Lamarsh
Prob 3.39

Partial cross sections

Several cross sections for U-235 at 1 MeV are as follows:

$\sigma_{el} = 4.0$ b $\quad \sigma_{inel} = 1.4$ b $\quad \sigma_f = 1.2$ b $\quad \sigma_a = 1.3$ b

where we will assume that all the remaining cross sections for any neutron-producing and charged-particle reactions are negligible.

Under these conditions what are the total cross section and the capture-to-fission ratio in U-235 at 1 MeV?

— — — —

$\sigma_T = \sigma_a + \sigma_s$

where $\sigma_a = \sigma_c + \sigma_f$ and $\sigma_s = \sigma_{el} + \sigma_{inel}$

∴ from data given

$\sigma_s = 4.0 + 1.4 = 5.4$b

$\boxed{\sigma_T = 1.3 + 5.4 = 6.7\text{b}}$

ans

also $\sigma_c = \sigma_a - \sigma_f$

$= 1.3 - 1.2 = 0.1$b

and $\boxed{\alpha = \frac{\sigma_c}{\sigma_f} = \frac{0.1}{1.2} = 0.083}$

ans

Lamarsh 3rd Ed Prob 3.3

Beam Target Interactions

mono energetic beam	$I = 4 \times 10^{10}\ n/cm^2\text{-sec}$
target area	$A = 1\ cm^2$
Target thickness	$\tau = 0.1\ cm$
Target density	$N = 4.8 \times 10^{-2}\ at/b\text{-cm}$
Total cross section	$\sigma_\tau = 4.5\ b$

(a) What is macroscopic cross section?

$$\Sigma_\tau = N\sigma_\tau = 4.8 \times 10^{-2} \frac{at}{b\text{-cm}} * 4.5\ b$$

$$\boxed{\Sigma_\tau = 0.216\ cm^{-1}}$$

(b) How many neutron interactions per second occur in target?

collision rate density

$$F_\tau = N\sigma_\tau I = \Sigma_\tau I$$

$$F_\tau = (0.216\ cm^{-1})(4 \times 10^{10}\ n/cm^2\text{-sec})$$

or $\boxed{F_\tau = 8.64 \times 10^9\ coll./cm^3\text{-sec}}$

Now because the target is so thin, we can assume that the beam attenuation is small

$$\therefore \frac{\#\ coll}{sec} = \begin{pmatrix}\text{collision} \\ \text{density}\end{pmatrix}\begin{pmatrix}\text{Target} \\ \text{vol}\end{pmatrix}$$

$$= \left(8.64 \times 10^9 \frac{coll}{cm^3\text{-sec}}\right)(0.1\ cm^3)$$

$V = A\tau$ ← THICKNESS

$= (1\ cm^2)(0.1\ cm)$

$= 0.1\ cm^3$

$$= \boxed{8.64 \times 10^8 \frac{coll}{sec}}$$

$\boxed{3.10}$ wide beam with ϕ_0 = initial intensity / flux
material has $\sigma_a >> \sigma_s$
Target area = A   Thickness = L

Derive an expression for the rate at which neutrons are absorbed within the target.

---

since $\sigma_a >> \sigma_s$

Then $\sigma_t = \sigma_a + \sigma_s \simeq \sigma_a$ } ∴ all the interactions lead to a loss of the neutron (absorption)

The absorption rate is given by

$$R(x) = \Sigma_a \phi(x) \qquad (\text{cm}^{-1})\left(\frac{n}{\text{cm}^2\text{-s}}\right) = \frac{\text{abs}}{\text{cm}^3\text{-sec}}$$

$$R_T = \int_v \Sigma_a \phi(\vec{r})\, d\vec{r}$$

here $d\vec{r} = A\,dx$

$$R_T = \Sigma_a A \int_0^L \phi(x)\, dx$$

{ wide beam implies constant w.r.t y and z direction

∴ all we need is $\phi(x)$, but from previous discussion (and from development in class and the book)

$$\phi(x) = \phi_0 e^{-\Sigma_t x} = \phi_0 e^{-\Sigma_a x}$$

{ spatial distribution of beam intensity

∴ $$R_T = \Sigma_a A \phi_0 \int_0^L e^{-\Sigma_a x}\, dx$$

$$= \Sigma_a A \phi_0 \left[\frac{-1}{\Sigma_a} e^{-\Sigma_a x}\right]_0^L$$

$$\boxed{R_T = A\phi_0 \left[1 - e^{-\Sigma_a L}\right]} \qquad \left(\frac{\text{abs}}{\text{sec}}\right)$$

ans

↑ units

$(\text{cm}^2)\left(\frac{\text{abs}}{\text{cm}^2\text{-s}}\right)$

ok

A beam of 2 MeV neutrons is incident on a slab of heavy water (D_2O). The total cross section of deuterium and oxygen at this energy are 2.6 b and 1.6 b, respectively. Note $\rho_{D_2O} \approx 1.1 g/cc$

(a) What is the macroscopic total cross section of D_2O at 2 MeV?

assume nominal density of about 1.10 g/cm³

$$\frac{1.1\ g\ D_2O}{cm^3} \times \frac{2(2.014)\ g\ of\ D}{20.028\ g\ D_2O} \times \frac{0.6022 \times 10^{24}\ at\ of\ D}{2.014\ g\ of\ D}$$

about 10% larger than normal H_2O

$$N_D = 6.615 \times 10^{22}\ at/cm^3$$

$$\boxed{N_D = 6.615 \times 10^{-2}\ at/b\text{-}cm}$$

$MW_D = 2.014$
$MW_O = 16.0$
$MW_{D_2O} = 20.028$

and

$$\boxed{N_O = \frac{1}{2} N_D = 3.307 \times 10^{-2}\ at/b\text{-}cm}$$

$$\therefore \Sigma_t = N_D \sigma_{tD} + N_O \sigma_{tO}$$

$$= (6.615 \times 10^{-2})(2.6) + (3.307 \times 10^{-2})(1.6)$$

$$\boxed{\Sigma_t = 0.225\ cm^{-1}} \longrightarrow \boxed{0.172 + 0.053 = 0.225}$$

components

(b) How thick must the slab be to reduce the intensity of the uncollided beam by a factor of 10?

$$\frac{I}{I_0} = 0.1 = e^{-\Sigma_t L}$$

or $\ln 0.1 = -\Sigma_t L \implies L = -\frac{\ln 0.1}{\Sigma_t}$

or $L = -\frac{\ln 0.1}{0.225\ cm^{-1}} \cong \boxed{10.2\ cm}$

(c) If an incident neutron has a collision in the slab, what is the relative probability that it collides with deuterium?

$$\frac{\Sigma_{tD}}{\Sigma_{t\,TOT}} = \frac{(6.615 \times 10^{-2})(2.6)}{0.225} = \frac{0.172}{0.225} \cong \boxed{0.764}$$

$$\boxed{\cong 76\text{-}77\%}$$

Lamarsh 3rd Ed. Prob 3.11 | macroscopic cross sections

given: SS304 $\rho_{ss} = 7.86\ g/cm^3$

composition (w/o)

carbon	.08
chromium	19
nickel	10
iron	70.92

compute Σ_a for SS304 at .0253 eV

Data from Japan Nuclear Data Center www.ndc.jaea.go.jp

nuclide	Molecular wt	σ_a (barns) at .0253 eV
carbon	12.011	0.0039
chromium	51.996	3.14
Nickel	58.693	4.47
iron	55.845	2.57

and $\boxed{\Sigma_{a_{ss}} = \sum_i N_i \sigma_{a_i}}$

∴ all we need are the nuclide densities

$$\text{Carbon} = 7.86 \frac{g\ of\ ss}{cm^3} \times \frac{.0008\ g\ of\ C}{g\ of\ ss} \times \frac{.6022\ at\ of\ C}{12.011\ g\ of\ C} \frac{cm^2}{b}$$

$$= (7.86)(.0008)\left(\frac{.6022}{12.011}\right)$$

$$= \boxed{3.153-4 \ \frac{at\ of\ C}{b\text{-}cm}}$$

$$\text{chromium} = (7.86)(.19)\left(\frac{.6022}{51.996}\right) = \boxed{1.730-2 \ \frac{at\ of\ Cr}{b\text{-}cm}}$$

$$\text{nickel} = (7.86)(.10)\left(\frac{.6022}{58.693}\right) = \boxed{8.064-3 \ \frac{at\ of\ Ni}{b\text{-}cm}}$$

$$\text{iron} = (7.86)(.7092)\left(\frac{.6022}{55.847}\right) = \boxed{6.011-2 \ \frac{at\ of\ Fe}{b\text{-}cm}}$$

$$\therefore \Sigma_{a_{ss}} = \sum_i N_i \sigma_{a_i}$$

$$= (3.153-4)(.0039) + (1.730-2)(3.14) + (8.064-3)(4.47) + (6.011-2)(2.57)$$

$$\boxed{\Sigma_{a_{ss}} = 0.245\ cm^{-1}}$$ ans

Duderstadt + Hamilton chapt #2 Prob 2.6 and 2.10

Mean Free Paths; Beam Target Interactions

Engineer's Computation Pad — No. 937 811E — STAEDTLER

(a) How many mean free paths thick must a shield be designed in order to attenuate an incident neutron beam by a factor of 1000?

$$\frac{I(x)}{I_0} = \frac{1}{1000} = e^{-\Sigma_t x}$$

let $\bar{x}$ = # of mean free paths, where $\bar{x} = \frac{x}{\lambda} = \Sigma_t x$

so $$\frac{I(\bar{x})}{I_0} = \frac{1}{1000} = e^{-\bar{x}}$$

or $$\bar{x} = -\ln 0.001 = \boxed{6.91}$$

a shield that is > 6.9 mfp will attenuate the beam by greater than a factor of 1000

(b) Boron is a common material used to shield against thermal neutrons. Estimate the thickness of natural boron at a density of 2.3 g/cm³ needed to attenuate an incident thermal neutron beam to 0.1% of its original intensity.

$$\frac{2.3\ \text{g of Bnat}}{\text{cm}^3} * \frac{0.6022\times10^{24}\ \text{at of Bnat/gmole}}{10.811\ \text{g of Bnat/gmole}} * 10^{-24}\frac{\text{cm}^2}{\text{b}} = N_{Bnat}$$

$$N_{Bnat} = 0.128\ \frac{\text{at}}{\text{b-cm}}$$

at E_0

$\sigma_a = 759\text{b}$ $\quad\therefore\quad \Sigma_a = (0.128)(759) = \boxed{97.2\ \text{cm}^{-1}}$

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$\sigma_s = 3.6\text{b}$ let's ignore this

now we desire Δx for $I/I_0 = 0.001$

$$\frac{I}{I_0} = 0.001 = e^{-\Sigma_a \Delta x}$$

or $$\Delta x = -\frac{\ln I/I_0}{\Sigma_a} = \frac{-\ln 0.001}{97.2}$$

$$\boxed{\Delta x = 0.071\ \text{cm}}$$

(c) Convert your result from Part b into mean free paths and comment on the result.

$$\bar{\Delta x} = \frac{\Delta x}{\lambda} = \Sigma_a \Delta x = (97.2)(0.071) = \boxed{6.91}$$

and, of course, this is the same result as in Part a. That is, we need ~ 6.9 mfp to attenuate by a factor of 1000...

Consider an isotropic point source and sample geometry where the sample is a 0.2 cm thick aluminum foil with a 5 cm^2 area that faces the source. The thermal neutron source strength is 10^8 neutrons/s. The aluminum sample density is 2.7 g/cm^3 and the total cross section for thermal neutrons is about 1.67 barns.

a. Estimate the uncollided flux at the sample location and the total Al interaction rate (reactions/sec) if the sample is placed 1 m from the source.

b. Re-compute the uncollided flux at the sample and the Al interaction rate if an 8 cm thick shield with $\Sigma_t = 0.5$ cm^{-1} is placed between the source and Al sample (assume that the total distance of 1 m between the sample and the source is unchanged from Part a).

Note: Neglect any neutron collisions in the air (that is, assume that the air represents a vacuum).

(a) No shield

$$\phi^o = \frac{S}{4\pi r^2} = \frac{10^8}{4\pi (100\ cm)^2} = \boxed{7.958\times10^2\ n/cm^2\text{-}s}$$

ans

$$R = N \sigma \phi^o$$

$$N = \frac{2.7\,g}{cm^3} \times \frac{6.022\times10^{23}\ at/gmole}{27\ g/gmole}$$

sample

$$= 6.022\times10^{22}\ at/cm^3$$

$$V = A\Delta x = (5)(0.2) = 1\ cm^3$$

$$\therefore \boxed{N = 6.022\times10^{22}\ \text{atoms of Al in sample}}$$

$$\sigma = 1.67b \times 10^{-24}\ \frac{cm^2}{b}$$

$$\sigma = 1.67\times10^{-24}\ cm^2$$

$$\Sigma_{t_{sample}} = 0.10\ cm^{-1}$$

$$R = (6.022\times10^{22})(1.67\times10^{-24})(7.958\times10^2)$$

$$\boxed{R = 80.0\ \frac{\text{reactions}}{s}}$$

ans

(b) with shield

$$\phi^o = \underbrace{\frac{S}{4\pi r^2}}_{\text{flux at } r \text{ due to geometric attenuation}} \underbrace{e^{-\Sigma_t \Delta x}}_{\text{reduction factor due to shield}}$$

Δx ← shield thickness

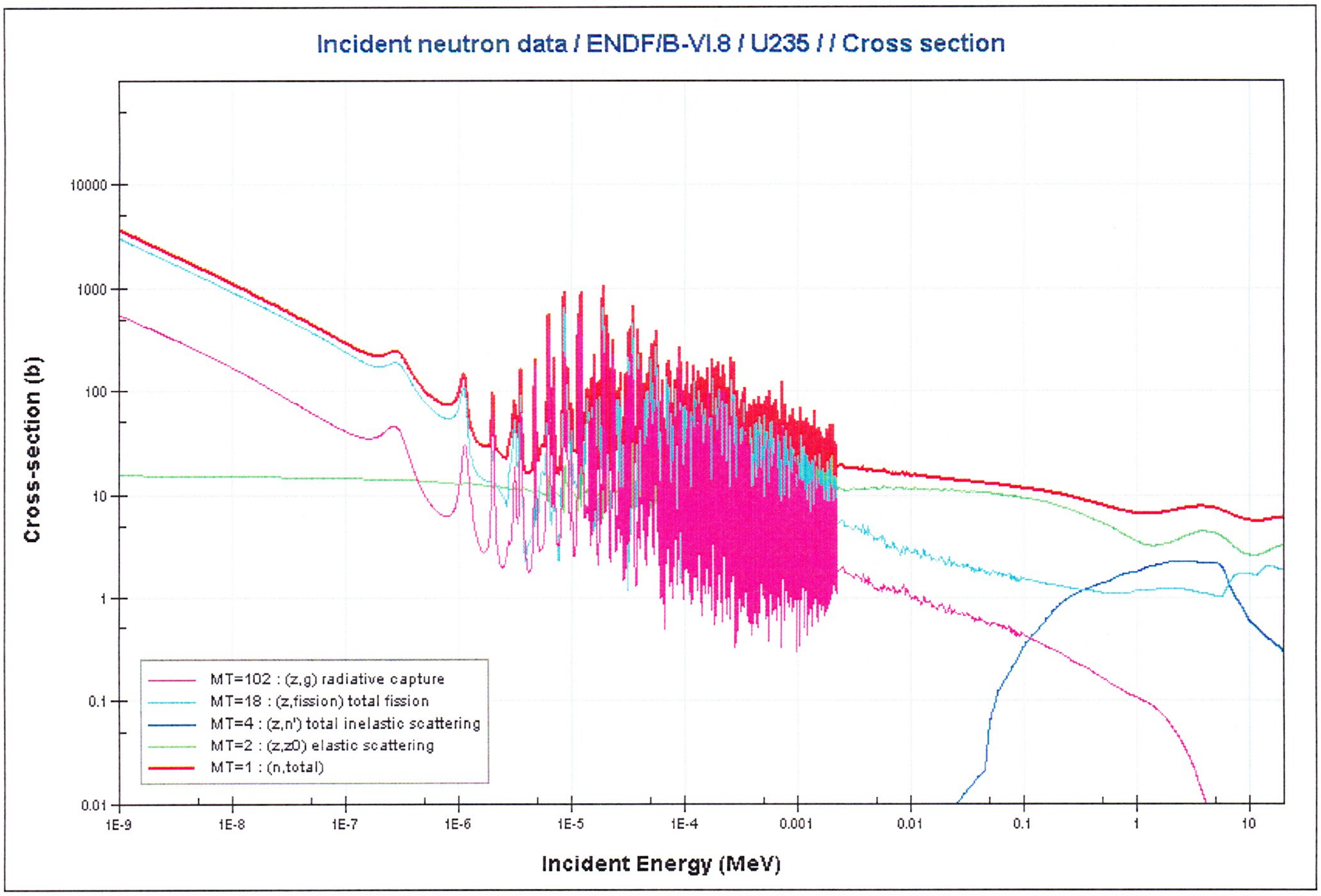
for $\Sigma_t = 0.5\ cm^{-1}$ and $\Delta x = 8\,cm$

$$e^{-(0.5)(8)} = e^{-4} = 0.0183 \leftarrow \text{reduction factor}$$

$$\therefore \phi^o_{\substack{with\\shield}} = 0.0183\ \phi^o_{\substack{no\\shield}} = \boxed{14.58\ n/cm^2\text{-}s}$$

$$R_{\substack{with\\shield}} = 0.0183\ R_{\substack{no\\shield}} = \boxed{1.47\ \frac{\text{reactions}}{sec}}$$

same location

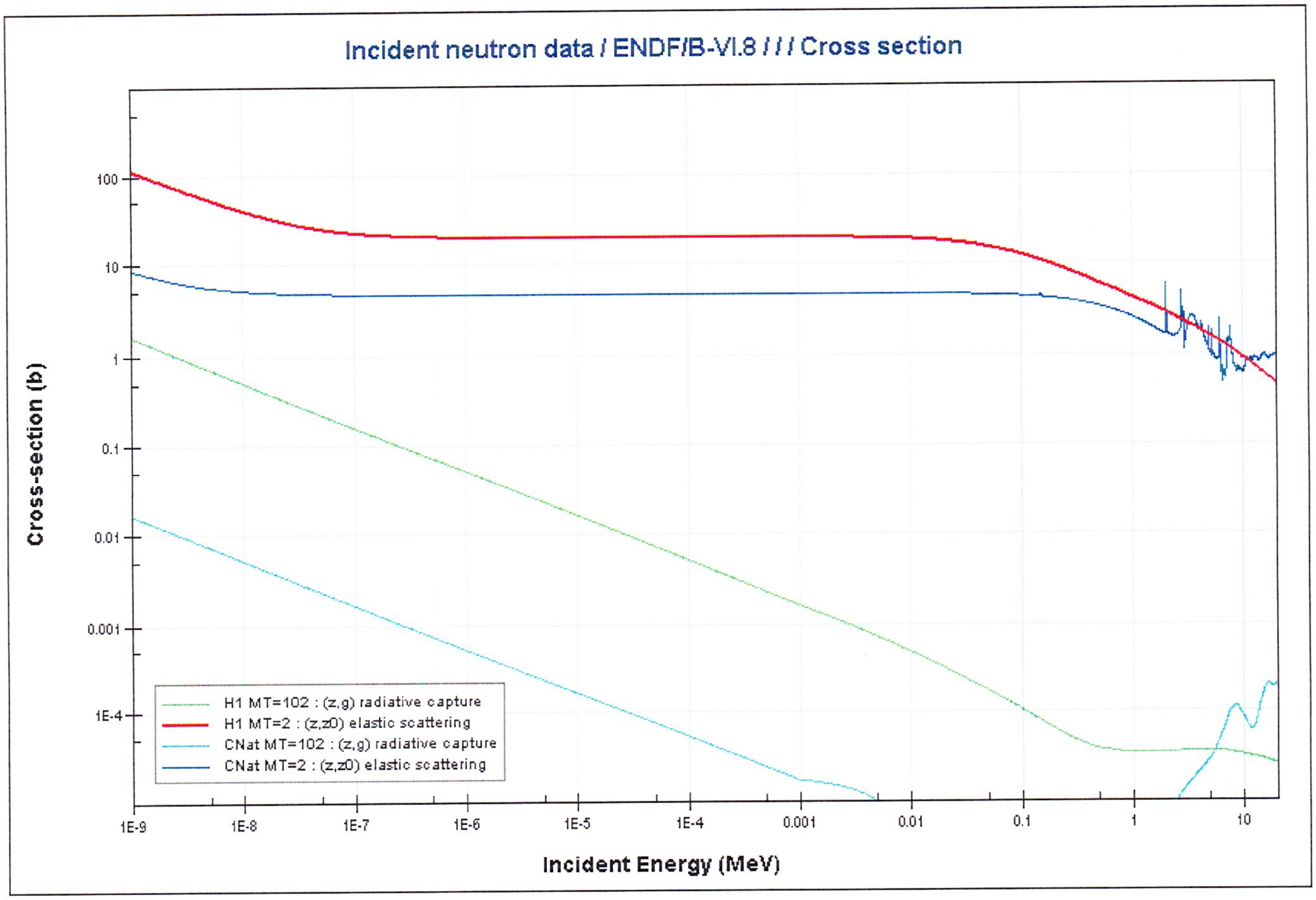


Typical Fuel Material Cross Sections:

U235 Total (1), Elastic (2), Inelastic (4), Fission (18), and Capture (102)

Notes:

1. Three general regions: 1/v region for capture & fission and nearly constant potential scattering at low energy, resonance region at intermediate energies, and slowly varying cross sections at high energies.
2. The fission cross section is larger than parasitic capture.
3. Total cross section is the sum of the scattering (elastic and inelastic) and absorption (capture, fission, …) cross sections.
4. The inelastic cross section is a threshold reaction.

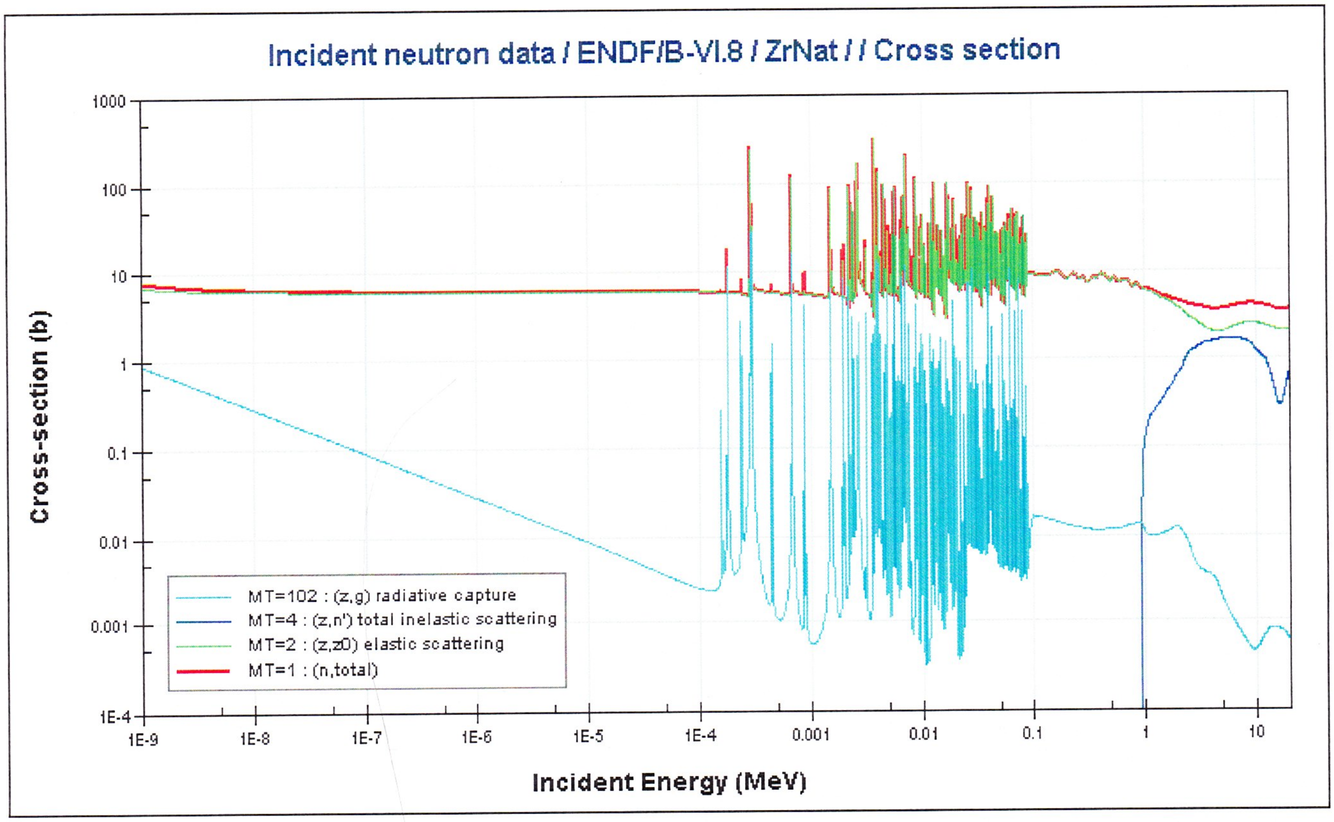


Typical Moderator Material Cross Sections:

H-1 Elastic (2) and Capture (102) and C Elastic (2) and Capture (102)

Notes:

1. The scattering cross section is much larger than the capture cross section.
2. Carbon (graphite) is a better moderator than H-1 (water) because of the lower parasitic capture cross section.
3. Essentially negligible resonance structure – only occurs at very high energies for low A materials.

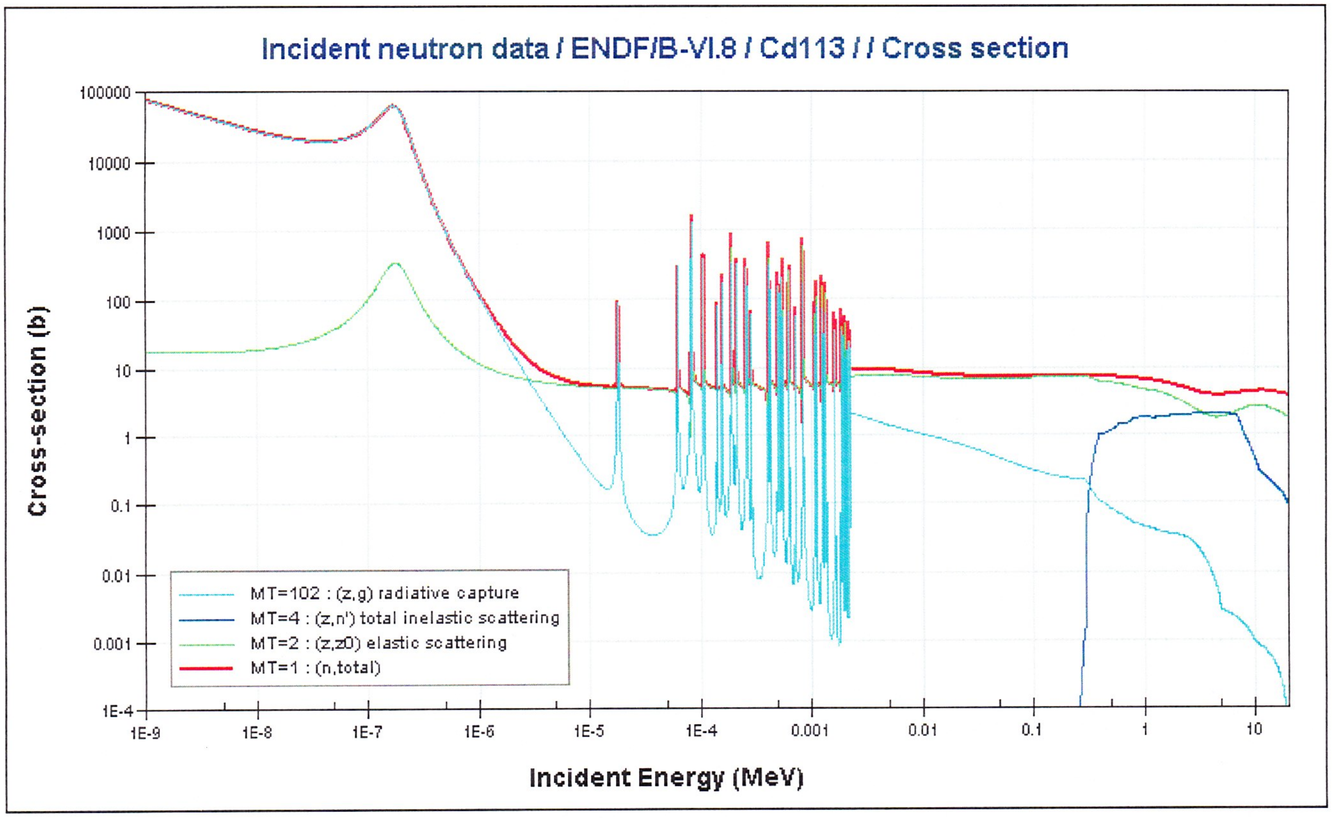


Typical Structural Material Cross Sections:

$\mathrm{Zr_{nat}}$ Total (1), Elastic (2), Inelastic (4), and Capture (102)

Notes:

1. Total cross section is relatively low and the parasitic capture component is small over most of the energy range of interest (except for a few narrow resonances).
2. Three general regions: 1/v region for capture and nearly constant potential scattering at low energy, resonance region at intermediate to high energies, and slowly varying cross sections at very high energies.
3. Total is the sum of the scattering (elastic and inelastic) and absorption (capture, fission, …) cross sections.
4. Inelastic scattering is a threshold reaction.

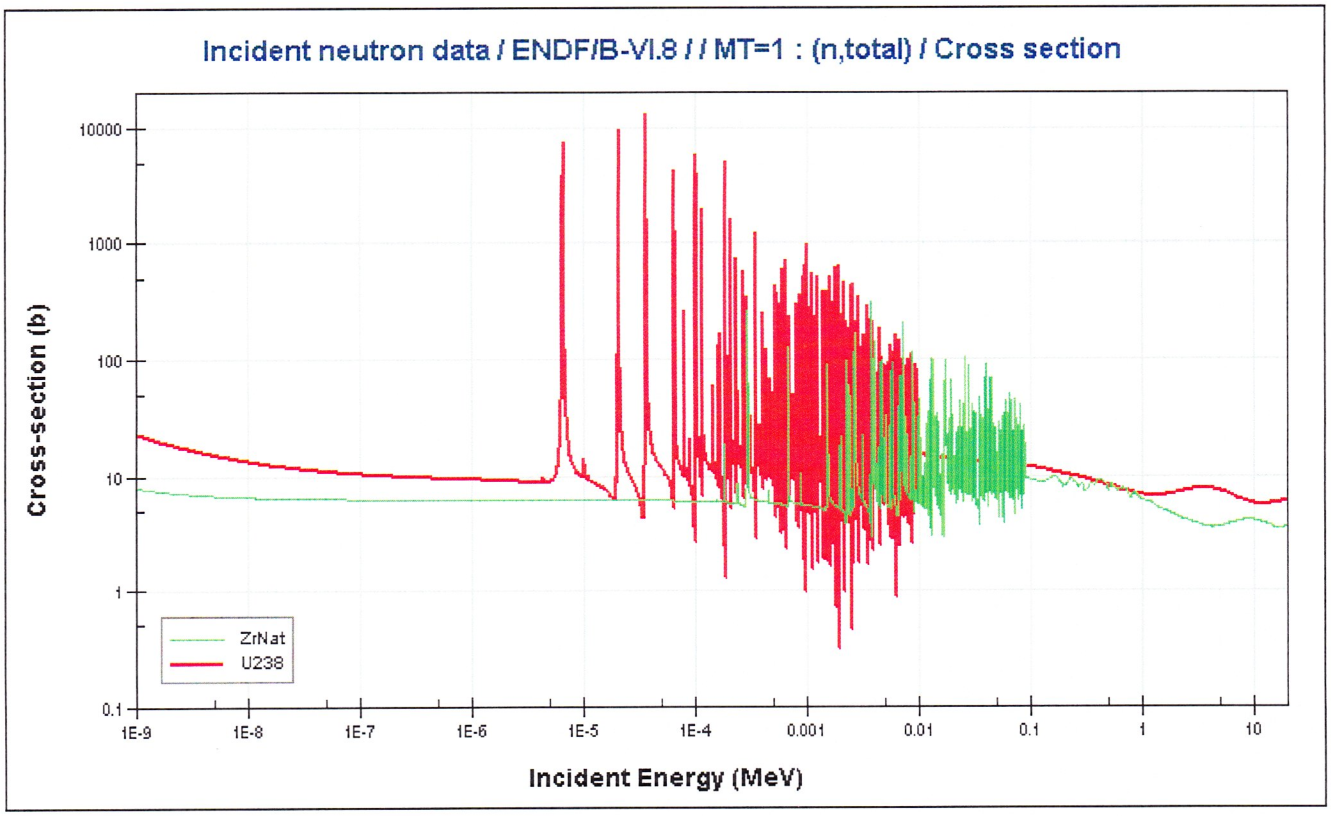


Typical Poison Material Cross Sections:

Cd-113 Total (1), Elastic (2), Inelastic (4), Capture (102), ~~and n, Alpha~~ (107)

Notes:

1. The absorption cross is very large in the low energy region (total ≈ absorption). Notice also that Cd-113 **is not** a pure 1/v absorber below 1 eV.
2. The n,γ cross section is the dominant absorption component.
3. Scattering is negligible below 1 eV (that is, 10^{-6} MeV).
4. Usual threshold inelastic scattering becomes important at very high energies.

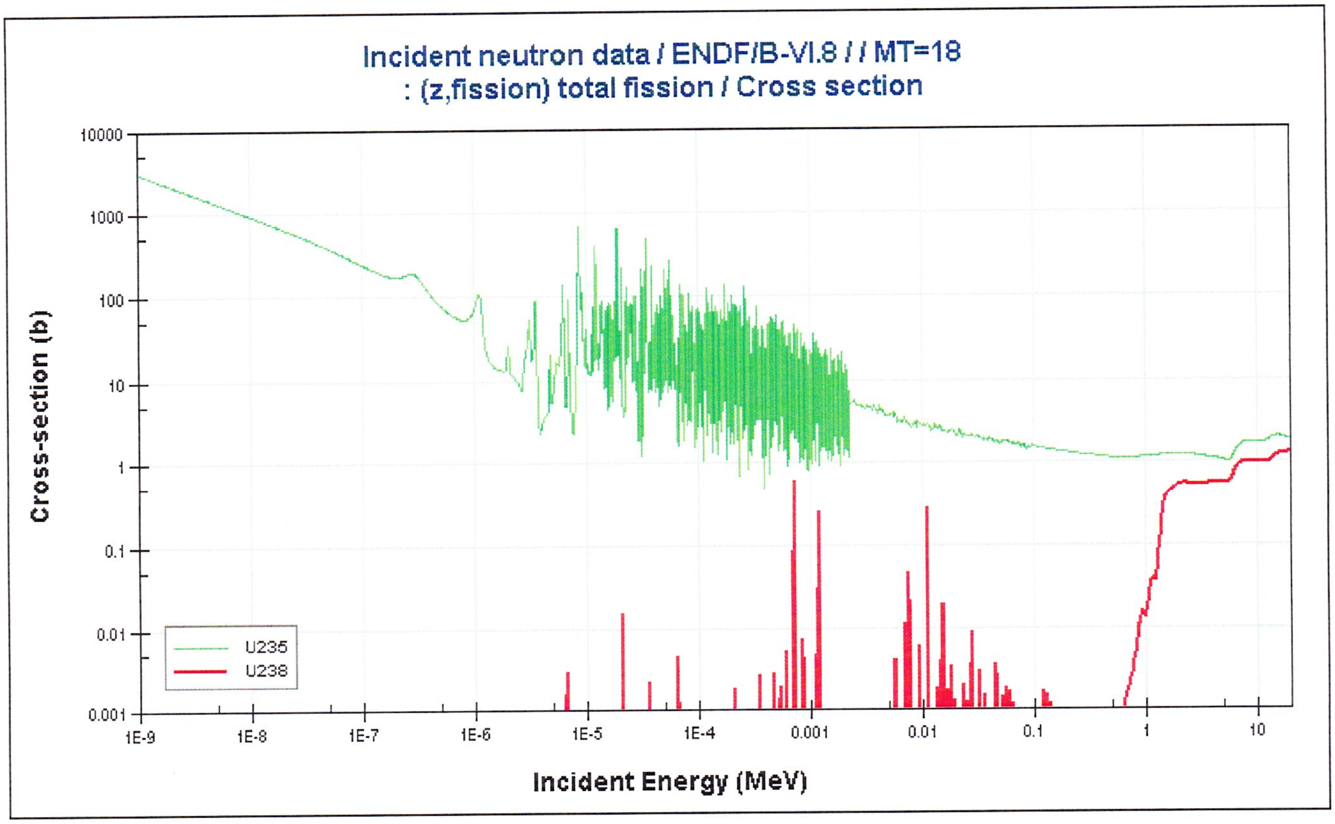


Highlight Location of Resonance Region:

Zr_{nat} Total (1) and U238 Total (1)

Notes:

1. The resonance region begins at a lower energy for the heavy isotopes because of the increased number of energy levels in the nucleus.
2. For low A materials, the resonance region only occurs at very high energies. For H-1, there are no resonances. For zirconium (with A ≈ 90-94), the resonance region extends down to about 300-400 eV -- but the lowest resonance in U238 is even lower (in the 6 – 7 eV range).

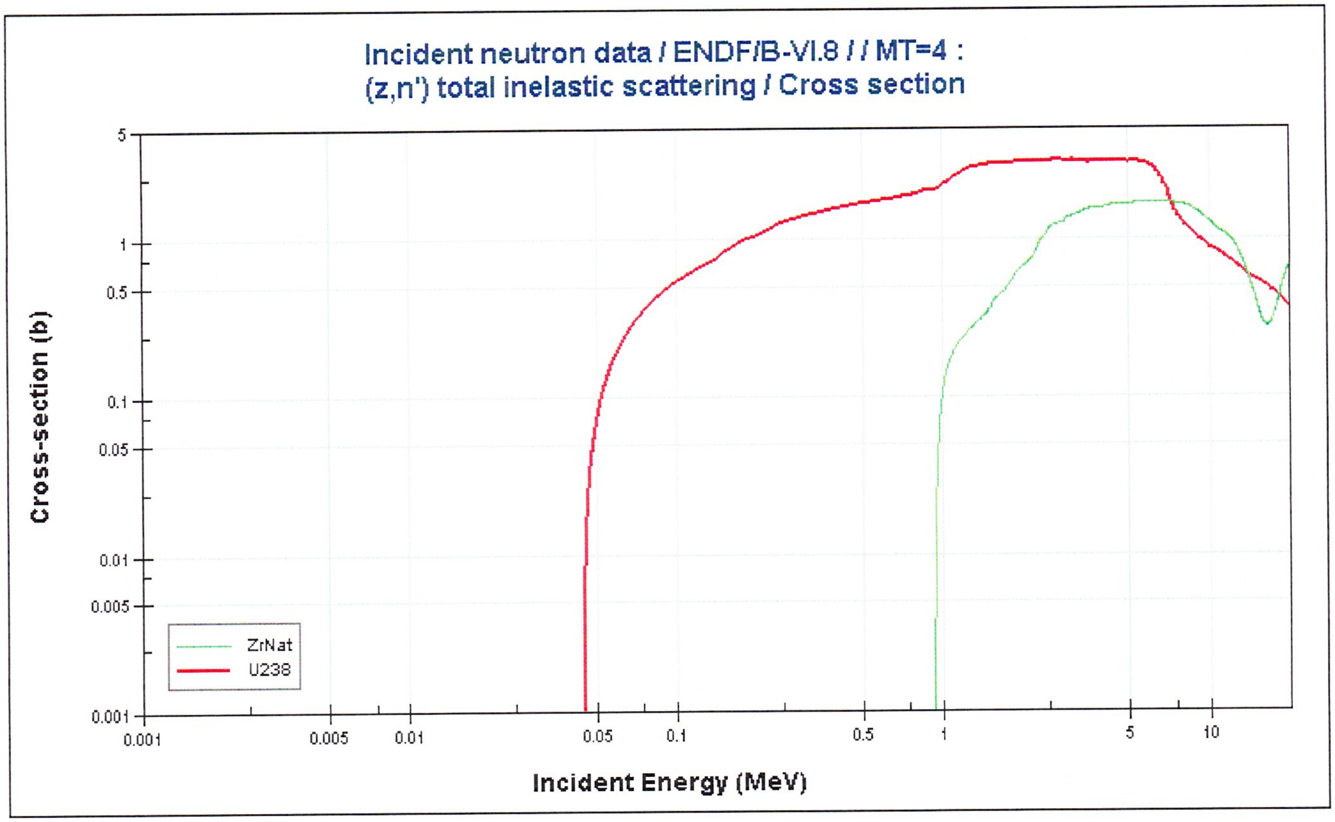


Highlight Fission Cross Section for Fissile and Non-Fissile (but fissionable) Materials:

U235 Fission (18) and U238 Fission (18)

Notes:

1. Fissile materials can fission with neutrons at any energy. However, the fission cross section typically follows the general three-region behavior: 1/v region at low energies, resonance region at intermediate energies, and slowly varying cross sections at high energies.
2. Fissionable materials that are not fissile, typically require a threshold energy before a sizable fission cross section exists (i.e. a threshold reaction except for a few very thin resonances).



Highlight Threshold Inelastic Scattering Cross Sections:

Zr_{nat} Inelastic (4) and U238 Inelastic (4)

Notes:

1. Inelastic scattering is a threshold reaction.
2. Inelastic scattering is more important in heavy isotopes because the threshold occurs at lower energies.