ENGY.3310 Fundamentals of Nuclear Science and Engineering

Spring 2016

HW #7: Radioactive Decay Calculations

Problem 1 Basic Definitions (5 points)

The activity of a radioactive nuclide is found to decrease by 30% in one week (168 hours). What are the values of its decay constant, half-life, and mean lifetime?

Problem 2 More Basic Definitions (5 points)

Tritium (H-3) decays by β^- decay with a half-life of 12.26 years.

a. To what nucleus does ^{3}H decay?
b. What is the mass of 1 mCi of tritium?

Problem 3 Po-210 RTG (10 points)

Polonium-210 decays to the ground state of Pb206 by emission of a 5.305 MeV α particle with a half-life of 138 days. What mass of Po210 is required to produce 1 MW of thermal power?

Problem 4 Pu-238 RTG (10 points)

The radioisotope generator SNAP-9 was fueled with 475 g of ^{238}PuC, which has a density of 12.5 g/cm^3. The Pu-238 has a half-life of 89 years and emits 5.6 MeV of energy per disintegration, all of which may be assumed to be absorbed in the generator. The thermal efficiency of the system is 5.4 %.

Compute the following:

a. the fuel efficiency in curies per watt thermal.

b. the specific power in watts thermal per gram of fuel

c. the power density in watts thermal per cm^3

d. the initial total electrical power of the generator, and

e. the expected lifetime of the generator if the minimum electrical power needed to perform properly is 12 watts electric.

Problem 5 $A \rightarrow B \rightarrow C$ with Constant Production of A (10 points)

In class we considered the classical radioactive decay problem of A decaying to B and B decaying to C, where C is a stable isotope, or

$$A \rightarrow B \rightarrow C$$

Let's call this situation **Case 1**, where the balance equations and resultant expressions for $n_A(t)$, $n_B(t)$, and $n_C(t)$, as well as all the detailed derivations, are given in the Lecture Notes. This case starts with some initial amount of nuclide A, but there are no production paths for this nuclide.

Now, to generalize the Case 1 situation slightly, let's assume that isotope A is produced in a nuclear reactor at a constant rate R_A (this might be from neutron capture in some material, for example), or,

$$\begin{array}{l} R_A \\ \downarrow \\ A \rightarrow B \rightarrow C \end{array}$$

We will call this situation **Case 2** and note that the only difference from the transmutation scheme in Case 1 is that nuclide A has a constant production rate, R_A (for simplicity, we assume that the neutron interaction rates with nuclides A, B, and C are small, so that the only loss mechanisms are due to radioactive decay).

Your job is to solve this problem analytically by deriving formal expressions for the atom densities $n_A(t)$, $n_B(t)$, and $n_C(t)$ where there is a non-zero production rate for nuclide A. Note here that this problem is only a slight generalization of Case 1 and the solution to Case 2 should reduce to that for Case 1 if $R_A = 0$. The same basic techniques that were used for Case 1 are required here -- however, the process gives slightly more complicated expressions because of the additional source term in the original balance equation. Write the final expression as concisely as possible and show that they do indeed reduce to the Case 1 equations when $R_A = 0$. Be careful with the integrations and the algebra…

Problem 6 The Ba-140 Decay Chain (10 points)

For the Ba-140 decay chain,

$$^{140}\text{Ba} \xrightarrow[12.8\text{ d}]{\beta^-} {}^{140}\text{La} \xrightarrow[40.2\text{ h}]{\beta^-} {}^{140}\text{Ce}$$

numerically evaluate and plot the analytical solutions that were derived in the previous problem (use the equations from the Lecture Notes and/or from the previous problem for the specific case given here). Carefully plot and label the solutions and rationalize that the time-dependent profiles behave as expected. Generate results for both cases described below. Also comment on the concentration profiles $n_A(t)$, $n_B(t)$, and $n_C(t)$ for Case 1 versus Case 2. Do these make sense physically? Are the asymptotic values realistic? You should provide your answers to these questions and other general comments about your solution strategy and graphical results as part of the overall documentation for this problem.

Case 1

Use a normalized initial concentration of unity for Ba-140 (i.e. $n_{Ao} = 1$) and assume zero initial conditions for the remaining isotopes (i.e. $n_{Bo} = n_{Co} = 0$). This case has no production of Ba-140.

Case 2

Solve the same problem as Case 1 but now assume that Ba-140 is produced at a constant rate. For specificity, let this production rate be five times the initial activity of Ba-140. With a normalized concentration, this can be written as

$$R_A = 5\alpha_{A_o} = 5\lambda_A n_{A_o} = 5\lambda_A$$

where n_{Ao} is the initial concentration of Ba-140. With everything normalized to n_{Ao}, this simply gives $n_{Ao} = 1$ and $R_A = 5\lambda_A$.

Note: Do the numerical simulations for both cases out to about 70-80 days (since this will give near equilibrium conditions for both cases) . You may also want to plot your results on both a linear and logarithmic scale for the concentration axis -- might shed some light on what is happening here…

Problem 7 The Ba-140 Decay Chain via a Numerical ODE Solver (optional 10 extra pts)

Using an available ODE solver within Matlab, Mathcad, or some other available package, numerically solve the problems described in the previous problem. Compare your numerical solutions to the analytical results. Briefly discuss the solution methodology and the relative ease of solution for the numerical and analytical techniques.