Shultis + Faw 2E
Prob 5.7

Radioactive Decay Terminology

The activity of a radioisotope is found to decrease by 30% in one week (168 hrs). What are the values of its decay constant, half-life, and mean lifetime?

---

for pure decay

$$\alpha(t) = \alpha_0 e^{-\lambda t}$$

a 30% decrease

$$\frac{\alpha(168\text{ hrs})}{\alpha_0} = 0.7 = e^{-168\lambda}$$

$$\ln 0.7 = -168\lambda$$

$$\text{or } \lambda = -\frac{\ln 0.7}{168} = \boxed{2.123 \times 10^{-3}\ \text{hr}^{-1}} \quad \text{or } 3.567 \times 10^{-1}\ \text{wk}^{-1}$$

then

$$t_{1/2} = \frac{\ln 2}{\lambda} = \boxed{326.5\ \text{hr}} \quad \text{or } 1.943\ \text{wks}$$

$$\bar{t} = \frac{1}{\lambda} = \boxed{471.0\ \text{hrs}} \quad \text{or } 2.804\ \text{wks}$$

ans

Lamarsh 3rd Ed

Radioactive Decay Terminology

$\boxed{2.29}$ given: Tritium = ^3H decay by β^-

$T_{1/2} = 12.26$ yr

$M = 3.016$

(a) what is daughter of ^3H decay?

in β^- decay $n \rightarrow p + e^- + \bar{\nu}$ (conceptual)

or $^3_1H \rightarrow \boxed{^3_2He^+} + e^- + \bar{\nu}$

daughter product

(b) what is the mass of 1 mCi of Tritium

$$\text{activity} \equiv \alpha = 1\ mCi * \frac{3.7 \times 10^{10}\ \text{decay/sec}}{Ci} \times \frac{10^{-3}\ Ci}{mCi}$$

$$= 3.7 \times 10^7\ dps$$

also $\alpha = \lambda N$ where $\lambda = \dfrac{\ln 2}{T_{1/2}}$

$$\therefore \lambda = \frac{\ln 2}{12.26\ yr} * \frac{1\ yr}{365\ day} * \frac{1\ day}{86400\ s}$$

$$= 1.7928 \times 10^{-9}\ sec^{-1}$$

and $N = \dfrac{\alpha}{\lambda} = \dfrac{3.7 \times 10^7\ at/s}{1.7928 \times 10^{-9}\ /s} = 2.0638 \times 10^{16}$ atoms

finally grams of ^3H $= 2.0638 \times 10^{16}$ at of ^3H $* \dfrac{3.016\ g\ of\ ^3H}{0.6022 \times 10^{24}\ at\ of\ ^3H}$

$$= \boxed{1.0335 \times 10^{-7}\ g\ of\ ^3H}$$

$$= \boxed{0.103\ \mu g}$$ ans

Polonium-210 decays to the ground state of Pb 206 by the emission of a 5.305 MeV α particle with a half life of 138 days. What mass of Po 210 is required to produce 1 MW of thermal power?

$$\text{Power} = \left(\frac{\text{energy}}{\text{decay}}\right)\left(\frac{\text{decay}}{\text{sec}}\right) \qquad \text{but } \alpha = \lambda N$$

$$= \frac{5.305 \text{ MeV}}{\text{decay}} * \frac{1.602\times10^{-13} \text{ J}}{\text{MeV}} * \underbrace{\frac{\ln 2}{138 \text{ day}} * \frac{1 \text{ day}}{86400 \text{ sec}}}_{\lambda = 5.813\times10^{-8} s^{-1}} * \underset{\#\text{ of atoms}}{N}$$

$$= 4.941\times10^{-20} N \text{ watts}$$

∴ To produce $P = 1$ MW

$$N = \frac{10^6 \text{ W}}{4.941\times10^{-20} \text{ W}} = 2.024\times10^{25} \text{ atoms}$$

now

$$m_{Po210} = N * \frac{M}{N_A}$$

$$= \left(2.024\times10^{25} \text{ at}\right)\left(\frac{210 \text{ g/gmole}}{0.6022\times10^{24} \text{ at/gmole}}\right)$$

$$m_{Po210} = \boxed{7058 \text{ g}} = \boxed{7.058 \text{ kg}}$$

ans

2.36. Given: SNAP-9 RTG with 475 g of ^{238}PuC

$\rho_{PuC} = 12.5\ g/cm^3$

$T_{1/2}$ of Pu238 = 89 yrs (α-emitter)

$E_\alpha = 5.6$ MeV per disintegration

$\eta = .054$ (Thermal to electrical efficiency)

(a.) Calc fuel efficiency in Ci/Wth (curies per watt thermal)

$$N_{Pu} = \frac{12.5\ g\ \text{of PuC}}{cm^3} * \frac{238\ g\ \text{of Pu}}{250\ g\ \text{of PuC}} * \frac{0.6022\times10^{24}\ \text{at of Pu}}{238\ g\ \text{of Pu}}$$

$$= .030\times10^{24}\ \frac{\text{at of Pu}}{cm^3}$$

$$Vol = \frac{mass}{density} = \frac{475\ g\ \text{of PuC}}{12.5\ g\ \text{of PuC}/cm^3} = 38\ cm^3\ \text{of PuC}$$

$$\therefore \boxed{N_{Pu} = 1.144\times10^{24}\ \text{atoms of Pu}}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{89\ yrs} * \frac{1\ yrs}{3.1536\times10^7\ sec}$$

$$= 2.470\times10^{-10}\ sec^{-1}$$

$$= 7.788\times10^{-3}\ yrs^{-1}$$

$$\alpha_{Pu} = \lambda N_{Pu}$$

$$= (2.470\times10^{-10})(1.144\times10^{24})$$

$$\boxed{\alpha = 2.825\times10^{14}\ dps}$$

and $1\ Ci = 3.7\times10^{10}$ dps

$$\therefore \boxed{\alpha_{Pu} = 7.636\times10^{3}\ Ci}$$

also

$$Power = 2.825\times10^{14}\ \frac{decays}{sec} * \frac{5.6\ MeV}{decay} * \frac{1.602\times10^{-13}\ J}{MeV} * \frac{1\ W}{1\ J/s}$$

$$\boxed{Power = 253.4\ watts}$$

$$\therefore \text{fuel eff} = \frac{7636\ Ci}{253.4\ watts} = \boxed{30\ Ci/watt}$$ ans

$\boxed{36.}$ cont.

(b) Calc specific power in Watt / g of fuel

$\therefore$ specific power $= \dfrac{253.4 \text{ W}_{th}}{475 \text{ g of PuC}} = \boxed{0.533 \text{ W}_{th}/\text{g of PuC}}$ ans

(c) Calc power density in watt / cm^3

$\therefore$ power density $= \dfrac{253.4 \text{ W}}{38 \text{ cm}^3} = \boxed{6.67 \text{ W}_{th}/\text{cm}^3}$ ans

(d) Calc total electrical power of the generator

$$P_{elec} = \eta * P_{th}$$

$$= (.054)(253.4 \text{ watts})$$

$\therefore$ $\boxed{P_{elect} = 13.7 \text{ watts}}$ ans

---

added by JRW

(e) What is the expected lifetime of the generator if the minimum electrical power needed to perform properly is 12 watts?

→ Since Pu238 is radioactive and there is no production, the activity simply decreases exponentially, or

$$\alpha(t) = \alpha_0 e^{-\lambda t}$$

Since the power is directly proportional to activity, we have

$$P(t) = P_0 e^{-\lambda t}$$

$$\therefore \quad \frac{P(t)}{P_0} = e^{-\lambda t} \quad \text{or} \quad \ln \frac{P(t)}{P_0} = -\lambda t$$

or $t = -\dfrac{1}{\lambda} \ln \dfrac{P(t)}{P_0}$ and in our case $t_{lifetime} = -\dfrac{89 \text{ yrs}}{\ln 2} \ln\left(\dfrac{12}{13.7}\right)$

$\boxed{t_{lifetime} \cong 17 \text{ yrs}}$ ans

Case 1 Determine the densities $A(t)$, $B(t)$, and $C(t)$ for the decay chain

$$A \to B \to C$$

where C is stable and the initial densities are A_0, B_0, & C_0.

For this case

$$① \quad \frac{dA}{dt} = -\lambda_A A \qquad \text{(no production)}$$

$$② \quad \frac{dB}{dt} = \lambda_A A - \lambda_B B$$

$$③ \quad \frac{dC}{dt} = \lambda_B B \qquad \text{(no loss)}$$

① $\frac{dA}{dt} = -\lambda_A A$: simply integrate to give

$$\frac{dA}{A} = -\lambda_A dt \quad \Rightarrow \quad \boxed{A(t) = A_0 e^{-\lambda_A t}}$$

ans

$$\ln \frac{A}{A_0} = -\lambda_A t$$

② $$\frac{dB}{dt} + \lambda_B B = \lambda_A A(t) = \lambda_A A_0 e^{-\lambda_A t}$$

1st order ODE, with integrating factor I.F. $= e^{\int \lambda_B dt} = e^{\lambda_B t}$

$$\therefore \quad \frac{d}{dt}\left(B e^{\lambda_B t}\right) = \left(\frac{dB}{dt} + \lambda_B B\right) e^{\lambda_B t} = \lambda_A A_0 e^{(\lambda_B - \lambda_A)t}$$

Integrating both sides

$$B(t) e^{\lambda_B t} - B_0 = \frac{\lambda_A A_0}{\lambda_B - \lambda_A}\left[e^{(\lambda_B - \lambda_A)t} - 1\right]$$

or

$$\boxed{B(t) = B_0 e^{-\lambda_B t} + \frac{\lambda_A A_0}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]}$$

ans

(3) $$\frac{dC}{dt} = \lambda_B B = \lambda_B B_o e^{-\lambda_B t} + \frac{\lambda_A \lambda_B A_o}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

Integrating both sides

$$C(t) - C_o = -B_o e^{-\lambda_B t}\Big|_0^t - \frac{\lambda_B A_o}{\lambda_B - \lambda_A} e^{-\lambda_A t}\Big|_0^t + \frac{\lambda_A A_o}{\lambda_B - \lambda_A} e^{-\lambda_B t}\Big|_0^t$$

or

$$\boxed{C(t) = C_o + B_o\left[1 - e^{-\lambda_B t}\right] + \frac{\lambda_B A_o}{\lambda_B - \lambda_A}\left[1 - e^{-\lambda_A t}\right] + \frac{\lambda_A A_o}{\lambda_B - \lambda_A}\left[e^{-\lambda_B t} - 1\right]}$$

ans

— — — —

Note: at large t, all the exponential terms approach zero

$$\therefore \quad C_\infty = C_o + B_o + \underbrace{\frac{\lambda_B A_o}{\lambda_B - \lambda_A} - \frac{\lambda_A A_o}{\lambda_B - \lambda_A}}_{A_o}$$

or $C_\infty = C_o + B_o + A_o$ as expected

ok

Case 2 Determine the densities $A(t)$, $B(t)$, and $C(t)$ for the following decay chain

$$R_A \searrow A \longrightarrow B \longrightarrow C$$

where A is produced from fission with constant production rate R_A and nuclide C is stable

- - - - - -

for this case

① $\dfrac{dA}{dt} = R_A - \lambda_A A$ $\quad (R_A = \text{const})$

② $\dfrac{dB}{dt} = \lambda_A A - \lambda_B B$

③ $\dfrac{dC}{dt} = \lambda_B B$ $\quad$ (C is stable)

① $\dfrac{dA}{dt} + \lambda_A A = R_A$ ← 1st order ODE

$$I.F. = e^{\int \lambda_A dt} = e^{\lambda_A t}$$

$$\therefore \; e^{\lambda_A t}\left[\frac{dA}{dt} + \lambda_A A\right] = \frac{d}{dt}\left[e^{\lambda_A t} A\right] = R_A e^{\lambda_A t}$$

integrating $\quad e^{\lambda_A t} A \Big|_0^t = R_A \left[\frac{1}{\lambda_A} e^{\lambda_A t}\right]_0^t$

$$e^{\lambda_A t} A(t) - A(0) = \frac{R_A}{\lambda_A}\left[e^{\lambda_A t} - 1\right]$$

now multiply through by $e^{-\lambda_A t}$

$$\boxed{A(t) = A_0 e^{-\lambda_A t} + \frac{R_A}{\lambda_A}\left[1 - e^{-\lambda_A t}\right]}$$ ans.

② $\dfrac{dB}{dt} + \lambda_B B = \lambda_A A(t)$ ← same form as ① except RHS is no longer constant

$$\therefore \; e^{\lambda_B t} B \Big|_0^t = \lambda_A \int_0^t A(t) e^{\lambda_B t} dt$$

$$= \lambda_A\left[A_0 \int_0^t e^{(\lambda_B - \lambda_A)t} dt + \frac{R_A}{\lambda_A}\int_0^t e^{\lambda_B t} dt - \frac{R_A}{\lambda_A}\int_0^t e^{(\lambda_B - \lambda_A)t} dt\right]$$

$$e^{\lambda_B t} B(t) - B_0 = \frac{\lambda_A A_0}{\lambda_B - \lambda_A}\left[e^{(\lambda_B - \lambda_A)t} - 1\right] + \frac{R_A}{\lambda_B}\left[e^{\lambda_B t} - 1\right] - \frac{R_A}{\lambda_B - \lambda_A}\left[e^{(\lambda_B - \lambda_A)t} - 1\right]$$

$$\therefore \; \boxed{B(t) = B_0 e^{-\lambda_B t} + \frac{\lambda_A A_0}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right] + \frac{R_A}{\lambda_B}\left[1 - e^{-\lambda_B t}\right] - \frac{R_A}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]}$$

ans

③ $$\frac{dC}{dt} = \lambda_B B(t)$$ ← simply integrate

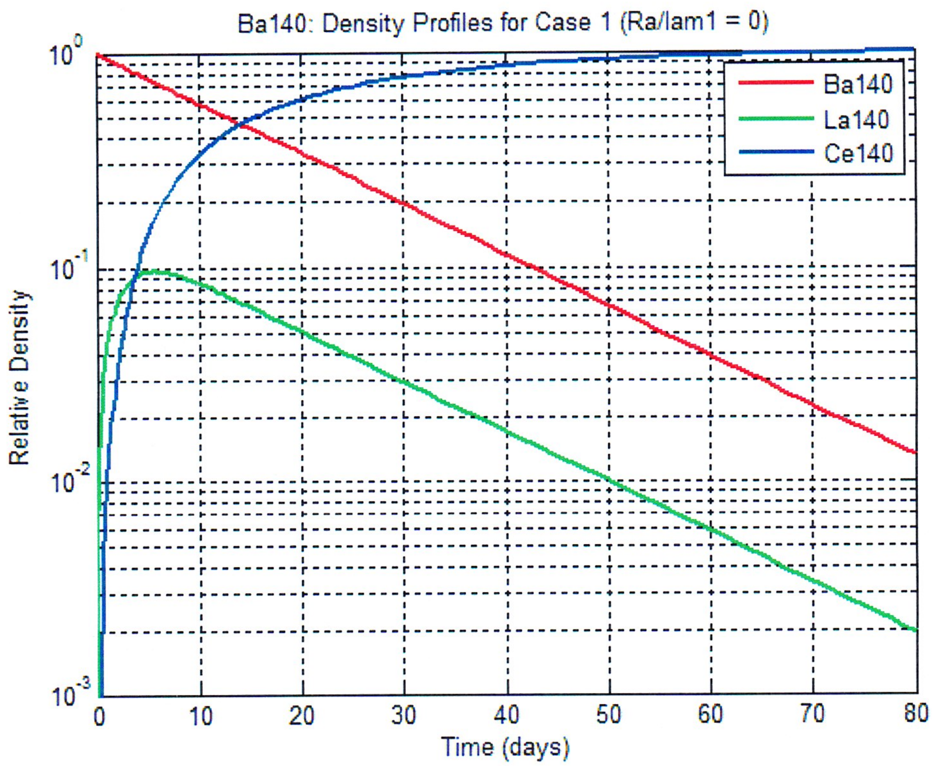
$$C(t) - C_0 = \lambda_B \int_0^t B(t)\,dt$$

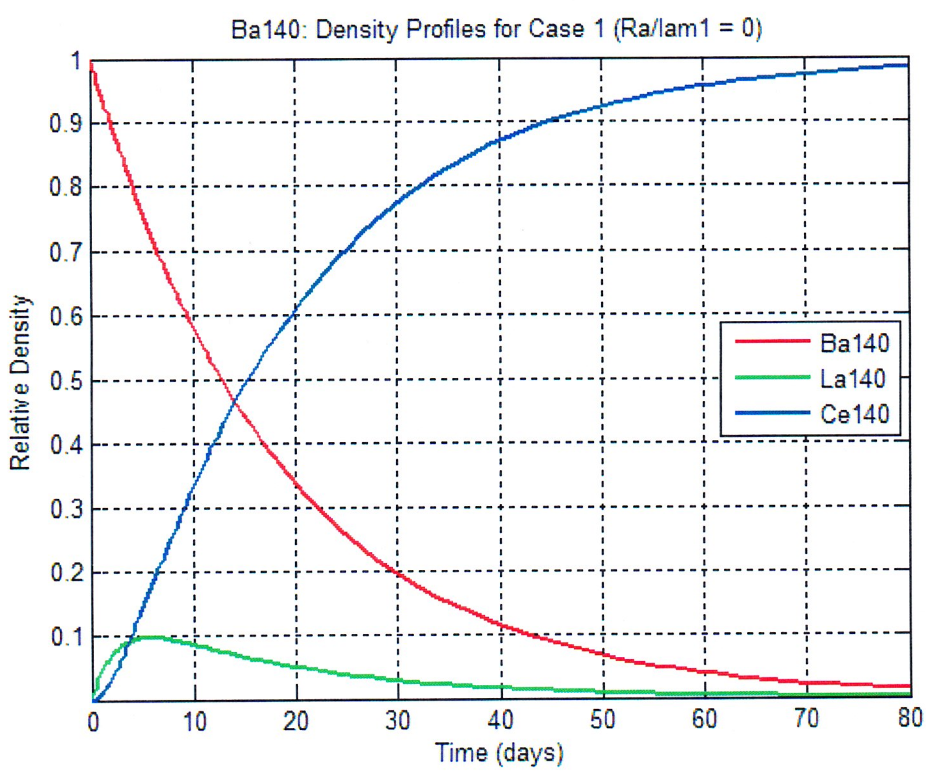
integrating each term gives

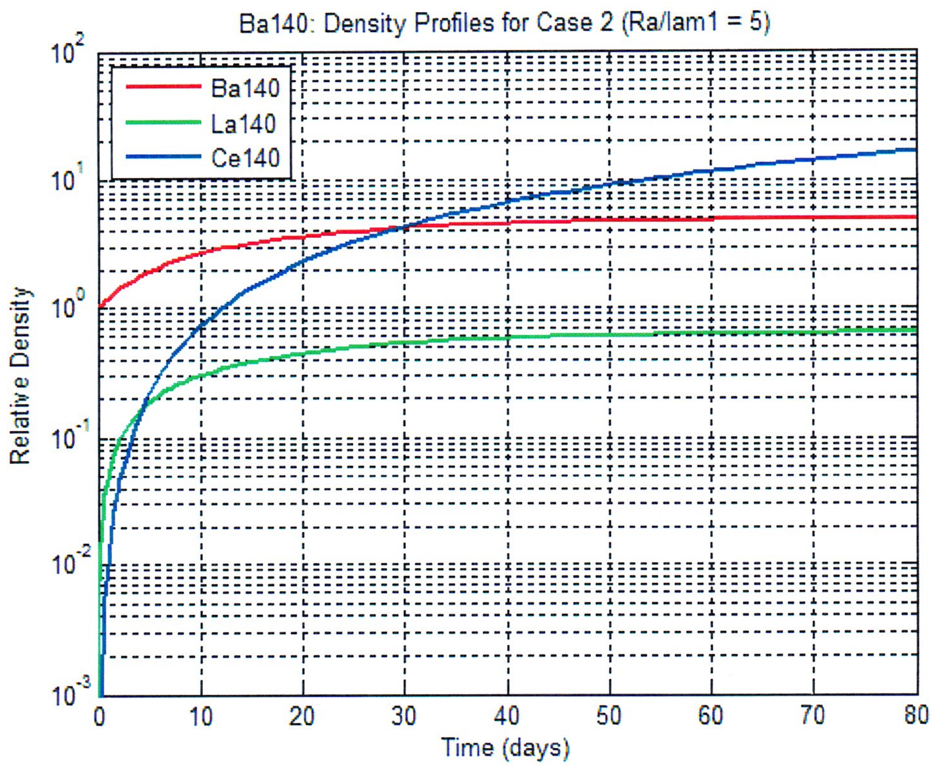
$$C(t) = C_0 + \lambda_B B_0 \left(\frac{-1}{\lambda_B}\right)\left[e^{-\lambda_B t} - 1\right]$$
$$+ \frac{\lambda_A \lambda_B A_0}{\lambda_B - \lambda_A}\left[\left(\frac{-1}{\lambda_A}\right)\left(e^{-\lambda_A t} - 1\right) - \left(\frac{-1}{\lambda_B}\right)\left(e^{-\lambda_B t} - 1\right)\right]$$
$$+ R_A t - R_A\left(\frac{1}{-\lambda_B}\right)\left(e^{-\lambda_B t} - 1\right)$$
$$- \frac{\lambda_B R_A}{\lambda_B - \lambda_A}\left[\left(-\frac{1}{\lambda_A}\right)\left(e^{-\lambda_A t} - 1\right) - \left(\frac{-1}{\lambda_B}\right)\left(e^{-\lambda_B t} - 1\right)\right]$$

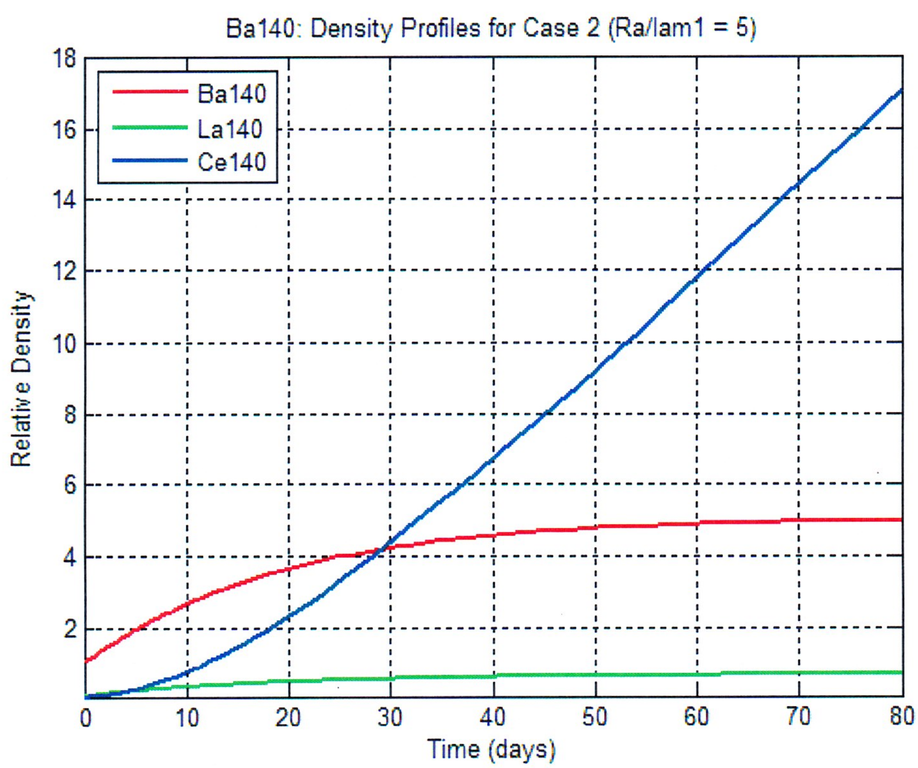
simplifying and combining terms gives

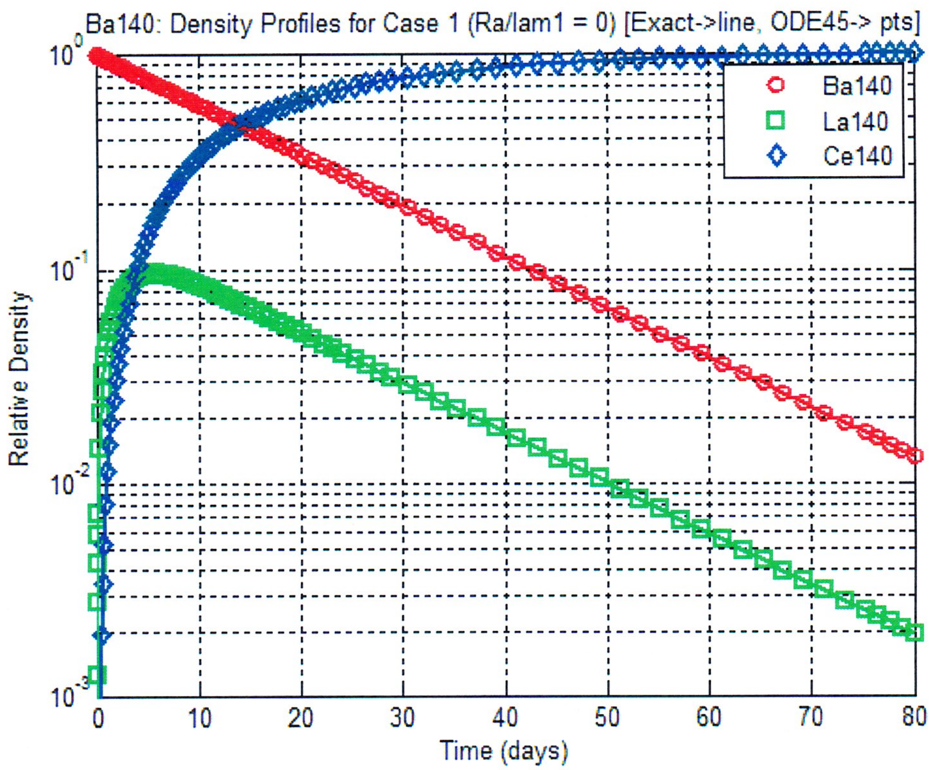
$$\boxed{\begin{aligned} C(t) &= C_0 + B_0\left(1 - e^{-\lambda_B t}\right) \\ &+ \frac{\lambda_B}{\lambda_B - \lambda_A}\left(A_0 - R_A/\lambda_A\right)\left[1 - e^{-\lambda_A t}\right] \\ &- \frac{\lambda_A}{\lambda_B - \lambda_A}\left(A_0 - R_A/\lambda_A\right)\left[1 - e^{-\lambda_B t}\right] \\ &+ R_A t - \left(R_A/\lambda_B\right)\left[1 - e^{-\lambda_B t}\right] \end{aligned}}$$ ans

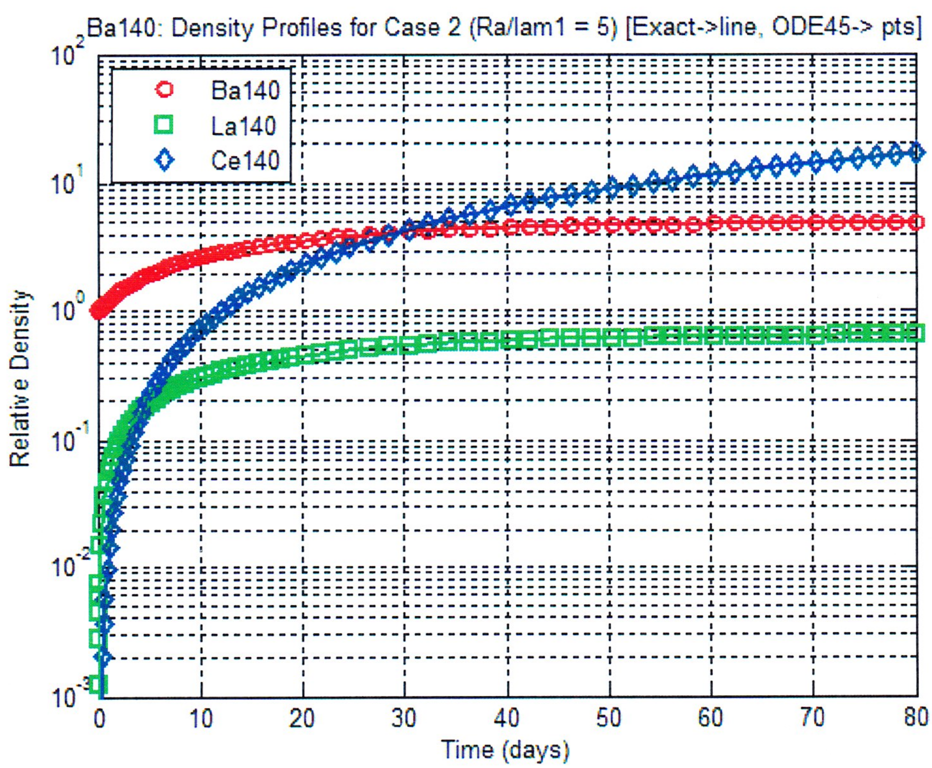

Ba140: Density Profiles for Case 1 (Ra/lam1 = 0)
10^0
10^{-1}
10^{-2}
10^{-3}
Relative Density
0
10
20
30
40
50
60
70
80
Time (days)
Ba140
La140
Ce140


Ba140: Density Profiles for Case 1 (Ra/lam1 = 0)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Relative Density
0
10
20
30
40
50
60
70
80
Time (days)
Ba140
La140
Ce140


Ba140: Density Profiles for Case 2 (Ra/lam1 = 5)
Ba140
La140
Ce140
Relative Density
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
50
60
70
80
Time (days)


Ba140: Density Profiles for Case 2 (Ra/lam1 = 5)
Ba140
La140
Ce140
Relative Density
18
16
14
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
Time (days)


Ba140: Density Profiles for Case 1 (Ra/lam1 = 0) [Exact->line, ODE45-> pts]
10^0
10^{-1}
10^{-2}
10^{-3}
Relative Density
0
10
20
30
40
50
60
70
80
Time (days)
Ba140
La140
Ce140


Ba140: Density Profiles for Case 2 (Ra/lam1 = 5) [Exact->line, ODE45-> pts]
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
Relative Density
0
10
20
30
40
50
60
70
80
Time (days)
Ba140
La140
Ce140

```
%
%   BA140.M    Simulation of Ba140 Chain (with & without constant production)
%
%   This program solves the first order differential equations associated
%   with the radioactive decay chain A -> B -> C, where C is stable.
%
%   Two separate cases are treated, with and without constant production of
%   isotope A, and these cases should be compared to expected behavior for
%   the physical situations described here.
%
%   Specific decay data for the Ba140 decay chain are used and, for the case with
%   constant production of Ba140, we assume that Ra = 5*initial activity.
%
%   The ODE45 routine is also used for numerical integration of the equations
%   that result (written in matrix form for ease of manipulation).  The ODE function
%   file is written in matrix form as an anonymous function.  The numerical results
%   are compared to the analytical solutions developed by straightforward integration
%   of the defining ODEs.
%
%   --> all nuclide densities are normalized to Ao = 1
%
%   File prepared by J. R. White, UMass-Lowell (Oct. 2014)
%

      clear all,    close all,    nfig = 0;
%
%   nuclide order:  Ba140 -> 1 = A    La140 -> 2 = B    Ce140 -> 3 = C
%
%   halflifes in hours, converted to decay constants (1/hr)
      hl1 = 12.8*24;    lam1 = log(2)/hl1;
      hl2 = 40.2;       lam2 = log(2)/hl2;
%
%   set time interval and initial conditions
      to = 0;                  % initial time for simulation (hours)
      tf = 24*80;              % final time (80 days -- should be near equilibrium)
      Ao = 1; Bo = 0; Co = 0;  % initial conditions (normalized to unity for first
nuclide)
      No = [Ao Bo Co]';        % initial condition vector
      tol = 1e-4;              % max error tolerance for ODE routine
      options = odeset('RelTol',tol);
%
%   CASE 1  (no production)
%
%   numerical solution
      Ra = 0;                          % production rate of first component (1/hr)
      FF = [Ra  0  0]';                % forcing function in state eqns
      AA = [-lam1     0       0 ;      % state matrix in state eqns
             lam1   -lam2     0 ;
               0     lam2     0];
      ftn = @(t,n)  AA*n+FF;
      [t1,N1n] = ode45(ftn,[to,tf],No,options);
      tp = t1/24;                      % scale time for plotting in days

%   analytical solution (evaluate on same time grid)
      N1a = zeros(size(N1n));
      c1 = lam1/(lam2-lam1);   c2 = Ra/lam1;   c3 = Ra/lam2;   c4 = lam2/(lam2-lam1);
```

```
    v1 = exp(-lam1*t1);   v2 = exp(-lam2*t1);
    N1a(:,1) = Ao*v1 + c2*(1-v1);
    N1a(:,2) = Bo*v2 + c1*Ao*(v1-v2) + c3*(1-v2) - c1*c2*(v1-v2);
    N1a(:,3) = Co + Bo*(1-v2) + c4*(Ao-c2)*(1-v1) ...
                  - c1*(Ao-c2)*(1-v2) + Ra*t1 - c3*(1-v2);
%
%  plot analytical results on a logarithmic scale
    nfig = nfig +1;   figure(nfig)
    semilogy(tp,N1a(:,1),'r-',tp,N1a(:,2),'g-',tp,N1a(:,3),'b-','LineWidth',2)
    title(['Ba140: Density Profiles for Case 1 (Ra/lam1 = ',num2str(c2),')'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140')
%
%  plot analytical results on a linear scale
    nfig = nfig +1;   figure(nfig)
    plot(tp,N1a(:,1),'r-',tp,N1a(:,2),'g-',tp,N1a(:,3),'b-','LineWidth',2)
    title(['Ba140: Density Profiles for Case 1 (Ra/lam1 = ',num2str(c2), ')'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140','Location','East')
%
%  plot both numerical and analytical results on a logarithmic scale
    nfig = nfig +1;   figure(nfig)
    semilogy(tp,N1n(:,1),'ro',tp,N1n(:,2),'gs',tp,N1n(:,3),'bd', ...
             tp,N1a(:,1),'r-',tp,N1a(:,2),'g-',tp,N1a(:,3),'b-','LineWidth',2)
    title(['Ba140: Density Profiles for Case 1 (Ra/lam1 = ', ...
           num2str(c2), ') [Exact->line, ODE45-> pts]'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140')
%
%  CASE 2  (constant production of first component)
%
%  numerical solution
    Ra = 5*lam1;                     % production rate of first component (1/hr)
    FF = [Ra  0  0]';                % forcing function in state eqns
    AA = [-lam1    0      0 ;        % state matrix in state eqns
           lam1   -lam2   0 ;
             0     lam2   0];
    ftn = @(t,n)  AA*n+FF;
    [t2,N2n] = ode45(ftn,[to,tf],No,options);
    tp = t2/24;   % scale time for plotting to days
%
%  analytical solution (evaluate on same time grid)
    N2a = zeros(size(N2n));
    c1 = lam1/(lam2-lam1);   c2 = Ra/lam1;  c3 = Ra/lam2;   c4 = lam2/(lam2-lam1);
    v1 = exp(-lam1*t2);   v2 = exp(-lam2*t2);
    N2a(:,1) = Ao*v1 + c2*(1-v1);
    N2a(:,2) = Bo*v2 + c1*Ao*(v1-v2) + c3*(1-v2) - c1*c2*(v1-v2);
    N2a(:,3) = Co + Bo*(1-v2) + c4*(Ao-c2)*(1-v1) ...
                  - c1*(Ao-c2)*(1-v2) + Ra*t2 - c3*(1-v2);

%  plot analytical results on a logarithmic scale
    nfig = nfig +1;   figure(nfig)
    semilogy(tp,N2a(:,1),'r-',tp,N2a(:,2),'g-',tp,N2a(:,3),'b-','LineWidth',2)
```

```
    title(['Ba140: Density Profiles for Case 2 (Ra/lam1 = ',num2str(c2), ')'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140','Location','NorthWest')

%  plot analytical results on a linear scale
    nfig = nfig +1;   figure(nfig)
    plot(tp,N2a(:,1),'r-',tp,N2a(:,2),'g-',tp,N2a(:,3),'b-','LineWidth',2)
    title(['Ba140: Density Profiles for Case 2 (Ra/lam1 = ',num2str(c2), ')'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140','Location','NorthWest')
%
%  plot results on a logarithmic scale
    nfig = nfig +1;   figure(nfig)
    semilogy(tp,N2n(:,1),'ro',tp,N2n(:,2),'gs',tp,N2n(:,3),'bd', ...
             tp,N2a(:,1),'r-',tp,N2a(:,2),'g-',tp,N2a(:,3),'b-','LineWidth',2)
    title(['Ba140: Density Profiles for Case 2 (Ra/lam1 = ', ...
           num2str(c2), ') [Exact->line, ODE45-> pts]'])
    xlabel('Time (days)'),ylabel('Relative Density'),grid
    vv = axis;   vv(3) = 0.001;   axis(vv)
    legend('Ba140','La140','Ce140','Location','NorthWest')
%
% end simulation
```