine all the constants into a new constant

$$C_2 = C_1\left(\frac{m}{2}\right)^{1/2} m$$

or $\boxed{f(v) = C_2 v^2 e^{-mv^2/2kT}}$

(b) Using the $f(v)$ function from Part a, compute the most probable speed and determine the energy associated with this speed.

Here we need to compute $\frac{df}{dv}$ and set this to zero.

to facilitate this

let $z = v^2$ and $a = \frac{m}{2kT}$

$$\therefore \quad f(v) = C_2 z e^{-az}$$

and

$$\frac{df}{dv} = C_2 z \underbrace{\frac{d}{dv}\left(e^{-az}\right)}_{\frac{d}{dz}\left(e^{-az}\right)\frac{dz}{dv}} + C_2 \frac{dz}{dv} e^{-az}$$

2v

cancels so not really needed

$$\therefore \quad \frac{df}{dv} = C_2 z\left(-a\, e^{-az}\right)\left(\frac{dz}{dv}\right) + C_2 \frac{dz}{dv} e^{-az} = 0$$

or $-za + 1 = 0$ or $z = \frac{1}{a}$ or $\boxed{v_p^2 = \frac{2kT}{m}}$

thus, the most probable speed is

$$\boxed{v_p = \sqrt{\frac{2kT}{m}}}$$

Now since $E = \frac{1}{2} m v^2$

$$E_{\substack{assoc \\ with\ v_p}} = \frac{1}{2} m \left(\frac{2kT}{m} \right) = kT$$

$$\therefore \boxed{E_{\substack{assoc \\ with\ v_p}} = kT}$$

← often called E_0, the thermal energy

Evaluate at 20°C → 293.15K

$$E_0 = kT = \left(8.6173 \times 10^{-5} \frac{eV}{K} \right) (293.15 K)$$

$$= \boxed{0.0253 \text{ eV}}$$

The first resonance in the cross section of aluminum occurs at 5.8 keV. The absorption cross section at 0.0253 eV is about 0.23 b and the scattering cross section is roughly 1.37 b. Based on these facts, estimate σ_a, σ_s, and σ_t at 100 eV. State any assumptions.

Note: Do not "look up" the Al cross sections in JANIS or other resource -- the idea here is to estimate the cross sections at 100 eV based on your understanding of the typical behavior of $\sigma(E)$.

The 1st statement says that the resonance region does not start until well into the keV region.

Thus, in the range of interest, we can assume
1. $\frac{1}{v}$ behavior for $\sigma_a(E)$.
2. potential scattering – that is $\sigma_s(E) \approx$ const.

Also $\sigma_t(E) = \sigma_a(E) + \sigma_s(E)$.

$\therefore$ at 100 eV

$$\sigma_a(E) = \frac{\sigma_a(E_0)\, v_0}{v} = \sigma_a(E_0)\sqrt{\frac{E_0}{E}}$$

$$\therefore \quad \sigma_a(100\text{ eV}) = 0.23\text{ b}\sqrt{\frac{0.0253}{100}}$$

$$= \boxed{0.0037\text{ b}} \quad \text{ans}$$

assume $\frac{1}{v}$ behavior

and $\sigma_s(E) = \sigma_s(E_0)$ = constant in potential scat region

assumption

$$\therefore \quad \sigma_s(100\text{ eV}) = \boxed{1.37\text{ b}} \quad \text{ans}$$

and
$$\sigma_t(100\text{ eV}) = \sigma_a(100\text{ eV}) + \sigma_s(100\text{ eV})$$
$$= 0.0037 + 1.37$$
$$= \boxed{1.374\text{ b}} \quad \text{ans}$$

Note: From JANIS with ENDF B 6.8

at $E_0 = 0.0253$ eV $\quad \sigma_a = 0.232$ b, $\sigma_s = 1.374$ b, $\sigma_t = 1.606$ b

at $E = 100$ eV $\quad \sigma_a = 0.0037$ b, $\sigma_s = 1.347$ b, $\sigma_t = 1.351$ b

ok $\qquad$ small change $\qquad$ not too bad

The control rods for a certain reactor are made of an alloy of cadmium (5 w/o), indium (15 w/o), and silver (80 w/o) — note That Ag is a 1/v absorber.

Compute the thermal absorption rate per gram of control rod at a temperature of 400°c if the 2200 m/s flux is 5×10^{13} n/cm²-s

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non $\frac{1}{v}$ factor — macro abs x sec for component i at E_0

$$F_a(T) = \sum_i g_{a_i}(T)\, \Sigma_{a_i}(E_0)\, \phi_0 \leftarrow \text{2200 m/s flux}$$

sum over all components

material temp

from table 3.2 at T = 400°c

$g_a|_{cd} = 2.56$ $\quad g_a|_{In} = 1.10$ $\quad g_a|_{Ag} = 1.00$ ← pure $\frac{1}{v}$

from Table II.3 at $E_0 = 0.0253$ eV

$\sigma_a|_{cd} = 2450$ b $\quad \sigma_a|_{In} = 193.5$ b $\quad \sigma_a|_{Ag} = 63.6$ b

$1\text{ b} = 10^{-24}$ cm²

Now compute N per gram of rod

Cd $\quad \dfrac{0.05 \text{ g cd}}{\text{g of rod}} * \dfrac{0.6022 \times 10^{+24} \text{ at of Cd/gmole}}{112.40 \text{ g of cd/gmole}} = 2.679 \times 10^{20} \dfrac{\text{at cd}}{\text{g of rod}}$

In $\quad 0.15 * \dfrac{0.6022 \times 10^{24}}{114.82} = 7.867 \times 10^{20} \dfrac{\text{at. In}}{\text{g of rod}}$

Ag $\quad 0.80 * \dfrac{0.6022 \times 10^{24}}{107.87} = 4.466 \times 10^{21} \dfrac{\text{at Ag}}{\text{g of rod}}$

$$\therefore F_a|_{400c} = \Big[(2.56)(2.679 \times 10^{20})(2450 \times 10^{-24}) + (1.10)(7.867 \times 10^{20})(193.5 \times 10^{-24}) + (1.00)(4.466 \times 10^{21})(63.6 \times 10^{-24})\Big] * 5 \times 10^{13} \frac{n}{cm^2\text{-}s}$$

units = $\dfrac{\text{at. cm}^2}{\text{g of rod}}$

$$= (1.680 + 0.167 + 0.284)(5 \times 10^{13})$$

$$= \boxed{1.066 \times 10^{14} \text{ abs/s per gram of rod material}}$$

ans

Two hypothetical nuclei have the following atomic weights and critical energies for fission as follows

	MW	E_{crit}
^{A}Z	241.0600	5.5 MeV
^{A+1}Z	242.0621	6.5 MeV

Is nuclide ^{A}Z fissile? Explain/justify your answer...

For ^{A}Z to be fissile, then the binding energy of the last neutron in ^{A+1}Z, BE_n, must be greater than the critical energy for fission in ^{A+1}Z.

$$^{A}Z + n \longrightarrow {}^{A+1}Z$$

$$BE_n = \left[M(^{A}Z) + M_n - M(^{A+1}Z) \right] 931.5 \text{ MeV/amu}$$

$$= \begin{array}{r} 241.0600 \\ + \quad 1.00867 \\ - \ 242.0621 \\ \hline \end{array}$$

$$(0.00657 \text{ amu}) * 931.5 \frac{\text{MeV}}{\text{amu}} = \boxed{6.12 \text{ MeV}}$$

Thus, since $BE_n < E_{crit}$, then ^{A}Z is <u>NOT</u> fissile.

Note: Based on the values given here, ^{A}Z is fissionable with a threshold energy for fission of about

$$E_{thres} = E_{crit} - BE_n$$

$$= 6.5 - 6.1 \approx \boxed{0.4 \text{ MeV}}$$

Fission can be induced when γ-rays are absorbed in a heavy nucleus. What energy γ-rays are necessary to induce fission in U235, U238, and Pu239?

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The critical energy for fission for these isotopes are listed below (from Table 3.3 in Lamarsh)

Element	Crit. Energy (MeV)
U235	5.75
U238	5.85
Pu239	5.5

* these are the minimum γ-ray energies needed for γ,f reaction

Since the photo-fission event [the (γ, f) reaction] does not have the usual compound nucleus formation associated with neutron capture, there is no additional energy associated with the binding energy of the last nucleon.

Thus, the minimum energy for the incident γ-rays to cause fission is simply the critical energy noted above.

Lamarsh 3E
Prob 3.30

Definition / Use of Lethargy

$\boxed{3-30}$ given: neuts at 2 MeV slow down to 1 eV

→ Find # of collisions required on average for Hydrogen and Graphite

$$n = \frac{\text{ave \# coll to go from } E_0 \to E}{} = \frac{\text{change in lethargy}}{\text{ave. change in lethargy per coll.}} = \frac{\Delta u}{\xi}$$

$$\therefore \boxed{n = \frac{\Delta u}{\xi}}$$

in scattering from 2 MeV to 1 eV

$$\Delta u = \ln \frac{E_1}{E_2} = \ln \frac{2\times 10^6}{1} = \boxed{14.509}$$

$$\xi = 1 + \frac{\alpha}{1-\alpha} \ln \alpha \quad \text{with } \alpha = \left(\frac{A-1}{A+1}\right)^2$$

For hydrogen $A = 1$ $\alpha = 0$ $\xi = \underline{1}$

$$\therefore \boxed{n = \frac{\Delta u}{\xi} = 14.509}$$ ans

~ 15 collisions

For graphite $A = 12$ $\alpha = \left(\frac{11}{13}\right)^2 = .7160$

$$\xi = 1 + \frac{.7160}{1-.7160} \ln 0.7160$$

or $\boxed{\xi = 0.1578}$

note
$\xi = \frac{2}{A + 2/3} = \frac{2}{12.666}$
$= .1579$
not bad

$$\therefore n = \frac{\Delta u}{\xi} = \frac{14.509}{0.1578} = \boxed{91.95}$$ ans

~ 92 collisions

A 1 MeV neutron elastically scatters with a C-12 nucleus that is initially at rest. If the neutron scattering angle in the center-of-mass system is 45 degrees, determine the following quantities:

a. Energy of the neutron after collision.

$$E_n' = \frac{A^2 + 2A\cos\theta_c + 1}{(A+1)^2} E \qquad \theta_c = 45^\circ$$

$$\cos\theta_c = 0.7071$$

$$= \frac{12^2 + 24(0.7071) + 1}{(13)^2} E$$

$$= \frac{161.970}{169}(1\,\text{MeV}) = \boxed{0.958\ \text{MeV}}$$

Energy of the C-12 nucleus after collision

conservation of energy says that the total energy before and after collision must be the same (recall that the nucleus remains in the ground state, so all the energy here is kinetic energy)

$$\therefore E_{C\text{-}12}' = E_{TOT} - E_n'$$

$$= 1 - 0.958 = \boxed{0.042\ \text{MeV}}$$

b. Scattering angle of the neutron in the lab coordinate syst.

$$\cos\theta_L = \frac{1 + A\cos\theta_c}{\sqrt{A^2 + 2A\cos\theta_c + 1}}$$

$\cos 45^\circ = 0.7071$ rad

$$= \frac{1 + 12(0.7071)}{\sqrt{12^2 + 24(0.7071) + 1}} = \frac{9.4852}{\sqrt{161.970}}$$

$$\cos\theta_L = 0.7453 \quad \therefore \boxed{\theta_L = 41.8^\circ}$$

c. The change in lethargy of the neutron

$$\Delta u = \ln\frac{E}{E'} = \ln\frac{1}{0.958} = \boxed{0.0429}$$